ELEMENTS
OF
PHILOSOPHY,
THE
FIRST SECTION,
CONCERNING
BODY.

Written in Latine by

THOMAS HOBBES

OF

MALMESBURY.

And now translated into

ENGLISH.

To which are added Six Lessons to the Professors of Mathematicks of the Institution of S^r. *HENRY SAVILE*, in the University of *OXFORD*.

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THE TRANSLATOR To the Reader.

IF when I had finished my Translation of this first Section of the Elements of Philosophy, I had presently committed the same to the Press, it might have come to your hands sooner then now it doth. But as I undertook it with much diffidence of my own ability to perform it well; so I thought fit before I published it, to pray Mr. *Hobbes* to view, correct and order it according to his own minde and pleasure. Wherefore, though you find some places enlarged, others altered, and two Chapters (the 18th and 20th) almost wholly changed, you may neverthelesse remain assured, that as now I present it to you, it

it doth not at all vary from the Authours own ſenſe and meaning. As for the ſix Leſſons to the *Savilian* Profeſſors at *Oxford*, they are not of my tranſlation, but were written, as here you have them in Engliſh, by Mr. *Hobbes* himſelf; and are joyned to this Book, becauſe they are chiefly in defence of the ſame.

The Authors Epistle Dedicatory,

TO THE

RIGHT HONORABLE,

My most Honored LORD,

WILLIAM

Earl of Devonshire.

THis *first Section of the* Elements *of* Philosophy, *the Monument of my Service,& your Lordships bounty, though (after the third Section published) long deferred, yet at last finished, I now present (my most Excellent Lord) and dedicate to your Lordship. A little Book, but full; and great enough, if men count* well *for* great; *and to an attentive Reader versed in the Demonstrations of* Mathematicians *that is, to your Lordship, clear & easy to understand; and almost new throughout, without any offensive Novelty. I know that that part of Philosophy wherein are considered Lines and Figures, has been delivered to us notably improved by the Anci-*

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ents; and withall a most perfect pattern of the Logique by which they were enabled to finde out and demonstrate such excellent Theoremes as they have done. I know also that the Hypothesis of the Earths Diurnal Motion was the invention of the Ancients; but that both it, and Astronomy (that is, Cœlestial Physiques) springing up together with it, were by succeeding Philosophers strangled with the snares of Words. And therefore the beginning of Astronomy (except Observations) I think is not to be derived from farther time then from Nicolaus Copernicus; who in the Age next preceding the present, revived the opinion of Pythagoras, Aristarchus & Philolaus. After him, the Doctrine of the Motion of the Earth being now received, & a difficult Question thereupon arising concerning the Descent of Heavy Bodies, Galilæus in our time striving with that difficulty, was the first that opened to us the ga[illegible] Natural Philosophy Universal, which is the kn[illegible]ledge of the Nature of Motion. So that neither can the Age of Natural Philosophy be reckoned higher then to him. Lastly, the Science of Mans Body, the most profitable part of Natural

ral Science, was first discovered with admirable sagacity by our Countryman Doctor Harvey*, principal Physician to King* James *and King* Charles *, in his Books of the* Motion of the Blood *and of the* Generation of Living Creatures*; who is the onely man I know, that conquering envy, hath established a new Doctrine in his life time. Before these, there was nothing certain in Natural Philosophy but every mans Experiments to himself, and the Natural Histories, if they may be called certain, that are no certainer then Civil Histories. But since these, Astronomy & Natural Philosophy in general have for so little time been extraordinarily advanced by* Joannes Keplerus, Petrus Gassendus,*&* Marinus Mersennus*; & the Science of Humane Bodies in special by the wit & industry of Physicians (the onely true Natural Philosophers) especially of our most Learned Men of the Colledge of Physicians in* London. *Natural Philosophy is therefore but young; but Civil Philosophy yet much younger, as being no older (I say it provoked, & that my Detractors may know how little they have wrought upon me) then my own Book* de Cive. *But what? Were there no Philosophers Natural nor Civil among the*

ancient Greeks? *There were men so called; Witness* Lucian, *by whom they are derided; Witness divers Cities, from which they have been often by publique Edicts banished. But it follows not that there was Philosophy. There walked in old* Greece *a certain Phantasme, for superficial gravity (though full within of fraud & filth) a little like Philosophy; which unwary men thinking to be it, adhered to the Professors of it, some to one, some to another (though they disagreed among themselves) and with great Salary put their children to them to be taught, in stead of Wisdome, nothing but to dispute; & neglecting the Laws, to determine every Question according to their own fancies. The first Doctors of the Church next the Apostles, born in those times, whilest they endeavored to defend the Christian Faith against the* Gentiles *by Natural Reason, began also to make use of Philosophy, & with the Decrees of Holy Scripture to mingle the Sentences of Heathen Philosophers; & first some harmless ones of* Plato; *but afterwards also many foolish & false ones out of the Physicks & Metaphysicks of* Aristotle; *& bringing in the Enemies betrayed unto them the Cittadel of Christianity. From that time, in stead of the Worship of God, there entred a thing called* School-Di-

vinity, *walking on one foot firmly, which is the Holy Scripture, but halted on the other rotten foot, which the Apostle* Paul *called* Vain, *& might have called* Pernicious Philosophy; *for it hath raised an infinite number of Controversies in the Christian World concerning Religion, & from those Controversies Wars. It is like that* Empusa *in the* Athenian *Comick Poet, which was taken in* Athens *for a Ghost that changed shapes, having one brazen leg, but the other was the leg of an Ass, & was sent (as was believed) by* Hecate, *as a signe of some approaching evil fortune. Against this* Empusa *I think there cannot be invented a better Exorcisme, then to distinguish between the Rules of Religion, that is, the Rules of Honoring God, which we have from the Laws, and the* Rules *of Philosophy, that is, the Opinions of private men; & to yeild what is due to* Religion *to the Holy Scripture, and what is due to Philosophy to* Natural *Reason. And this I shall do, if I but handle the Elements of Philosophy truly & clearly, as I endevour to do. Therefore having in the 3d Section wch I have published & dedicated to your* Lordship *long since, reduced all Power Ecclesiastical and Civil by strong Arguments of* Reason, *without repugnance to Gods Word, to one and the same Soveraign Authority; I intend now,*

by putting into a clear Method the true Foundations of Natural Philosophy, to fright and drive away this Metaphysical Empusa; *not by skirmish, but by letting in the light upon her. For I am confident (if any confidence of a Writing can proceed from the Writers fear, circumspection & diffidence) that in the three former parts of this Book, all that I have said is sufficiently demonstrated from Definitions; & all in the fourth part, from Suppositions not absurd. But if there appear to your* Lordship *any thing less fully demonstrated then to satisfie every Reader, the cause was this, that I professed to write not all to all, but some things to Geometricians onely. But that your* Lordship *will be satisfied I cannot doubt.*

There remains the second Section, which is concerning Man. *That part thereof where I handle the* Optiques, *contayning six Chapters, together with the Tables of the Figures belonging to them, I have already written & engraven lying by me above these six years. The rest shall, as soon as I can, be added to it; though by the contumelies & petty injuries of some unskilful men, I know already by experience how much greater thanks will be due, then payed me, for telling Men the truth of what*

Men

Men are. But the burthen I have taken on me I mean to carry through; not striving to appease, but rather to revenge my self of Envy, by encreasing it. For it contents me that I have your Lordships *favour; which, (being all you require) I acknowledge; and for which, with my prayers to Almighty God for your* Lordships *safety, I shall (to my power) be always thankefull.*

London, *April* 23, 1655.

YOUR LORDSHIPS

most humble Servant

Thomas Hobbes.

The Authors Epistle

To the Reader.

THink not (courteous Reader) that the Philosophy the Elements whereof I am going to set in order, is that which makes Philosophers Stones, nor that which is found in the Metaphsique Codes. But that it is the Natural Reason of Man busily flying up and down among the Creatures, & bringing back a true report of their Order, Causes & Effects. Philosophy therefore, the Childe of the World and your own Mind, is within your self; perhaps not fashioned yet, but like the World its Father, as it was in the beginning, a thing confused. Do therefore as the Statuaries do, who by hewing off that which is superfluous, do not make but find the Image. Or imitate the Creation. If you will be a Philosopher in good earnest, let your Reason move upon the Deep of your own Cogitations and Experience. Those things that lie in Confusion must be set asunder, distinguished, and every one stampt with its own name set in order; that is to say, your Method must resemble that of the Creation. The order of the Creation was, *Light*, *Distinction of Day and Night*, the *Firmament*, the *Luminaries*, *Sensible Creatures*, *Man*; and after the Creation, the *Commandement*. Therefore the order of Contemplation will be, *Reason*, *Definition*, *Space*, *the Starres*, *Sensible Quality*, *Man*; and after Man is grown up, *Subjection to Command*. In the first part of this Section which is entitled Logique, I set up the light of Reason. In the Second (which hath for title the Grounds of Philosophy) I distinguish the most common Notions by accurate definition, for the avoiding of confusion and obscurity. The third part concerns the Expansion of Space, that is, Geometry. The fourth contains the Motion of the Starres, together with the doctrine of Sensible Qualities.

To the Reader.

In the ſecond Section (if it pleaſe God) ſhall be handled Man. In the third Section the doctrine of Subjection is handled already. This is the Method I followed; and if it like you you may uſe the ſame; for I do but propound, not commend to you any thing of mine. But whatſoever ſhall be the Method you will like, I would very fain commend Philoſophy to you, that is to ſay, the ſtudy of Wiſdome, for want of which we have all ſuffered much dammage lately. For even they that ſtudy Wealth, do it out of love to Wiſdome; for their Treaſures ſerve them but for a Looking-glaſs, wherin to behold and contemplate their owne Wiſdome. Nor do they that love to be employed in publike buſineſs, aime at any thing but place wherein to ſhew their Wiſdome. Neither do Voluptuous men neglect Philoſophy, but onely becauſe they know not how great a pleaſure it is to the Mind of Man to be raviſhed in the vigorous and perpetual embraces of the moſt beauteous World. Laſtly, though for nothing elſe, yet (becauſe the Mind of Man is no leſs impatient of Empty Time, then Nature is of Empty Place) to the end you be not forced for want of what to do, to be troubleſome to men that have buſineſs, or take hurt by falling into idle Company, but have ſomewhat of your own wherewith to fill up your time, I recommend unto you the Study of Philoſophy. *Farewell.*

T. H.

The Titles of the CHAPTERS.

The first Part, or Logique.

COMPUTATION OR LOGIQUE.

Chap. I. Of Philoſophy.

1. The Introduction. 2 The Definition of Philoſophy explained. 3 Ratiocination of the Mind. 4 Properties what they are. 5 How Properties are known by Generation, & contrarily. 6 The Scope of Philoſophy. 7 The Utility of it. 8 The Subject. 9 The Parts of it. 10 The Epilogue.

PHILOSOPHY ſeems to me to be amongſt men now, in the ſame manner as Corn and Wine are ſaid to have been in the world in ancient time. For from the beginning there were Vines and Ears of Corn growing here and there in the fields; but no care was taken for the planting and ſowing of them. Men lived therefore upon Akorns; or if any were ſo bold as to venture upon the eating of thoſe unknown and doubtfull fruits, they did it with danger of their health. In like manner, every man brought Philoſophy, that is, Naturall Reaſon, into the world with him; for all men can reaſon to ſome degree, and concerning ſome things: but where there is need of a long ſeries of Reaſons, there moſt men wander out of the way, and fall into Error for want of Method, as it were for want of ſowing and planting, that is, of improving their Reaſon. And from hence it comes to paſſe, that they who content themſelves with daily experience, which may be likened to feeding upon Akorns, and either reject, or not much regard Philoſo-

phy, are commonly esteemed, and are indeed, men of sounder judgement, then those, who from opinions, though not vulgar, yet full of uncertainty, and carelesly received, do nothing but dispute and wrangle, like men that are not well in their wits. I confesse indeed, that that part of Philosophy by which Magnitudes and Figures are computed, is highly improved. But because I have not observed the like advancement in the other parts of it, my purpose is, as far forth as I am able, to lay open the few and first Elements of Philosophy in generall, as so many Seeds, from which pure and true Philosophy may hereafter spring up by little and little.

I am not ignorant how hard a thing it is to weed out of mens mindes such inveterate opinions as have taken root there, and been cōfirmed in them by the authority of most eloquent Writers; especially, seeing true (that is accurate) Philosophy, professedly rejects not only the paint and false colours of Language, but even the very ornaments and graces of the same; and the first Grounds of all Science, are not only not beautifull, but poore, aride, and in appearance deformed. Neverthelesse, there being certainly some men, though but few, who are delighted with Truth and strength of Reason in all things, I thought I might do well to take this pains for the sake even of those few. I proceed therefore to the matter, and take my beginning from the very Definition of Philosophy, which is this.

2 PHILOSOPHY *is such knowledge of Effects or Appearances, as we acquire by true Ratiocination from the knowledge we have first of their Causes or Generation: And again, of such Causes or Generations as may be from knowing first their Effects.*

For the better understanding of which Definition, we must consider; first, that although Sense and Memory of things, which are common to Man and all living Creatures, be Knowledge, yet because they are given us immediately by Nature, and not gotten by Ratiocination, they are not Philosophy.

Secondly, Seeing Experience is nothing but Memory; and Prudence, or Prospect into the future time, nothing but Expectation of such things as we have already had experience of, Prudence also is not to be esteemed Philosophy.

By RATIOCINATION, I mean *Computation.* Now to compute

pute

pute, is either to collect the sum of many things that are added together, or to know what remains when one thing is taken out of another. *Ratiocination* therefore is the same with *Addition* and *Substraction*; and if any man adde *Multiplication* and *Division*, I will not be against it, seeing Multiplication is nothing but Addition of equals one to another, and Division nothing but a Substraction of equals one from another, as often as is possible. So that all Ratiocination is comprehended in these two operations of the minde, Addition and Substraction.

3 But how by the *Ratiocination* of our Minde, we Adde and Substract in our silent thoughts, without the use of words, it will be necessary for me to make intelligible by an example or two. If therefore a man see something a far off and obscurely, although no appellation had yet been given to any thing, he will notwithstanding have the same Idea of that thing, for which now by imposing a name on it, we call it *Body*. Again, when by comming neerer, he sees the same thing thus and thus, now in one place and now in another, he will have a new Idea thereof, namely that, for which we now call such a thing *Animated*. Thirdly, when standing neerer he perceives the figure, hears the voice, and sees other things, which are signes of a Rationall minde, he has a third Idea, though it have yet no appellation, namely, that for which we now call any thing *Rationall*. Lastly, when by looking fully and distinctly upon it he conceaves all that he has seen as one thing, the Idea he has now, is compounded of his former Ideas, which are put together in the Minde, in the same order, in which these three single names *Body*, *Animated*, *Rationall*, are in speech compounded into this one name *Body-Animated-Rationall*, or *Man*. In like manner, of the severall conceptions of *four sides*, *equality of sides*, *and right angles*, is compounded the conception of a *Square*. For the mind may conceive a figure of foure sides without any conception of their equality; & of that equality without conceiving a right angle; and may joyne together all these single conceptions into one conception or one Idea of a Square. And thus we see how the Conceptions of the mind are compounded. Again, whosoever sees a man standing neer him, conceives the whole Idea of that man; and if as he goes away he follow him with his eyes onely, he will lose the Idea of those things which

were ſignes of his being Rationall, whileſt neverthelesse the Idea of a Body-Animated remaines ſtill before his eies; ſo that the Idea of Rationall is ſubſtracted from the whole Idea of Man, that is to ſay of Body-Animated-Rationall, and there remaines that of Body-Animated; & a while after at a greater diſtance the Idea of Animated will be loſt, & that of Body only will remain; ſo that at laſt, when nothing at all can be ſeen, the whole Idea will vaniſh out of ſight. By which examples, I thinke it is manifeſt enough, what is the internall Ratiocination of the Mind, without words.

We muſt not therefore thinke that Computation, that is, Ratiocination, has place onely in numbers; as if man were diſtinguiſhed from other living Creatures (which is ſaid to have been the opinion of *Pythagoras*) by nothing but the faculty of numbring; for *Magnitude*, *Body*, *Motion*, *Time*, *Degrees of Quality*, *Action*, *Conception*, *Proportion*, *Speech and Names* (in which all the kinds of Philoſophy conſiſt) are capable of Addition and Subſtraction. Now ſuch things as we adde or ſubſtract, that is, which we put into an account, we are ſaid to *conſider*, in Greek λογίζεσθαι; in which language alſo συλλογίσθαι ſignifies to *Compute*, *Reaſon* or *Reckon*.

4 But *Effects* and the *Appearances* of things to ſenſe, are faculties or Powers of Bodies, which make us diſtinguiſh them from one another; that is to ſay, conceive one Body to be equall or unequall, like or unlike to another Body; as in the example above, when by coming neer enough to any Body, we perceive the Motion and Going of the ſame, we diſtinguiſh it thereby from a Tree, a Column and other fixed Bodies; and ſo that motion, or going is the *Property* thereof, as being proper to living creatures, and a faculty by which they make us diſtinguiſh them from other Bodies.

5 How the knowledge of any Effect may be gotten from the knowledge of the Generation thereof, may eaſily be underſtood by the example of a Circle: For if there be ſet before us a plain figure having as neer as may be the figure of a Circle, we cannot poſſibly perceive by ſenſe whether it be a true Circle or no; then which neverthelesse nothing is more eaſie to be known, to him that knowes firſt the Generation of the propounded figure. For let it be known that the figure was made by the circumduction of

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a Body whereof one end remained unmoved, and we may reason thus; a Body carried about, retaining alwayes the same length, applies it selfe first to one *Radius*, then to another, to a third, a fourth, and successively to all; and therefore the same length, from the same point, toucheth the circumference in every part thereof; which is as much to say as all the *Radii* are equal. We know therefore that from such generation proceeds a figure, from whose one middle point all the extreame points are reached unto by equal *Radii*. And in like manner, by knowing first what figure is set before us, we may come by Ratiocination to some Generation of the same, though perhaps not that by which it was made, yet that by wch it might have been made; for he that knows that a Circle has the property above declared, will easily know whether a Body carried about, as is said, will generate a Circle or no.

6 The *End* or *Scope* of Philosophy, is, that we may make use to our benefit of effects formerly seen; or that by applicatiō of Bodies to one another, we may produce the like effects of those we conceive in our minde, as far forth as matter, strength & industry will permit, for the commodity of humane life. For he inward glory and triumph of mind that a man may have, for the mastering of some difficult and doutfull matter, or for the discovery of some hidden truth, is not worth so much paines as the study of Philosophy requires; nor need any man care much to teach another what he knowes himselfe, if he think that will be the onely benefit of his labour. The end of Knowledge is Power; and the use of Theoremes (which among Geometricians serve for the finding out of Properties) is for the construction of Problemes; and lastly, the scope of all speculation is the performing of some action, or thing to be done.

7 But what the *Utility* of Philosophy is, especially of Natural Philosophy and Geometry, will be best understood by reckoning up the chief commodities of which mankind is capable; and by comparing the manner of life of such as enjoy them, with that of others which want the same. Now the greatest commodities of mankind are the Arts, namely of measuring Matter and Motion; of moving ponderous Bodies; of Architecture; of Navigation; of making instruments for all uses; of calculating the Cœlestiall Motions, the Aspects of the Stars, and the parts of Time; of Geography,

graphy &c. By which Sciences, how great benefits men receive, is more easily understood then expressed. These benefits are enjoyed by almost all the people of Europe, by most of those of Asia, and by some of Africa; but the Americans, and they that live neer the Poles do totally want them. But why? Have they sharper wits then these? Have not all men one kinde of soule, and the same faculties of mind? What then makes this difference, except Philosophy? Philosophy therefore is the cause of all these benefits. But the Utility of Morall and Civil Philosophy is to be estimated not so much by the commodities we have by knowing these Sciences, as by the calamities we receive from not knowing them. Now all such calamities as may be avoided by humane industry arise from warre, but chiefly from Civil warre; for from this proceed Slaughter, Solitude, and the want of all things. But the cause of warre is not that men are willing to have it; for the Will has nothing for Object but Good, at least that which seemeth good. Nor is it from this, that men know not that the effects of war are evil; for who is there that thinks not poverty and losse of life to be great evils? The cause therefore of Civill warre, is that men know not the causes neither of Warre nor Peace, there being but few in the world that have learned those duties which unite and keep men in peace, that is to say, that have learned the rules of civill life sufficiently. Now the knowledge of these rules is Morall Philosophy. But why have they not learned them, unlesse for this reason that none hitherto have taught them in a clear and exact method? For what shall we say? Could the ancient Masters of Greece, Egypt, Rome, and others perswade the unskillfull multitude to their innumerable opinions concerning the nature of their Gods, which they themselves knew not whether they were true or false, and which were indeed manifestly false & absurd; & could they not perswade the same multitude to civill duty, if they themselves had understood it? Or shall those few writings of Geometricians which are extant, be thought sufficient for the taking away of all controversy in the matters they treat of, and shall those innumerable and huge Volumes of *Ethicks* be thought unsufficient, if what they teach had been certain and well demonstrated? What then can be imagined to be the cause that the writings of

those

those men have increased science, and the writings of these have increased nothing but words, saving that the former were written by men that knew, and the later by such as knew not the doctrine they taught onely for ostentation of their wit and eloquence? Neverthelesse, I deny not but the reading of some such books is very delightfull; for they are most eloquently written, and containe many cleer, wholsome and choice sentences; which yet are not universally true, though by them universally pronounced. From whence it comes to passe, that the circumstances of times, places and persons being changed, they are no lesse frequently made use of to confirme wicked men in their purposes, then to make them understand the precepts of Civill duties. Now that which is chiefly wanting in them, is a true and certaine rule of our actions, by which we might know whether that we undertake be just or unjust. For it is to no purpose to be bidden in every thing to do Right, before there be a certain Rule and measure of Right established; which no man hitherto hath established. Seeing therefore from the not knowing of Civill duties, that is, from the want of Morall science proceed Civill warres, and the greatest calamities of mankind, we may very well attribute to such science the production of the contrary commodities. And thus much is sufficient, to say nothing of the prayses and other contentment proceeding from Philosophy, to let you see the Utility of the same in every kinde thereof.

8 The *Subject* of Philosophy, or the matter it treats of, is every Body of which we can conceive any generation, and which we may by any consideration thereof compare with other Bodies; or which is capable of composition and resolution; that is to say, every Body, of whose Generation or Properties we can have any knowledge.. And this may be deduced from the Definition of Philosophy, whose profession it is to search out the Properties of Bodies from their Generation, or their Generation from their Properties; and therefore where there is no Generation nor Property, there is no Philosophy. Therefore it excludes *Theology*, I meane the doctrine of God, Eternal, Ingenerable, Incomprehensible, and in whom there is nothing neither to divide nor compound, nor any Generation to be conceived.

It excludes the doctrine of *Angels*; and all such things as are thought

thought to be neither Bodies, nor properties of Bodies; there being in them no place neither for composition, nor division, nor any capacity of more and lesse; that is to say, no place for Ratiocination.

It excludes *History*, as well *Naturall* as *Politicall*, though most usefull (nay necessary) to Philosophy; because such Knowledge is but Experience, or Authority, and not Ratiocination.

It excludes all such Knowledge as is acquired by Divine Inspiration, or Revelation, as not derived to us by Reason, but by Divine grace in an instant, and as it were by some sense supernaturall.

It excludes, not onely all Doctrines which are false, but such also as are not well grounded; for whatsoever we know by right Ratiocination, can neither be false nor doubtfull; and therefore *Astrology*, as it is now held forth, and all such Divinations rather then sciences, are excluded.

Lastly, the doctrine of *Gods Worship* is excluded from Philosophy, as being not to be known by naturall reason, but by the authority of the Church; and as being the object of Faith, and not of Knowledge.

9 The principall parts of Philosophy are two. For two chief kinds of Bodies, and very different from one another, offer themselves to such as search after their Generation & Properties; One whereof being the worke of Nature, is called a *Naturall Body*; the other is called a *Commonwealth*, and is made by the wills and agreement of men. And from these spring the two parts of Philosophy called *Naturall* and *Civill*. But seeing that for the knowledge of the Properties of a Common-wealth, it is necessary first to know the Dispositions, Affections and Manners of men, Civill Philosophy is againe commonly divided into two parts; whereof one which treats of mens Dispositions and Manners is called *Ethicks*, and the other which takes cognisance of their Civil Duties is caled *Politicks* or simply *Civill Philosophy*. In the first place therefore (after I have set downe such Premisses as appertaine to the nature of Philosophy in general) I will discourse of *Bodies Naturall*; in the second, of the *Dispositions and Manners of men*; and in the third, of the *Civill Duties of Subjects*.

10 To conclude, seeing there may be many who will not like this

this my Definition of Philosophy, and will say that from the liberty which a man may take of so defining as seemes best to himselfe, he may conclude any thing from any thing (though I thinke it no hard matter to demonstrate, that this Definition of mine agrees with the sense of all men;) yet lest in this point there should be any cause of dispute betwixt me and them, I here undertake no more then to deliver the Elements of that Science, by which the Effects of any thing may be found out from the known Generation of the same, or contrarily the Generation from the Effects; to the end that they who search after other Philosophy, may be admonished to seeke it from other Principles.

Chap. II.

Of Names.

1 The necessity of sensible Moniments or Marks for the help of Memory, A Marke defined. 2 The necessity of Marks for the signification of the conceptions of the Mind. 3 Names supply both those necessities. 4 The Definition of a Name. 5 Names are Signes not of Things, but of our Cogitations. 6 What it is we give Names to. 7 Names Positive and Negative. 8 Contradictory Names. 9 A Common Name. 10 Names of the first and second Intention. 11 Universall, Particular, Individuall, and Indefinite Names. 12 Names Univocall and Equivocal. 13 Absolute and Relative Names. 14 Simple and Compounded Names. 15 A Prædicament described. 16 Some things to be noted concerning Prædicaments.

1 HOw unconstant and fading mens thoughts are, and how much the recovery of them depends upon chance, there is none but knows by infallible experience in himself. For no man is able to remember Quantities without sensible and present Measures, nor Colours without sensible and present Patterns, nor Number without the Names of Numbers disposed in order and learned by heart. So that whatsoever a man has put together in his mind by ratiocination without such helps, will presently slip from him, and not be revocable but by beginning his ratiocination anew. From which it followes, that for the acquiring of Philosophy some sensible Moniments are necessary, by which our past thoughts may be not onely reduced, but also registred every one in its own order. These Moniments I call Marks, namely, sensible things taken at pleasure, that by the sense of them such thoughts may be recalled to our mind, as are like those thoughts for which we took them.]

2 Again,

2 Again, though some one man, of how excellent a wit soever, should spend all his time, partly in reasoning and partly in inventing Marks for the help of his memory, and advancing himself in learning; who sees not, that the benefit he reapes to himselfe will not be much, and to others none at all? For unlesse he communicate his notes with others, his science will perish with him. But if the same notes be made common to many, and so one mans inventions be taught to others, sciences will thereby be encreased to the generall good of mankind. It is therefore necessary for the acquiring of Philosophy that there be certain Signes, by which what one man finds out may be manifested and made known to others. Now those things we call SIGNES, are the *Antecedents of their Consequents, and the Consequents of their Antecedents, as often as we observe them to go before or follow after in the same manner.* For example, a thick Cloud is a Signe of Rain to follow; and Rain a Signe, that a Cloud has gone before, for this reason onely, that we seldom see Clouds without the Consequence of Rain, nor Rain at any time but when a Cloud has gone before. And of Signs some are *Naturall*, whereof I have already given an example; others are *Arbitrary*, namely, those we make choice of at our own pleasure; as a bush hung up, signifies that Wine is to be sold there; a stone set in the ground, signifies the bound of a field; and words so and so connected, signifie the Cogitations and Motions of our Minde. The difference therefore betwixt Markes and Signes is this, that we make those for our own use, but these for the use of others.

3 Words so connected, as that they become signes of our Thoughts, are called SPEECH, of which every part is a *Name*. But seeing (as is said) both Markes and Signes are necessary for the acquiring of Philosophy, (Marks by which we may remember our own thoughts, and Signes by which we may make our thoughts known to others,) Names do both these offices; but they serve for Marks before they be used as Signes. For though a man were alone in the world, they would be usefull to him in helping him to remember; but to teach others, (unlesse there were some others to be taught,) of no use at all. Again, Names, though standing singly by themselves, are Marks, because they serve to recall our own thoughts to mind; but they cannot be Signes, otherwise then by being disposed and ordered in Speech,

as parts of the ſame. For example, a man may begin with a word, whereby the hearer may frame an Idea of ſomething in his mind, which neverthelesse he cannot conceive to be the Idea which was in the mind of him that ſpake, but that he would ſay ſomething which began with that word, though perhaps not as by it ſelfe, but as part of another word. So that the nature of a name conſiſts principally in this, that it is a Mark taken for Memories ſake; but it ſerves alſo by accident, to ſignifie and make known to others what we remember our ſelves; and therefore I will define it thus:

4 *A* NAME *is a Word taken at pleaſure to ſerve for a Mark, which may raiſe in our Mind a thought like to ſome thought we had before, and which being pronounced to others, may be to them a Sign of what thought the ſpeaker had or had not before in his mind.* And it is for brevities ſake that I ſuppoſe the Originall of Names to be Arbitrary, judging it a thing that maybe aſſumed as unqueſtionable. For cōſidering that new Names are daily made, and old ones laid aſide; that diverſe Nations uſe different Names, and how impoſſible it is either to obſerve ſimilitude, or make any compariſōn betwixt a Name and a Thing, how can any man imagine that the Names of Things were impoſed from their natures? For though ſome Names of living creatures and other things, which our firſt Parents uſed, were taught by God himſelfe; yet they were by him arbitrarily impoſed, and afterwards both at the Tower of Babel, and ſince in proceſſe of time, growing every where out of uſe, are quite forgotten, and in their roomes have ſucceeded others, invented and received by men at pleaſure. Moreover, whatſoever the common uſe of words be, yet Philoſophers, who were to teach their knowledge to others, had alwayes the liberty, and ſometimes they both had and will have a neceſſity, of taking to themſelves ſuch Names as they pleaſe for the ſignifying of their meaning, if they would have it underſtood. Nor had Mathematicians need to aske leave of any but themſelves to name the figures they invented *Parabolas, Hyperboles, Ciſſoeides, Quadratrices, &c.* or to call one Magnitude A, another B.

5 But ſeeing Names ordered in ſpeech (as is defined) are ſignes of our Conceptions, it is manifeſt they are not ſignes of the Things themſelves; for that the ſound of this word *Stone* ſhould be the

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signe of a Stone, cannot be understood in any sense but this, that he that heares it, collects that he that pronouunces it thinkes of a Stone. And therefore that disputation, whether Names signifie the Matter or Form, or something compounded of both, and other like subtleties of the *Metaphysicks*, is kept up by erring men, and such as understand not the words they dispute about.

6 Nor indeed is it at all necessary that every Name should be the Name of some Thing. For as these, a *Man*, a *Tree*, a *Stone*, are the Names of the Things themselves; so the Images of a Man, of a Tree and of a stone, which are represented to men sleeping, have their Names also, though they be not Things, but onely fictions and Phantasmes of things. For we can remember these; and therefore it is no lesse necessary that they have Names to mark and signifie them, then the Things themselves. Also this word *Future* is a Name, but no future thing has yet any being, nor do we know whether that which we call Future, shall ever have a being or no. Neverthelesse, seeing we use in our mind to knit together things Past with those that are Present, the Name *Future* serves to signifie such knitting together. Moreover, that which neither is, nor has been, nor ever shall or ever can be, has a name, namely, *That which neither is, nor has been, &c.* Or more briefly this, *Impossible*. To conclude, this word *Nothing*, is a name, which yet cannot be the name of any thing. For when (for example) we substract 2 and 3 from 5, and so nothing remaining we would call that substraction to mind, this speech *Nothing remains*, and in it the word *Nothing* is not unusefull. And for the same reason we say truly *Lesse then Nothing* remaines, when we substract More from Lesse; for the minde feigns such remaines as these for Doctrines sake, and desires as often as is necessary, to call the same to memory. But seeing every name has some relation to that which is named, though that which we name be not alwaies a thing that has a being in Nature, yet it is lawfull for Doctrines sake to apply the word *Thing*, to whasoever we name; as if it were all one whether that thing be truly existent or be onely feigned.

7 The first distinction of Names, is that some are *Positive*, or *Affirmative*; others *Negative*, which are also called *Privative* and *Indefinite*. *Positive* are such as we impose for the likenesse, Equality

lity or Identity of the things we consider; *Negative* for the diversity, Unliknesse, or Inequality of the same. Examples of the former are *a Man, a Philosopher*; for *a Man* denotes any one of a multitude of men, and *a Philosopher* any one of many Philosophers by reason of their similitude; Also *Socrates* is a Positive name, because it signifies alwayes one and the same man. Examples of *Negatives* are such Positives as have the Negative particle *Not* added to them, as *Not-Man, Not-Philosopher*. But Positives were before Negatives; for otherwise there could have been no use at all of these. For when the name of *White* was imposed upon certain things, and afterwards upon other things the names of *Black, Blew, Transparent, &c.* the infinite dissimilitudes of these with *White* could not be comprehended in any one Name, save that which had in it the Negation of White, that is to say, the Name *Not-White*, or some other equivalent to it, in which the word *White* is repeated, such as *Unlike to white, &c.* And by these *Negative names*, we take notice our selves, and signifie to others what we have not thought of.

8 Positive and Negative names are *Contradictory* to one another, so that they cannot both be the name of the same thing. Besides, of Contradictory names, one is the name of any thing whatsoever; for whatsoever is, is either Man or Not-man, White or Not-white, and so of the rest. And this is so manifest, that it needs no further proofe or explication; for they that say *the same thing cannot both be, and not be*, speak obscurely; but they that say, *Whatsoever is, either is, or is not*, speake also absurdly and ridiculously. The certainty of this Axiome, viz. *Of two Contradictory Names, one is the Name of any thing whatsoever, the other not*, is the originall and foundation of all Ratiocination, that is, of all Philosophy; and therefore it ought to be so exactly propounded, that it may be of it selfe cleare and perspicuous to all men; as indeed it is, saving to such, as reading the long discourses made upon this subject by the Writers of *Metaphysicks* (which they beleeve to be some egregious learning) thinke they understand not, when they do.

9 Secondly, of Names, some are *Common* to many things, as a *Man, a Tree*; others *Proper* to one thing, as *he that writ the Iliad, Homer, this man, that man*. And a Common name, being the name of many things severally taken, but not collectively of all together (as Man

is

is not the name of all mankind, but of every one, as of Peter, Iohn, and the rest severally) is therefore called an *Universall name*; and therefore this word *Universall* is never the name of any thing existent in nature, nor of any Idea or Phantasme formed in the mind, but alwayes the name of some word or name; so that when *a Living creature*, *a Stone*, *a Spirit*, or any other thing is said to be *Universal*, it is not to be understood, that any Man, Stone, &c. ever was or can be Univer sall, but onely that these words, *Living creature*, *Stone*, *&c.* are *Universall names*, that is, Names common to many things; and the Conceptions answering them in our minde, are the Images and Phantasmes of severall Living Creatures, or other things. And therfore for the understanding of the extent of an Universal name, we need no other faculty but that of our imagination, by which we remember that such names bring sometimes one thing; sometimes another into our minde. Also of Common Names some are more, some lesse Common. *More Common*, is that which is the name of more things; *Lesse Common*, the name of fewer things. As *Living-Creature* is more Common then *Man*, or *Horse* or *Lion*, because it comprehends them all; and therefore a more Common name, in respect of a lesse Common, is called the *Genus* or a *Generall name*; and this in respect of that, the *Species*, or a *Speciall Name*.

10 And from hence proceeds the third distinction of Names, which is, that some are called names of the *First*, others of the *Second Intention*. Of the *first Intention* are the names of Things, a *Man*, *Stone*, *&c.* of the *second* are the names of names and speeches, as *Universall*, *Particular*, *Genus*, *Species*, *Syllogisme*, and the like. But it is hard to say why those are called names of the *First*, and these of the *Second Intention*, unlesse perhaps it was first intended by us to give names to those things which are of daily use in this life, and afterwards to such things as appertaine to science, that is, that our Second Intention was to give names to Names. But whatsoever the cause hereof may be, yet this is manifest, that *Genus*, *Species*, *Definition*, *&c.* are names of Words and Names onely; and therefore to put *Genus* and *Species* for Things, and *Definition* for the nature of any thing, as the Writers of *Metaphysicks* have done, is not right, seeing they be only significations of what we thinke of the nature of Things.

11 Fourthly, some Names are of *certaine* and *determined*, others

of

of *uncertaine* and *undetermined* ſignification. Of *determined* and *certain* ſignification is, firſt, that name which is giuen to any one thing by it ſelf, and is called an *Individuall Name*; as *Homer*, *this tree*, *that living Creature*, &c. Secondly that which has any of theſe words *All*, *Every*, *Both*, *Either*, or the like added to it; and it is therefore called an Univerſall Name, becauſe it ſignifies every one of thoſe things to which it is Common; and of *certaine* ſignification for this reaſon, that he which heares, conceives in his minde the ſame thing that he which ſpeakes would have him conceive. Of *Indefinite* ſignification is, firſt, that Name which has the word *ſome*, or the like added to it, and is called a *Particular name*; Secondly a Common Name ſet by it ſelfe without any note either of Univerſality or Particularity, as *Man*, *Stone*, and is called an *Indefinite Name*; but both *Particular* and *Indefinite* names are of uncertaine ſignification, becauſe the Hearer knowes not what thing it is the Speaker would have him conceive; and therefore in Speech, Particular and Indefinite names are to be eſteemed equivalent to one another. But theſe words, *All*, *Every*, *Some*, &c. which denote Univerſality and Particularity, are not Names, but parts onely of Names; So that *Every Man*, and *That Man which the Hearer conceives in his mind*, are all one; and *Some Man*, and *That Man which the Speaker thought of* ſignifie the ſame. From whence it is evident, that the uſe of ſignes of this kind, is not for a mans own ſake, or for his getting of knowledge by his own private meditation (for every man has his own, Thoughts ſufficiently determined without ſuch helpes as theſe) but for the ſake of others; that is, for the teaching and ſignifying of our Conceptions to others; nor were they invented onely to make us remember, but to make us able to diſcourſe with others.

12 Fifthly, Names are uſually diſtinguiſhed into *Univocall*, and *Equivocall*. *Univocall* are thoſe which in the ſame train of Diſcourſe ſignifie alwayes the ſame thing; but *Equivocall* thoſe which meane ſometimes one thing, and ſometimes another. Thus, the Name *Triangle* is ſaid to be *Univocall*, becauſe it is alwayes taken in the ſame ſenſe; and *Parabola* to be *Equivocall*, for the ſignification it has ſometimes of Allegory or Similitude, and ſometimes of a certaine Geometricall figure. Alſo every *Metaphor* is by profeſſion *Equivocall*. But this diſtinction belongs not ſo much to Names, as to thoſe that uſe Names; for ſome uſe them properly and accurately

curately for the finding out of truth; others draw them from their proper sense, for Ornament, or Deceipt.

13 Sixtly, of Names, some are *Absolute*, others *Relative*. *Relative* are such as are imposed for some Comparison, as *Father*, *Sonne*, *Cause*, *Effect*, *Like*, *Unlike*, *Equal*, *Unequal*, *Master*, *Servant*, *&c*. And those that signifie no Comparison at all are *Absolute Names*. But as it was noted above, that Universality is to be attributed to Words & Names onely, and not to Things; so the same is to be said of other distinctions of Names; for no Things are either *Univocall* or *Equivocall*, or *Relative* or *Absolute*. There is also another distinction of Names into *Concrete* and *Abstract*; but because Abstract Names proceed from Proposition, and can have no place where there is no Affirmation, I shall speake of them hereafter.

14 Lastly there are *Simple* and *Compounded Names*. But here it is to be noted, that a name is not taken in Philosophy, as in Grammar, for one single word, but for any number of words put together to signifie one Thing; for among Philosophers *Sentient Animated Body*, passes but for one Name, being the Name of every living Creature; which yet, among Gramarians is accounted three Names. Also a *Simple Name* is not here distinguished from a *Compounded Name* by a Preposition, as in Grammar. But I call a *Simple Name*, that which in every kind is the most Common or most Universall; and that a *Compounded Name*, which by the joyning of another Name to it is made lesse Universall, and signifies that more conceptions then one were in the mind, for which that later Name was added. For example, in the conception of Man (as is shewn in the former Chap.) First, he is conceived to be something that has Extension, which is marked by the word *Body*. *Body* therefore is a *Simple Name*, being put for that first single Conception; Afterwards, upon the sight of such and such motion, another Conception arises for which he is called an *Animated Body*; and this I here call a *Compounded Name*, as I doe also the name *Animal*, which is equivalent to an *Animated Body*. And in the same manner an *Animated Rational Body*, as also a *Man*, which is equivalent to it, is a more Compounded Name. And by this we see how the Composition of Conceptions in the mind is answerable to the Composition of Names; for as in the minde one Idea or Phantasme succeeds to another, and to this a third; so to one Name is added another and another succes-

sively, and of them all is made one *Compounded* Name. Neverthelesse we must not thinke Bodies, which are without the Minde, are compounded in the same manner, namely that there is in Nature a Body, or any other imaginable Thing existent, which at the first has no Magnitude, and then by the addition of Magnitude, comes to have Quantity, and by more or lesse Quantity to have Density or Rarity, and again by the addition of Figure to be Figurate, and after this by the injection of Light or Colour, to become Lucid or Coloured; though such has been the Philosophy of many.

15 The Writers of *Logique* have endeavoured to digest the Names of all the kinds of Things into certaine *Scales* or Degrees, by the continual subordination of Names lesse Common, to Names more Common. In the Scale of *Bodies* they put in the first and highest place *Body* simply, and in the next place under it lesse Common Names, by which it may be more limited and determined, namely *Animated* and *Inanimated*, and so on till they come to *Individualls*. In like manner in the Scale of *Quantities* they assign the first place to *Quantity*, and the next to *Line*, *Superficies*, and *Solid*, which are Names of lesse latitude; and these Orders or Scales of Names they usually call *Prædicaments* and *Categories*; And of this Ordination not onely Positive but Negative Names also are capable; which may be exemplified by such Formes of the *Prædicaments* as follow.

The Form of the Prædicament of *Body*.

Not-Body; or Accident

- Body
  - Not animated
  - Animated
    - Not Living-Creature
    - Living-Creature
      - Not Man
      - Man
        - Not Peter
        - Peter

- Both Accident and Body are considered
  - Absolutely as,
    - Quantity, or so much
    - Quality, or such
  - or
  - Comparatively, which is called their Relation

The

The Forme of the Prædicament of *Quantity*.

- Quãtity
  - Not Continual, as Number,
  - Continuall
    - Of it ſelfe, as
      - Line
      - Superficies
      - Solid
    - By Accident, as ——————
      - Time, by Line
      - Motion, by Line and Time
      - Force, by Motion and Solid

Where it is to be noted, that *Line*, *Superficies* and *Solid* may be ſaid to be of ſuch and ſuch Quantity, that is, to be originally and of their own nature capable of Equality and Inequality; But we cannot ſay there is either Majority or Minority, or Equality, or indeed any Quantity at all, in *Time*, without the help of *Line* and *Motion*; nor in *Motion*, without *Line* and *Time*; nor in *Force*, otherwiſe then by *Motion* and *Solid*.

The Forme of the Prædicament of *Quality*.

- Quality
  - Perceptiõ by Senſe
    - Primary
      - Seeing
      - Hearing
      - Smelling
      - Taſting
      - Touching
    - Secondary
      - Imagination
      - Affection ——
        - Pleaſant
        - Unpleaſant
  - Senſible Quality
    - By Seeing, as Light and Colour
    - By Hearing, as Sound
    - By Smelling, as Odors
    - By Taſting, as Savours
    - By Touching, as Hardneſſe, Heat, Cold, &c.

The forme of the Prædicament of *Relation.*

- Relation of
  - Magnitudes, as Equality and Inequality
  - Qualities, as Likeneſſe and Unlikeneſſe
  - Order
    - Together
      - In Place
      - In Time
    - Not toge-ther
      - In Place
        - Former
        - Later
      - In Time
        - Former
        - Later

16 Concerning which Prædicaments it is to be noted in the firſt place, That as the diviſion is made in the firſt Prædicament into Contradictory Names, ſo it might have been done in the reſt. For as there, *Body* is divided into *Animated* and *Not-Animated*, ſo in the ſecond Prædicament *Continuall Quantity* may be divided into *Line* and *Not-line*, and again, *Not-line* into *Superficies* and *Not-Superficies*, and ſo in the reſt; but it was not neceſſary.

Secondly, it is to be obſerved, that of *Poſitive* Names the former comprehends the later; but of *Negatives* the former is comprehended by the later. For example *Living-Creature* is the Name of every Man, and therefore it comprehends the Name *Man*; but on the contrary *Not-Man* is the Name of every Thing which is Not-Living-Creature, and therefore the Name *Not-Living-Creature* which is put firſt, is comprehended by the later Name *Not-Man.*

Thirdly, we muſt take heed we do not thinke, that as Names, ſo the diverſities of Things themſelves may be ſearched out and determined by ſuch Diſtinctions as theſe; or that arguments may be taken from hence (as ſome have done ridiculouſly) to prove that the kinds of Things are not infinite.

Fourthly, I would not have any man thinke I deliver the Forms above for a true and exact Ordination of Names; for this cannot be performed as long as Philoſophy remains imperfect; Nor that by placing (for example) *Light* in the Prædicameut of *Qualities*, while another places the ſame in the Prædicament of Bodies, I pretend that either of us ought for this to be drawn from

from his opinion; for this is to be done onely by Arguments and Ratiocination, and not by disposing of words into *Classes*.

Lastly, I confesse I have not yet seen any great use of the Prædicaments in Philosophy. I beleeve *Aristotle* when he saw he could not digest the Things themselves into such Orders, might neverthelesse desire out of his owne Authority to reduce Words to such Formes, as I have done; but I doe it onely for this end, that it may be understood what this Ordination of Words is, and not to have it received or true, till it be demonstrated by good reason to be so.

CHAP.

CHAP. III.

Of Proposition.

1 Divers Kinds of Speech. 2 Proposition defined. 3 Subject, Prædicate and Copula what they are, and Abstract and Concrete what. The Use and Abuse of Names Abstract. 5 Proposition Universal and Particular. 6 Affirmative and Negative. 7 True and False. 8 True and False belongs to Speech, and not to Things. 9 Proposition Primary, not Primary, Definition, Axiome, Petition. 10 Proposition Necessary and Contingent. 11 Categoricall and Hypotheticall. 12 The same Proposition diversly pronounced. 13 Propositions that may be reduced to the same Categoricall Proposition, are Equipollent. 14 Universal Propositions converted by Contradictory Names, are Equipollent. 15 Negative Propositions are the same, whether the Negation be before or after the Copula. 16 Particular Propositions simply converted, are Equipollent. 17 What are Subaltern, Contrary, Subcontrary and Contradictory Propositions. 18 Consequence what it is. 19 Falsity cannot follow from Truth. 20 How one Proposition is the Cause of another.

1 FRom the Connection or Contexture of Names arise diverse kinds of Speech, whereof some signifie the Desires and Affections of Men; such are first *Interrogations*, which denote the desire of Knowing; as *Who is a good Man?* In which speech there is one Name expressed, & another desired and expected from him of whom we aske the same. Then *Prayers*, which signifie the desire of having something; *Promises*, *Threats*, *Wishes*, *Commands*, *Complaints*, and other significations of other Affections. Speech may also be Absurd and Insignificant; as when there is a succession of Words, to which there can be no succession of Thoughts in the mind to answer them; and this happens often to such, as understanding nothing in some subtil matter, doe neverthelesse, to make others beleeve they understand, speake of the same in-

incoherently; For the connection of incoherent Words, though it want the end of Speech (which is Signification) yet it is Speech; and is used by the Writers of *Metaphysicks* almost as frequently as Speech significative. In Philosophy there is but one kinde of Speech usefull, which some call in Latine *Dictum*, others *Enuntiatum & Pronunciatum*; but most men call it *Proposition*, and is the speech of those that Affirm or Deny, and expresseth Truth or Falsity.

2 *A* PROPOSITION *is a Speech consisting of two Names copulated, by which he that speaketh signifies he conceives the later Name to be the Name of the same thing whereof the former is the Name; or* (which is all one) *that the former Name is comprehended by the later.* For example, this speech *Man is a Living Creature*, in which two Names are copulated by the verb *Is*, is a *Proposition*, for this reason, that he that speakes it conceives both *Living Creature* and *Man* to be Names of the same thing, or that the former Name *Man* is comprehended by the later Name *Living Creature.* Now the former Name is commonly called the *Subject*, or *Antecedent*, or the *Contained Name*, and the later the *Prædicat*, *Consequent* or *Containing Name.* The signe of Connection amongst most Nations is either some word, as the word *is* in the Proposition *Man is a living Creature*, or some Case or Termination of a word, as in this Proposition, *Man walketh* (which is equivalent to this, *Man is walking*) the Termination by which it is said he *walketh*, rather then he *is walking*, signifieth that those two are understood to be copulated, or to be Names of the same Thing.

But there are, or certainly may be some Nations that have no word which answers to our Verbe *Is*, who neverthelesse forme Propositions by the position onely of one Name after another, as if instead of *Man is a Living Creature*, it should be said *Man a Living Creature*; for the very order of the Names may sufficiently shew their connection; and they are as apt and usefull in Philosophy, as if they were copulated by the Verbe *Is*.

3 Wherefore in every Proposition three things are to be considered, *viz.* the two Names, which are the *Subject* and the *Prædicate*, and their *Copulation*; both which Names raise in our Minde the Thought of one and the same Thing; but the Copulation makes us thinke of the Cause for which those Names were imposed on that Thing. As for example, when we say *a Body is moveable*, though we conceive the same thing to be designed by both

those

those Names, yet our Minde rests not there, but searches further what it is *to be a Body*, or *to be Moveable*, that is, wherein consists the difference betwixt these and other Things, for which these are so called, others are not so called. They therefore that seeke what it is *to be* any thing, as *to be Moveable*, *to be Hot*, *&c.* seek in Things the causes of their Names.

And from hence arises that distinction of Names (touched in the last Chap.) into *Concrete* and *Abstract*. For *Concrete* is the Name of any thing which we suppose to have a being, and is therefore called the *Subject*, in Latine *Suppositum*, and in Greek ὑποκείμενον; as *Body*, *Moveable*, *Moved*, *Figurate*, *a Cubit high*, *Hot*, *Cold*, *Like*, *Equal*, *Appius*, *Lentulus* and the like; and *Abstract* is that which in any Subject denotes the Cause of the Concrete Name, as *to be a Body*, *to be Moveable*, *to be Moved*, *to be Figurate*, *to be of such Quantity*, *to be Hot*, *to be Cold*, *to be Like*, *to be Equall*, *to be Appius*, *to be Lentulus*, *&c.* Or Names equivalent to these, which are most commonly called *Abstract* Names, as *Corporeity*, *Mobility*, *Motion*, *Figure*, *Quantity*, *Heat*, *Cold*, *Likenesse*, *Equality*, and (as Cicero has it) *Appiety* and *Lentulity*. Of the same kind also are Infinitives; for *to Live* and *to Move* are the same with *Life* and *Motion*, or *to be Living*, and *to be Moved*. But *Abstract Names* denote onely the Causes of *Concrete Names*, and not the Things themselves. For example, when we see any thing, or conceive in our Minde any Visible thing, that Thing appears to us, or is conceived by us, not in one Point, but as having Parts distant from one another, that is, as being extended and filling some space. Seeing therefore we call the Thing so conceived *Body*, the cause of that name is, that that Thing is *extended*, or the *Extension* or *Corporeity* of it. So when we see a Thing appeare sometimes here, sometimes there, and call it *Moved* or *Removed*, the Cause of that Name is that it *is Moved* or the *Motion* of the same.

And these Causes of Names, are the same with the Causes of our Conceptions, namely some Power or Action, or Affection of the Thing conceived, which some call the Manner by which any thing workes upon our senses, but by most men they are called *Accidents*; I say Accidents, not in that sense in which *Accident* is opposed to *Necessary*; but so, as being neither the Things themselves, nor parts thereof, do neverthelesse accompany the Things

Things in ſuch manner,that (ſaving *Extenſion*) they may all periſh and be deſtroyed, but can never be *abſtracted*.

4 There is alſo this difference betwixt *Concrete* and *Abſtract Names*, that thoſe were invented before Propoſitions, but theſe after; for theſe could have no being till there were Propoſitions, from whoſe *Copula* they proceed. Now in all matters that concerne this life, but chiefly in Philoſophy, there is both great Uſe and great Abuſe of *Abſtract Names*; and the Uſe conſiſts in this, that without them we cannot for the moſt part either Reaſon, or compute the Properties of Bodies; for when we would multiply, divide, adde or ſubſtract Heat, Light, or Motion, if we ſhould double or adde them together by *Concrete* Names, ſaying (for example) Hot is double to Hot, Light double to Light, or Moved double to Moved, we ſhould not double the Properties, but the Bodies themſelves that are Hot, Light, Moved, &c. which we would not doe. But the Abuſe proceeds from this, that ſome men ſeeing they can conſider, that is (as I ſaid before) bring into account the Increaſings and Decreaſings of Quantity, Heat and other Accidents, without conſidering their Bodies or Subjects (which they call *Abſtracting*, or making to exiſt apart by themſelves,) they ſpeake of Accidents, as if they might be ſeparated from all Bodies. And from hence proceed the groſſe errors of writers of *Metaphyſicks*; for, becauſe they can conſider Thought without the conſideration of Body, they inferre there is no need of a Thinking-Body; and becauſe Quantity may be conſidered without conſidering Body, they thinke alſo that Quantity may be without Body, and Body without Quantity; and that a Body has Quantity by the addition of Quantity to it. From the ſame fountaine ſpring thoſe inſignificant words, *Abſtract Subſtance*, *Separated Eſſence* and the like; as alſo that confuſion of words derived from the Latine Verb *Eſt*, as *Eſſence*, *Eſſentiality*, *Entity*, *Entitative*; beſides *Reality*, *Aliquiddity*, *Quiddity*, &c. which could never have been heard of among ſuch Nations as do not *Copulate* their Names by the Verbe *Is*, but by Adjective Verbs, as Runneth, Readeth, &c. or by the mere placing of one Name after another; and yet ſeeing ſuch Nations Compute and Reaſon, it is evident that Philoſophy has no need of thoſe words *Eſſence*, *Entity* and other the like barbarous Termes.

There are many Distinctions of Propositions, whereof the first is, that some are *Universall*, others *Particular*, others *Indefinite*, and others *Singular*; and this is commonly called the distinction of *Quantity*. An *Universall Proposition* is that whose subject is affected with the sign of an Universall Name, as *Every man is a Living Creature*. *Particular*, that whose subject is affected with the sign of a Particular Name, as *Some Man is learned*. An *Indefinite Proposition* has for its Subject a Common Name, and put without any sign, as *Man is a Living Creature*, *Man is Learned*. And a *Singular Proposition* is that whose Subject is a singular Name, as *Socrates is a Philosopher*, *This man is black*.

6 The Second Distinction is into *Affirmative* and *Negative*, and is called the Distinction of *Quality*. An *Affirmative Proposition* is that whose Prædicate is a Positive Name, as *Man is a Living Creature*. *Negative*, that whose Prædicate is a Negative Name, as *Man is Not-a-stone*.

7 The third Distinction is, that one is *True*, another *False*. A *True Proposition* is that, whose Prædicate containes, or comprehends its Subject, or whose Prædicate is the Name of every thing, of which the Subject is the Name; as *Man is a Living Creature* is therefore a True Proposition, because whatsoever is called *Man*, the same is also called *Living Creature*; and *Some Man is sick*, is True, because *sick* is the Name of *Some Man*. That which is not True, or that whose Prædicate does not containe its Subject, is called a *False Proposition*, as *Man is a Stone*.

Now these words *True*, *Truth*, and *True Proposition* are equivalent to one another; For Truth consists in Speech, and not in the Things spoken of; and though *True* be sometimes opposed to to *Apparent* or *Feigned*, yet it is alwayes to be referred to the Truth of Proposition; for the Image of a Man in a Glasse, or a Ghost, is therefore denyed to be a very Man, because this Proposition, *A Ghost is a man*, is not True; for it cannot be denied but that a Ghost is a very Ghost. And therefore Truth or Verity is not any Affection of the Thing, but of the Proposition concerning it. As for that which the Writers of *Metaphysicks* say, that *A thing*, *One thing*, and *a Very thing*, are equivalent to one another, it is but trifling and childish; for who does not know, that *A Man*, *One Man*, and a *Very Man*, signifie the same.

8 And

8 And from hence it is evident, that Truth and Falsity have no place but amongst such Living Creatures as use Speech. For though some brute Creatures, looking upon the Image of a Man in a Glasse, may be affected with it as if it were the Man himselfe, and for this reason feare it or fawne upon it in vain; Yet they doe not apprehend it as True or False, but onely as Like; and in this they are not deceived. Wherefore, as men owe all their True Ratiocination to the right understanding of Speech; So also they owe their Errors to the misunderstanding of the same; and as all the Ornaments of Philosophy proceed onely from Man, so from Man also is derived the ugly absurdity of False opinions. For Speech has something in it like to a Spiders Web (as it was said of old of *Solons* laws;) for by contexture of words tender and delicate Wits are insnared and stopt; but strong Wits breake easily through them.

From hence also this may be deduced, that the first Truths were arbitrarily made by those that first of all imposed Names upon Things, or received them from the imposition of others. For it is true (for example) that *Man is a Living Creature*; but it is for this reason, that it pleased men to impose both those Names on the same thing.

9 Fourthly, Propositions are distinguished into *Primary* and *not Primary*. *Primary* is that wherein the Subject is explicated by a Prædicate of many Names, as *Man, is a Body Animated Rationall*; for that which is comprehended in the Name *Man* is more largely expressed in the Names *Body*, *Animated*, and *Rationall* joyned together; and it is called *Primary*, because it is first in Ratiocination; for nothing can be proved, without understanding first the name of the Thing in question. Now *Primary Propositions* are nothing but Definitions, or parts of Definitions, and these onely are the principles of Demonstration, being Truths constituted arbitrarily by the Inventors of Speech, and therefore not to be demonstrated. To these Propositions, some have added others, which they call *Primary* and *Principles*; namely *Axiomes* and *Common Notions*; which (though they be so evident that they need no proofe, yet) because they may be proved, are not truly Principles; & the lesse to be received for such, in regard Propositions not intelligible, and sometimes manifestly false, are thrust on us under the Name of *Principles*

by the clamour of Men, who obtrude for evident to others all that they themſelves thinke True. Alſo certaine *Petitions* are commonly received into the number of Principles; as for example, *That a ſtreight line may be drawne between two points*, and other *Petitions* of the Writers of Geometry; and theſe are indeed the Principles of *Art* or *Conſtruction*, but not of *Science* and *Demonſtration.*

10 Fiftly, Propoſitions are diſtinguiſhed into *Neceſſary*, that is, neceſſarily True; and True, but not neceſſarily, which they call *Contingent*. A *Neceſſary Propoſition* is when nothing can at any time be conceived or feigned, whereof the Subject is the Name, but the Prædicate alſo is the Name of the ſame thing; as *Man is a Living-Creature* is a neceſſary Propoſition, becauſe at what time ſoever we ſuppoſe the name *Man* agrees with any thing, at that time the name *Living-Creature* alſo agrees with the ſame. But a *Contingent Propoſition* is that which at one time may be true, at another time falſe; as *Every Crow is Black*; which may perhaps be true now, but falſe hereafter. Again, in every *Neceſſary Propoſition*, the Prædicate is either equivalent to the Subject, as in this, *Man is a Rational Living-Creature*; or part of an equivalent Name, as in this, *Man is a Living-Creature*, for the Name *Rational Living-Creature*, or *Man*, is compounded of theſe two, *Rational* and *Living-Creature*. But in a *Contingent Propoſition* this cannot be; for though this were true, *Every Man is a Lyar*, yet becauſe the word *Lyar* is no part of a Compounded name equivalent to the Name *Man*, that Propoſition is not to be called *Neceſſary*, but *Contingent*, though it ſhould happen to be true alwayes. And therefore thoſe Propoſitions only are *Neceſſary*, which are of Sempiternal Truth, that is, true at all times. From hence alſo it is manifeſt, that Truth adheres not to Things, but to Speech onely; for ſome Truths are eternal; for it will be eternally true, *if Man, then Living-Creature*; but that any *Man*, or *Living-Creature* ſhould exiſt eternally, is not neceſſary.

11 A ſixth Diſtinction of Propoſitions is into *Categorical* and *Hypotheticall*. A *Categoricall Propoſition* is that which is ſimply or abſolutely pronounced, as *Every Man is a Living Creature*, *No Man is a Tree*; and *Hypotheticall* is that which is pronounced conditionally, as, *If any thing be a Man the ſame is alſo a Living Creature*, *If any thing be a Man the ſame is alſo Not-a-ſtone.*

A Categoricall Propoſition, and an *Hypothetical* anſwering it, doe both

both signifie the same, if the Propositions be Necessary; but not if they be Contingent. For example, if this, *Every Man is a Living-Creature*, be true, this also will be true, *If any thing be a Man, the same is also a Living-Creature*; but in Contingent Propositions, though this be true, *Every Crow is Black*; yet this, *If any thing be a Crow the same is Black*, is false. But an *Hypotheticall Proposition* is then rightly said to be true, when the Consequence is true; as *Every Man is a Living-Creature* is rightly said to be a true Proposition, because of whatsoever it is truly said *That is a Man*, it cannot but be truly said also *The same is a Living-Creature*. And therefore whensoever an *Hypotheticall Proposition* is true, the *Categoricall* answering it is not only true, but also necessary; which I thought worth the noting, as an argument, that Philosophers may in most things reason more solidly by *Hypotheticall* then *Categoricall Propositions*.

12 But seeing every Proposition may be & uses to be pronounced and written in many formes, and we are obliged to speake in the same manner as most men speake; yet they that learne Philosophy from Masters, had need to take heed they be not deceived by the Variety of expressions. And therefore whensoever they meet with any obscure Proposition, they ought to reduce it to its most Simple and Categoricall forme; in which the Copulative word *Is* must be expressed by it selfe, and not mingled in any manner either with the Subject or Prædicate, both which must be separated and cleerly distinguished one from another. For example, if this Proposition *Man can not sinne*, be compared with this, *Man cannot sinne*, their difference will easily appeare if they be reduced to these, *Man is able not to sinne*, and *Man is not able to sinne*, where the Prædicates are manifestly different. But they ought to doe this silently by themselves, or betwixt them and their Masters onely; for it will be thought both ridiculous and absurd, for a man to use such language publiquely. Being therefore to speake of *Equipollent Propositions*, I put in the first place all those for *Equipollent*, that may be reduced purely to one and the same Categoricall Proposition.

13 Secondly, that which is Categoricall and Necessary, is *Equipollent* to its Hypotheticall Proposition; as this Categorical, *A Right-lined Triangle has its three Angles equal to two Right angles*, to this Hypotheticall, *If any Figure be a Right-lined Triangle, the three Angles of it are equal to two Right Angles*.

14 Also

14 Also, any two Universall Propositions, of which the Termes of the one (that is, the Subject and Prædicate) are Contradictory to the Termes of the other, and their Order inverted, as these, *Every man is a Living Creature*, and *Every thing that is not a Living Creature, is not a Man*, are *Equipollent*. For seeing *Every Man is a Living Creature* is a true Proposition, the Name *Living Creature* containes the Name *Man*; but they are both Positive Names; and therefore (by the last Article of the præcedent Chapter) the Negative Name *Not Living Creature* containes the Negative Name *Not Man*; Wherefore *Every thing that is not a Living Creature is not a Man* is a true Proposition. Likewise these, *No Man is a Tree*, *No Tree is a Man*, are *Equipollent*. For if it be true that *Tree* is not the Name of any *Man*, then no one thing can be signified by the two Names *Man* and *Tree*, wherefore *No Tree is a Man* is a true Proposition. Also to this, *Whatsoever is not a Living Creature is not a Man*, where both the Termes are Negative, this other Proposition is *Equipollent*, *Onely a Living Creature is a Man*.

15 Fourthly, Negative Propositions, whether the particle of Negation be set after the *Copula* as some Nations doe, or before it, as it is in Latine and Greeke, if the Termes be the same, are *equipollent*; as for example, *Man is not-a-Tree* and *Man is not-a-Tree*, are equipollent, though *Aristotle* deny it. Also these, *Every Man is not a Tree*, and *No Man is a Tree* are equipollent, and that so manifestly, as it needs not be demonstrated.

16 Lastly, all Particular Propositions that have their Termes inverted, as these, *Some Man is Blind*, *Some Blind thing is a Man*, are *equipollent*; for either of the two Names, is the Name of some one and the same Man; and therefore in which soever of the two Orders they be connected, they signifie the same Truth.

17 Of Propositions that have the same Termes, and are placed in the same Order, but varied either by Quantity or Quality, some are called *Subaltern*, others *Contrary*, others *Subcontrary*, and others *Contradictory*.

Subaltern, are Universal and Particular Propositions of the same Quality; as, *Every Man is a Living Creature*, *Some Man is a Living Creature*; or, *No Man is Wise*, *Some Man is not Wise*. Of these if the Universal be true, the Particular will be true also.

Contrary, are Universal Propositions of different Quality; *as Every*

Every Man is happy, *No Man is happy*. And of these if one be true the other is false; also they may both be false, as in the example given.

Subcontrary, are Particular Propositions of different Quality; as *Some Man is learned*, *Some Man is not learned*; which cannot be both false, but they may be both true.

Contradictory are those that differ both in Quantity and Quality; as *Every Man is a Living Creature*, *Some Man is not a Living Creature*; which can neither be both true, nor both false.

18 A Proposition is said to *follow* from two other Propositions, when these being granted to be true, it cannot be denyed but the other is true also. For example, let these two Propositions, *Every Man is a Living Creature*, and *Every Living Creature is a Body*, be supposed true, that is, that *Body* is the Name of *Every Living Creature*, and *Living Creature*, the Name of *Every Man*. Seeing therefore, if these be understood to be true, it cannot be understood that *Body* is not the name of *Every man*, that is, that *Every Man is a Body*, is false, this Proposition will be said to *follow* from those two, or to be necessarily *inferred* from them.

19 That a true Proposition may follow from false Propositions, may happen sometimes; but false from true, never. For if these, *Every Man is a Stone*, and *Every Stone is a Living Creature*, (which are both false) be granted to be true, it is granted also that *Living Creature* is the name of *Every Stone*, and *Stone* of *Every Man*, that is, that *Living Creature* is the Name of *Every Man*; that is to say, this Proposition *Every Man is a Living Creature*, is true, as it is indeed true. Wherefore a true Proposition may sometimes follow from false. But if any two Propositions be true, a false one can never follow from them. For if true follow from false, for this reason onely, that the false are granted to be true, then truth from two truths granted will follow in the same manner.

20 Now seeing none but a true Proposition will follow from true, and that the understanding of two Propositions to be true, is the cause of understanding that also to be true which is deduced from them; the two Antecedent Propositions are commonly called the Causes of the inferred Proposition, or Conclusion. And from hence it is that Logicians say, the *Premisses* are Causes of the *Conclusion*; which may passe, though it be not properly spoken; for though

though Underſtanding be the cauſe of Underſtanding, yet Speech is not the cauſe of Speech. But when they ſay, the Cauſe of the Properties of any thing, is the Thing it ſelf, they ſpeake abſurdly. Eor example, if a Figure be propounded which is Triangular; Seeing every Triangle has all its angles together equal to two right angles, from whence it follows that all the angles of that Figure are equal to two right angles, they ſay for this reaſon, that that Figure is the Cauſe of that Equality. But ſeeing the Figure does not it ſelf make its angles, and therefore cannot be ſaid to be the *Efficient-Cauſe*, they call it the *Formall-Cauſe*; whereas indeed it is no Cauſe at all; nor does the Property of any Figure follow the Figure, but has its Being at the ſame time with it; only the Knowledge of the Figure goes before the Knowledge of the Properties; and one Knowledge is truly the Cauſe of another Knowledge, namely the *Efficient-Cauſe*.

And thus much concerning *Propoſition*; which in the Progreſs of Philoſophy is the firſt Step, like the moving towards of one Foot. By the due addition of another Step I ſhall proceed to *Syllogiſme*, and make a compleat Pace. Of which in the next Chapter.

CHAP.

CHAP. IV.
Of Syllogisme.

1 *The Definition of Syllogisme.* 2 *In a Syllogisme there are but three Termes.* 3 *Major, Minor and Middle Term; also Major and Minor Proposition, what they are.* 4 *The Middle Terme in every Syllogisme ought to be determined in both the Propositions to one and the same thing.* 5 *From two Particular Propositions nothing can be concluded.* 6 *A Syllogisme is the Collection of two Propositions into one Summe.* 7 *The Figure of a Syllogisme what it is.* 8 *What is in the mind answering to a Syllogisme.* 9 *The first Indirect Figure how it is made.* 10 *The second Indirect Figure how made.* 11 *How the third Indirect Figure is made* 12 *There are many Moods in every Figure, but most of them Uselesse in Philosophy.* 13 *An Hypotheticall Syllogisme when equipollent to a Categoricall.*

1. A Speech consisting of three Propositions, from two of which the third followes, is called a SYLLOGISME; and that which followes is called the *Conclusion*; the other two *Premisses*. For example this Speech, *Every man is a Living Creature, Every Living Creature is a Body*, therefore, *Every Man is a Body*, is a *Syllogisme*, because the third Proposition follows from the two first; that is, if those be granted to be true, this must also be granted to be true.

2 From two Propositions which have not one Terme common, no Conclusion can follow; and therefore no *Syllogisme* can be made of them. For let any two Premisses, *A man is a Living Creature*, *A Tree is a Plant*, be both of them true, yet because it cannot be collected from them that *Plant* is the Name of a *Man*, or *Man* the Name of a *Plant*, it is not necessary that this Conclusion, *A Man is a Plant* should be true. Corollary, Therefore in the *Premisses* of a *Syllogisme* there can be but three *Termes*.

Besides there can be no Terme in the *Conclusion*, which was not in the *Premisses*. For let any two Premisses be, *A Man is a Living Creature*, *A Living Creature is a Body*, yet if any other Terme be put

in the Conclusion, as *Man is two footed*; though it be true, it cannot follow from the Premisses, because from them it cannot be collected, that the Name *Two footed* belongs to a *Man*; and therefore againe, In every *Syllogisme* there can be but three *Termes*.

3 Of these Termes, that which is the *Predicate* in the Conclusion, is commonly called the *Major*; that which is the *Subject* in the Conclusion, the *Minor*, and the other is the *Middle Term*; as in this Syllogisme, *A Man is a Living Creature*, *A Living Creature is a Body*, therefore, *A Man is a Body*, *Body* is the *Major*, *Man* the *Minor* and *Living Creature* the *Middle Term*. Also of the Premisses, that in which the *Major Terme* is found, is called the *Major Proposition*, and that which has the *Minor Term* the *Minor Proposition*.

4 If the Middle Terme be not in both the Premisses determined to one and the same singular thing, no Conclusion will follow, nor Syllogisme be made. For let the *Minor Terme* be *Man*, the *Middle Terme Living Creature*, and the *Major Term Lyon*; and let the Premisses be *Man is a Living Creature*, *Some Living Creature is a Lyon*, yet it will not follow that, *Every* or *Any Man is a Lyon*. By which it is manifest, that in every Syllogisme, that Proposition which has the *Middle Terme* for its *Subject*, ought to be either *Universal* or *Singular*, but not *Particular* nor *Indefinite*. For example, this Syllogism, *Every Man is a Living Creature*, *some Living Creature is four-footed*, therefore *some Man is four-footed*, is therefore faulty, because the *Middle Term*, *Living Creature*, is in the first of the Premisses determined onely to *Man*, for there the Name of *Living Creature* is given to *Man onely*, but in the later Premisse it may be understood of some other Living Creature besides Man. But if the later Premisse had been *Universall*, as here, *Every Man is a Living Creature*, *Every Living Creature is a Body*, therefore *Every Man is a Body*, the Syllogisme had been true; for it would have followed that *Body* had been the Name of *Every Living Creature*, that is of *Man*, that is to say, the Conclusion *Every Man is a Body* had been true. Likewise when the *Middle Term* is a *Singular* Name, a Syllogisme may be made, I say a true Syllogisme, though uselesse in Philosophy, as this, *Some Man is Socrates*, *Socrates is a Philosopher*, therefore *Some Man is a Philosopher*; for the Premisses being granted, the Conclusion cannot be denyed.

5 And therefore of two Premisses, in both which the *Middle Terme*

Terme is *Particular*, a Syllogisme cannot be made; for whether the *Middle Terme* be the *Subject* in both the Premisses, or the *Predicate* in both, or the *Subject* in one and the *Predicate* in the other, it will not be necessarily determined to the same thing. For let the Premisses be,

Some Man is blind	In both which the *Middle Term*
Some Man is learned	is the *Subject*,

It will not follow, that *Blind* is the Name of any *learned Man*, or *Learned* the Name of any *Blind Man*, seeing the Name *Learned* does not containe the Name *Blind*, nor this that; and therefore it is not necessary that both should be Names of the same Man. So from these Premisses.

Every Man is a Living Creature	In both which the *Middle Terme*
Every Horse is a Living Creature	is the *Predicate*,

Nothing will follow. For seeing *Living Creature* is in both of them *Indefinite*, which is equivalent to *Particular*, and that *Man* may be one kind of *Living Creature*, and *Horse* another kind, it is not necessary that *Man* should the be Name of *Horse*, or *Horse* of *Man*. Or if the Premisses be,

Every Man is a Living Creature	In one of which the *Middle Terme* is the *Subject*, and in
Some Living Creature is four-footed	the other the *Predicate*,

The Conclusion will follow, because the Name *Living Creature* being not determined, it may in one of them be understood of *Man* in the other of *Not-Man*.

6 Now it is manifest from what has been said, that a Syllogisme is nothing but a Collection of the summe of two Propositions, joyned together by a common Term, which is called the *Middle Terme*. And as Proposition is the Addition of two Names, so Syllogisme is the adding together of three.

7 Syllogismes are usually distinguished according to their diversity of *Figures*, that is, by the diverse position of the Middle Term. And againe in Figure there is a distinction of certain

Moods, which consist of the differences of Propositions in *Quantity* & *Quality*. The first Figure is that, in which the Terms are placed one after another according to their latitude of Signification; in which order the *Minor Term* is first, the *Middle Term* next, and the *Major* last; as if the *Minor* Term be Man, the *Middle Term Living Creature* and the *Major Term Body*, then, *Man is a Living Creature, is a Body*, will be a Syllogisme in the first *Figure*; in which, *Man is a Living Creature*, is the *Minor* Proposition; the *Major*, *Living Creature is a Body*, and the *Conclusion* or sum of both *Man is a Body*. Now this *Figure* is called *Direct*, because the Termes stand in direct Order; and it is varied by *Quantity* and *Quality* into four *Moods*; of which the first is that wherein all the Terms are *Positive*, and the Minor Term *Universal*, as *Every Man is a Living Creature*, *Every Living Creature is a Body*; in which all the Propositions are *Affirmative* and *Universall*. But if the Major Term be a Negative Name, and the Minor an Universall Name, the *Figure* will be in the second *Mood*, as, *Every Man is a Living Creature*, *Every Living Creature is not a Tree*, in which the Major Proposition and Conclusion are both *Universall* and *Negative*. To these two are commonly addded two more, by making the Minor Term *Particular*. Also it may happen that both the Major and Middle Termes are *Negative* Terms, and then there arises another *Mood*, in which all the Propositions are Negative, and yet the Syllogisme will be good; as, if the Minor Term be *Man*, the Middle Term *Not a Stone*, and the Major Terme, *Not a Flint*, this Syllogisme, *No Man is a Stone*, *Whatsoever is not a Stone is not a Flint*, therefore *No Man is a Flint*, is true, though it consist of three Negatives. But in Philosophy, the Profession whereof is to establish Universall Rules concerning the Properties of Things, seeing the difference betwixt *Negatives* and *Affirmatives* is onely this, that in the former the Subject is affirmed by a Negative Name, and by a Positive in the later, it is superfluous to consider any other *Mood* in direct *Figure*, besides that, in which all the Propositions are both Universal and Affirmative.

8. The Thoughts in the mind answering to a Direct Syllogism, proceed in this manner; First, there is conceived a Phantasme of the thing named, with that Accident or Quality thereof for which it is in the Minor Proposition called by that name which

is

is the Subject; next, the Mind has a Phantasme of the same thing with that Accident or Quality for which it hath the name that in the same Proposition is the Predicate; Thirdly, the Thought returns of the same thing as having that Accident in it, for which it is called by the Name that is in the Predicate of the Major Proposition; and lastly, remembring that all those are the Accidents of one and the same thing, it concludes that those three Names are also Names of one and the same thing; that is to say, the Conclusion is true. For example, when this Syllogisme is made, *Man is a Living Creature*, *A Living Creature is a Body*, therefore *Man is a Body*, the Mind conceives first an image of a Man *speaking* or discoursing, and remembers that that which so appears, is called *Man*; then it has the image of the same Man *moving*, and remembers that that which appeares so is called *Living Creature*; thirdly, it conceives an image of the same Man as *filling some place* or space, and remembers that what appeares so is called *Body*; and lastly, when it remembers, that that thing which was extended, and moved and spake, was one and the same thing, it concludes that the three Names *Man*, *Living Creature*, and *Body*, are Names of the same thing, and that therefore *Man is a Living Creature* is a true Proposition. From whence it is manifest, that Living Creatures that have not the use of Speech, have no Conception or Thought in the Mind, answering to a Syllogisme made of Universall Propositions; seeing it is necessary to Thinke not only of the Thing, but also by turnes to remember the diverse Names, which for diverse considerations thereof are applied to the same.

9 The rest of the Figures arise either from the Inflexion, or Inversion of the first or direct Figure; which is done by changing the Major, or Minor, or both the Propositions into converted Propositions æquipollent to them.

From whence follow three other Figures, of which, two are *Inflected*, and the third *Inverted*. The first of these three is made by the Conversion of the Major Proposition. For let the Minor, Middle and Major Terms stand in direct order, thus, *Man is a Living Creature, Is not a Stone*, which is the first or direct Figure; the Inflexion will be by converting the Major Proposition in this manner,

Man

Man is a Living Creature,

A stone is not a Living Creature; And this is the second

Figure, or the first of the Indirect Figures; in which the Conclusion will be, *Man is not a stone.* For (having shewn in the last Chap. 14 Article, that Universall Propositions converted by contradiction of the Termes are æquipollent,) both those Syllogismes conclude alike; so that if the Major be read (like Hebrew) backwards, thus, *A Living Creature is not a Stone*, it will be direct again, as it was before.. In like manner this Direct Syllogisme, *Man is not a Tree, is not a Pear-tree*, will be made Indirect by converting the Major Proposition (by contradiction of the Termes) into another æquipollent to it, thus,

Man is not a Tree,

A Pear-tree is a Tree; for the same Conclusion will follow, *Man is not a Pear-tree.*

But for the Conversion of the Direct Figure into the first Indirect Figure, the Major Terme in the Direct Figure ought to be *Negative.* For though this Direct, *Man is a Living Creature, is a Body* be made Indirect, by converting the Major Proposition, thus,

Man is a Living Creature.

Not a Body is not a Living Creature, Therefore

Every Man is a Body; yet this Conversion appeares so obscure, that this Mood is of no use at all. By the Conversion of the Major Proposition, it is manifest, that in this Figure, the Mi dle Terme is alwayes the Predicate in both the Premisses.

10 The second Indirect Figure is made by converting the Minor Proposition, so as that the Middle Term is the Subject in both. But this never concludes Uniuersally, and therefore is of no use in Philosophy. Neverthelesse I will set down an example of it; by which this Direct *Every Man is a Living Creature,*
Every Living Creature is a Body,
by Conversion of the Minor Proposition will stand thus,

Some Living Creature is a Man,

Every Living Creature is a Body, Therefore

Some Man is a Body.

For *Every Man is a Living Creature*, cannot be converted into this, *Every Living Creature is a Man*; and therefore if this Syllogisme

gisme be restored to its Direct forme, the Minor Proposition will be *Some Man is a Living Creature*, and consequently the Conclusion will be *Some Man is a Body*, seeing the Minor Terme *Man*, which is the Subject in the Conclusion, is a Particular Name.

11 The third Indirect or Inverted Figure, is made by the Conversion of both rhe Premisses. For Example, this Direct Syllogisme,

Every Man is a Living Creature,
Every Living Creature is not a Stone, Therefore
Every Man is not a Stone; being Inverted will stand thus,
Every Stone is not a Living Creature
Whatsoever is not a Living Creature, is not a Man, Therefore
Every Stone is not a Man. Which Conclusion is the Converse of the Direct Conclusion, and æquipollent to the same.

The Figures therefore of Syllogisms, if they be numbred by the diverse scituation of the Middle Terme onely, are but three; in the first whereof the Middle Term has the Middle place; in the second, the last; and in the third, the first place. But if they be numbred according to the scituation of the Termes simply, they are four; for the first may be distinguished againe into two, namely into Direct and Inverted. From whence it is evident, that the controversie among Logicians concerning the fourth Figure, is a meer λογομαχία, or contention about the Name thereof; for as for the thing it selfe, it is plain, that the scituation of the Termes (not considering the Quantity or Quality by which the Moods are distinguished) makes four differences of Syllogismes, which may be called Figures, or have any other Name at pleasure.

12 In every one of these Figures there are many Moods, which are made by varying the Premisses according to all the differences they are capable of, by Quantity and Quality; as namely, in the Direct Figure there are six Moods; in the first Indirect Figure, four; in the second, fourteen; and in the third, eighteen. But because from the Direct Figure I rejected as superfluous all Moods besides that which consists of Universal Propositions, and whose Minor Proposition is Affirmative, I doe together with it reject the Moods of the rest of the Figures which are made by Conversion of the Premisses in the Direct Figure.

13 As

13. As it was shewed before, that in Necessary Propositions a Categoricall and Hypotheticall Proposition are æquipollent; so likewise it is manifest that a Categoricall and Hypotheticall Syllogisme are æquivalent. For every Categoricall Syllogisme, as this,

Every Man is a Living Creature,
Every Living Creature is a Body, Therefore
Every Man is a Body, is of equall force with this Hypotheticall Syllogisme.

If any thing be a Man, the same is also a Living Creature,
If any thing be a Living Creature, the same is a Body, Therefore
If any thing be a Man, the same is a Body.

In like manner, this Categorical Syllogisme in an Indirect Figure

No Stone is a Living Creature,
Every Man is a Living Creature, Therefore
No Man is a Stone, or
No Stone is a Man, is æquivalent to this Hypotheticall Syllogisme.

If any thing be a Man, the same is a Living Creature,
If any thing be a a Stone, the same is not a Living Creature, Therefore
If any thing be a Stone, the same is not a Man, or
If any thing be a Man, the same is not a Stone.

And thus much seemes sufficient for the nature of Syllogismes; (for the doctrine of Moods and Figures is cleerely delivered by others that have written largely and profitably of the same.) Nor are Precepts so necessary as Practice for the attaining of true Ratiocination; and they that study the Demonstrations of Mathematicians, will sooner learn true Logick, then they that spend time in reading the Rules of Syllogizing which Logicians have made; no otherwise then little Children learn to goe, not by Precepts, but by exercising their feet. This therefore may serve for the first Pace in the way to Philosophy.

In the next place I shall speak of the Faults and Errors, into which men that reason unwarily, are apt to fall; and of their Kinds and Causes.

CHAP.

CHAP. V.

Of Erring, Falsity and Captions.

1. Erring and Falsity how they differ. Error of the Mind by it selfe without the use of Words, how it happens. 2 A sevenfold Incoherency of Names, every one of which makes allwayes a false Proposition. 3. Examples of the first manner of Incoherency. 4 Of the second. 5 Of the third. 6 Of the fourth. 7 Of the fifth. 8 Of the sixth. 9 Of the seventh. 10 Falsity of Propositions detected by resolving the Terms with Definitions continued till they come to Simple Names, or Names that are the most Generall of their kind. 11 Of the fault of a Syllogisme consisting in the Implication of the Termes which the Copula 12 Of the fault which consists in Equivocation. 13 Sophisticall Captions are oftner faulty in the matter then in the forme of Syllogismes.

1 MEn are subject to *Erre* not onely in Affirming and Denying; but also in Perception, and in silent Cogitation. In Affirming and Denying, when they call any thing by a Name, which is not the Name thereof; as if from seeing the Sun first by reflection in Water, and afterwards again directly in the Firmament, we should to both those appearances give the Name of Sunne, and say there are two Sunnes; which none but men can doe; for no other Living Creatures have the use of Names. This kind of Error onely deserves the name of *Falsity*, as arising, not from sense, nor from the Things themselves but from pronouncing rashly; for Names have their constitution, not from the Species of Things, but from the Will and Consent of Men. And hence it comes to passe, that men pronounce *Falsely* by their own negligence, in departing from such appellations of things as are agreed upon, and are not deceived neither by the Things, nor by the Sense; for they do not perceive that the thing they see is called Sunne, but they give it that Name from their owne will and agreement. Tacite Errors, or the Errors of

Senſe and Cogitation, are made, by paſſing from one Imagination to the Imagination of another different thing; or by feigning that to be Paſt, or Future, which never was, nor ever ſhall be; as when by ſeeing the Image of the Sunne in Water, we imagine the Sunne it ſelfe to be there; or by ſeeing ſwords, that there has been or ſhall be fighting, becauſe it uſes to be ſo for the moſt part; or when from Promiſes we feigne the mind of the Promiſer to be ſuch and ſuch; or laſtly, when from any Signe we vainly imagine ſomething to be ſignified, which is not. And Errors of this ſort are common to all things that have ſenſe; and yet the Deception proceeds neither from our ſenſes, nor from the Things we perceive; but from our ſelves, while we feigne ſuch things as are but meer Images, to be ſomething more then Images. But neither Things, nor Imaginations of Things can be ſaid to be Falſe, ſeeing they are truly what they are; nor doe they as Signes promiſe any thing which they do not performe; for they indeed do not promiſe at all, but we from them; nor doe the Clouds, but we from ſeeing the Clouds, ſay it ſhall rain. The beſt way therefore to free our ſelves from ſuch Errors as ariſe from naturall Signes, is firſt of all, before we begin to reaſon concerning ſuch conjecturall things, to ſuppoſe our ſelves ignorant, and then to make uſe of our Ratiocination; for theſe Errors proceed from the want of Ratiocination; whereas Errors which conſiſt in Affirmation and Negation, (that is, the Falſity of Propoſitions) proceed only from Reaſoning amiſſe. Of theſe therefore, as repugnant to Philoſophy, I will ſpeake principally.

2 Errors which happen in Reaſoning, that is, in Syllogizing, conſiſt either in the Falſity of the Premiſſes, or of the Inference. In the firſt of theſe caſes, a Syllogiſme is ſaid to be faulty in the *Matter* of it; and in the ſecond caſe, in the *Forme*. I will firſt conſider the Matter, namely how many wayes a Propoſition may be falſe; and next the Forme, and how it comes to paſs, that when the Premiſses are True, the Inference is notwithſtanding Falſe.

Seeing therefore that Propoſition onely is True, (Chap. 3. Art. 7.) in which are copulated two Names of one and the ſame thing; and that alwayes Falſe, in which Names of different things are copulated; look how many wayes Names of different things

may be copulated, and ſo many wayes a Falſe Propoſition may be made.

Now all things to which we give Names, may be reduced to theſe four kinds, namely, *Bodies*, *Accidents*, *Phantaſmes*, and *Names* themſelves; and therefore in every true Propoſition it is neceſſary that the Names copulated, be both of them Names of *Bodies*, or both Names of *Accidents*, or both Names of *Phantaſmes*, or both Names of *Names*. For Names otherwiſe copulated are incoherent, and conſtitute a Falſe Propoſition. It may happen alſo that the Name of a *Body*, of an *Accident*, or of a *Phantaſme* may be copulated with the Name of a *Speech*. So that copulated Names may be Incoherent ſeven manner of wayes.

1 If the Name of a *Body*		the Name of an *Accident*.
2 If the Name of a *Body*		the Name of a *Phantaſme*.
3 If the Name of a *Body*		the Name of a *Name*.
4 If the Name of an *Accident*		the Name of a *Phantaſme*.
5 If the Name of an *Accident*	be copulated with	the Name of a *Name*.
6 If the Name of a *Phantaſme*		the Name of a *Name*.
7 If the Name of a *Body*, of an *Accident* or of a *Phantaſme*		the Name of a *Speech*.

Of all which I will give ſome examples.

3 After the firſt of theſe wayes Propoſitions are falſe, when Abſtract Names are copulated with Concrete Names; as (in Latine and Greek) *Eſſe eſt Ens*, *Eſſentia eſt Ens*, τὸ τί ἦν εἶναι (*i.*) *Quidditas eſt Ens*, and many the like which are found in Ariſtotles *Metaphyſicks*. Alſo, the *Underſtanding worketh*, the *Underſtanding underſtandeth*, the *Sight ſeeth*, A *Body* is *Magnitude*, A *Body* is *Quantity*, A *Body* is *Extenſion*, To *Be a Man* is *a Man*, *Whiteneſſe* is a *White thing*, *&c.* which is as if one ſhould ſay The *Runner* is the *Running*, or the *Walke Walketh*. Moreover, *Eſſence* is *ſeparated*, *Subſtance* is *Abſtracted*; and others like theſe, or derived from theſe (with which common Philoſophy abounds). For ſeeing no *Subject* of an *Accident*, (that is, No *Body*) is an *Accident*; no Name of an *Accident* ought to be given to a *Body*, nor of a *Body* to an *Accident*.

4 Falſe in the ſecond Manner are ſuch Propoſitions as theſe, A

A *Ghost is a Body*, or a *Spirit*, that is, a thinne Body; *Sensible Species fly up and down in the Air*, or *are moved hither and thither*, which is proper to Bodies; also, A *Shadow is Moved*, or *is a Body*; *Light is Moved*, or *is a Body*; *Colour is the Object of Sight*, *Sound* of *Hearing*; *Space* or *Place is Extended*; and innumerable others of this kind. For seeing *Ghosts*, *sensible Species*, a *Shadow*, *Light*, *Colour*, *Sound*, *Space*, *&c.* appeare to us no lesse sleeping then waking, they cannot be things without us, but onely *Phantasmes* of the mind that imagins them; and therefore the Names of these, copulated with the Names of *Bodies*, cannot constitute a true Proposition.

5 False Propositions of the third kind, are such as these; *Genus est Ens*, *Universale est Ens*, *Ens de Ente Prædicatur*. For *Genus*, and *Universale*, and *Prædicare* are Names of *Names*, and not of *Things*. Also *Number is Infinite*, is a false Proposition; for no number can be Infinite, but onely the word *Number* is then called an Indefinite Name when there is no determined Number answering to it in the Mind.

6 To the fourth kind belong such false Propositions as these, *An Object is of such Magnitude or Figure as appeares to the Beholders*; *Colour*, *Light*, *Sound*, *are in the Object*; and the like. For the same Object appeares sometimes *greater*, sometimes *lesser*, sometimes *square*, sometimes *round* according to the diversity of the Distance and Medium; but the true Magnitude and Figure of the thing seen is allwayes one and the same; so that the magnitude and figure which appeares, is not the true magnitude and figure of the Object, nor any thing but Phantasme; and therefore in such Propositions as these, the Names of *Accidents* are copulated with the Names of *Phantasmes*.

7 Propositions are false in the fifth manner, when it is said that *The Definition is the Essence of a thing*; *Whitenesse*, or some other Accident *is the Genus*, *or Universal*. For *Definition* is not the *Essence* of any thing, but a speech signifying what we conceive of the Essence thereof; and so also not Whitenesse it selfe, but the word *Whitenesse*, is a *Genus*, or an *Universall Name*.

8 In the sixth manner they Erre, that say the *Idea of any thing is Universal*; as if there could be in the Mind an Image of *a Man* which were not the Image of *some one Man*, but of *Man simply*, which is impossible; for every *Idea* is one, and of one thing; but they are de-

deceived in this, that they put the *Name* of the thing for the *Idea* thereof.

9 They erre in the seventh manner, that make this distinction between things that have being, that some of them *exist by themselves*, others *by Accident*; Namely, because *Socrates is a Man* is a Necessary Proposition, and *Socrates is a Musician* a Contingent Proposition, therefore they say some things *exist necessarily* or *by themselves*, others *contingently* or *by Accident*; whereby, seeing *Necessary, Contingent, By it selfe, By Accident*, are not Names of *Things*, but of *Propositions*, they that say *any thing that has being, exists by Accident*, copulate the Name of a *Proposition* with the Name of a *Thing*. In the same manner also they Erre, which place some Ideas in the Understanding, others in the Fancy; as if from the Understanding of this Proposition *Man is a Living Creature*, we had one Idea or Image of a Man derived from sense to the Memory, and another to the Understanding; wherein that which deceives them is this, that they think one *Idea* should be answerable to a *Name*, another to a *Proposition*; which is false; for *Preposition* signifies onely the order of those things one after another, which we observe in the same *Idea* of Man; so that this Proposition *Man is a Living Creature*, raises but one *Idea* in us, though in that *Idea* we consider that first, for which he is called *Man*, and next that for which he is called *Living Creature*. The Falsities of Propositions in all these several manners, is to be discovered by the Definitions of the Copulated Names.

10 But when Names of *Bodies* are copulated with Names of *Bodies*, Names of *Accidents* with Names of *Accidents*, Names of *Names* with Names of *Names*, and Names of *Phantasmes* with Names of *Phantasmes*, if we neverthelesse remaine still doubtfull whether such Propositions are true; we ought then in the first place to find out the Definition of both those Names, and againe the Definitions of such Names as are in the former Definition, and so proceed by a continuall Resolution till we come to a simple Name, that is, to the most Generall or most Universall Name of that kind; and if after all this the Truth or Falsity thereof be not evident, we must search it out by Philosophy, and Ratiocination, beginning from Definitions. For every Proposition Universally true, is either a Definition, or part of a Definition, or the evidence of it depends upon Definitions.

11 That

11 That fault of a Syllogisme which lyes hid in the Forme thereof, will allwayes be found either in the implication of the Copula with one of the Termes, or in the Æquivocation of some word; and in either of these wayes there will be *four Terms*, which (as I have shewne) cannot stand in a true Syllogisme. Now the implication of the Copula with either Terme, is easily detected by reducing the Propositions to plain and cleere Prædication; as (for example) if any man should argue thus,

The Hand toucheth the Pen,
The Pen toucheth the Paper, Therefore
The Hand toucheth the Paper; the Fallacy will easily appear by reducing it, thus,

The Hand, is, touching the Pen,
The Pen, is, touching the Paper, Therefore
The Hand, is, touching the Paper; where there are manifestly these four Termes, *The Hand, Touching the Pen, The Pen,* and *Touching the Paper.* But the danger of being deceived by Sophismes of this kind, does not seem to be so great, as that I need insist longer upon them.

12 And though there may be Fallacy in Æquivocal Terms, yet in those that be manifestly such there is none at all; nor in *Metaphors*; for they professe the transferring of Names from one thing to another. Neverthelesse sometimes Æquivocalls (and those not very obscure) may deceive; as in this argumentation, *It belongs to Metaphysicks, to treat of Principles; But the first Principles of all, is, that the same thing cannot both exist and not exist at the same time; and therefore it belongs to Metaphysicks to treat whether the same thing may both exist and not exist at the same time;* where the Fallacy lies in the Æquivocation of the word *Principle*; for whereas Aristotle in the beginning of his Metaphysicks sayes, that *the treating of Principles belongs to primary science,* he understands by *Principles, Causes* of things, and certaine *Existences* which he calls *Primary*; but where he sayes *a Primary Proposition is a Principle,* by *Principle* there he means the *beginning and cause of Knowledge*, that is the understanding of words, which if any man want, he is incapable of learning.

13 But the *Captions* of Sophists and Scepticks, by which they were wont of old to deride and oppose Truth, were faulty for the

the most part, not in the Forme, but in the Matter of Syllogisme; and they deceived not others oftner then they were themselves deceived. For the force of that famous argument of Zeno against Motion, consisted in this Proposition, *Whatsoever may be divided into parts infinite in number, the same is infinite*; which he without doubt thought to be true, yet neverthelesse is false. For to be divided into *infinite parts*, is nothing else but to be divided into *as many parts as any man will*. But it is not necessary that a Line should have parts infinite in number, or be infinite, because I can divide and subdivide it as often as I please; for how many parts soever I make, yet their number is finite; but because he that sayes *Parts*, simply, without adding *how many*, does not limit any number, but leaves it to the determination of the Hearer, therefore we say commonly a line may be divided infinitely; which cannot be true in any other sense.

And thus much may suffice concerniug *Syllogisme*, which is as it were the first Pace towards Philosophy; in which I have said as much as is necessary, to teach any man from whence all true argumentation has its force. And to enlarge this Treatise with all that may be heaped together, would be as superfluous, as if one should (as I said before) give a young child Precepts for the teaching of him to goe; for the Art of Reasoning is not so well learned by Precepts as by Practice, and by the reading of those books in which the Conclusions are all made by severe Demonstration. And so I pass on to the way of Philosophy, that is, to the Method of Study.

CHAP.

CHAP. VI.

Of Method.

1 *Method and Science defined.* 2 *It is more easily known concerning Singular then Universall things, That they are; and contrarily, it is more easily knowne concerning Universall then Singular things, Why they are, or what are their Causes.* 3 *What it is Philosophers seek to know.* 4 *The first Part, by which Principles are found out is purely Analyticall.* 5 *The highest Causes, and most Universall in every kind, are knowne by themselves.* 6 *Method from Principles found out, tending to Science simply, what it is.* 7 *That Method of Civill and Naturall Science which proceeds from Sense to Principles, is Analytical; and againe that which begins at Principles, is Syntheticall.* 8 *The Method of searching out, whether any thing propounded, be Matter or Accident.* 9 *The Method of seeking whether any Accident be in this, or in that Subject.* 10 *The Method of searching after the Cause of any Effect propounded.* 11 *Words serve to Invention, as Markes; to Demonstration, as Signes.* 12 *The Method of Demonstration is Syntheticall.* 13 *Definitions onely are Primary and Universal Propositions.* 14 *The Nature and Definition of a Definition.* 15 *The Properties of a Definition.* 16 *The Nature of a Demonstration.* 17 *The Properties of a Demonstration, and Order of things to be demonstrated.* 18 *The Faults of a Demonstration.* 19 *Why the Analyticall Method of Geometricians cannot be treated of in this place.*

1 For the understanding of *Method*, it will be necessary for me to repeat the definition of Philosophy, delivered above (Chap. 1. Art. 2.) in this manner, *Philosophy is the knowledge we acquire by true Ratiocination, of Appearances, or apparent Effects, from the knowledge we have of some possible Production or Generation of the same; and of such Production as has been or may be, from the knowledge we have of the Effects.* METHOD therefore in the Study of Philosophy, *is the shortest way of finding out Effects by their known*

known Causes, or of Causes by their known Effects. But we are then said to *know* any Effect, when we know, *that there be Causes of the same,* and *in what Subiect those Causes are,* and *in what Subiect they produce that Effect,* and *in what Manner they work the same.* And this is the Science of Causes, or as they call it of the διότι. All other Science, which is called the ὅτι, is either Perception by Sense, or the Imagination, or Memory remaining after such Perception.

The first Beginnings therefore of Knowledge, are the Phantasmes of Sense and Imagination; and that there be such Phantasmes we know well enough by Nature; but to know why they be, or from what Causes they proceed, is the work of Ratiocination; which consists (as is said above, in the 1. Chap. 2. Art.) in *Composition*, and Division or *Resolution*. There is therefore no Method, by which we find out the Causes of things, but is either *Compositive*, or *Resolutive*, or *partly Compositive*, and *partly Resolutive*. And the Resolutive is commonly called *Analyticall* Method, as the Compositive is called *Syntheticall*.

2 It is common to all sorts of Method, to proceed from known things to unknown; and this is manifest from the cited Definition of Philosophy. But in Knowledge by Sense, the whole object is more known, then any part thereof; as when we see a Man, the Conception or *whole Idea* of that *Man* is first or more known, then the *particular Ideas* of his being *figurate, animate, and rationall*; that is, we first see the whole Man, and take notice of his *Being*, before we observe in him those other Particulars. And therefore in any knowledge of the ὅτι, or that any thing *is*, the beginning of our search is from the *whole Idea*; and contrarily, in our knowledge of the διότι, or of the Causes of any thing, that is, in the *Sciences*, we have more knowledge of the Causes of the Parts, then of the Whole. For the Cause of the Whole is compounded of the Causes of the Parts; but it is necessary that we know the things that are to be compounded, before we can know the whole Compound. Now by Parts, I do not here mean Parts of the thing it self, but Parts of its Nature; as, by the Parts of Man I do not understand his Head, his Shoulders, his Arms, &c. but his Figure, Quantity, Motion, Sense, Reason, and the like; which Accidents being compounded or put together, constitute the whole Nature of Man, but not the man himselfe. And this is the meaning of that

common saying, namely, that some things are more knowne to *Us*, others more known to *Nature*; for I do not thinke that they which so distinguish, mean, that something is known to Nature, which is known to no man; and therefore, by those things that are more known to Us, we are to understand, things we take notice of by our Senses, and by more known to Nature, those we acquire the knowledge of by Reason; for in this sense it is, that the *Whole*, that is, those things that have Universal Names, (which for brevities sake I call *Universall*) are more knowne to us then the *Parts*, that is, such things as have Names lesse Universal, (which I therefore call *Singular*;) and the Causes of the Parts, are more known to Nature then the Cause of the Whole; that is, Universalls then Singulars.

3 In the Study of Philosophy men search after Science either Simply, or Indefinitely; that is, to know as much as they can, without propounding to themselves any limited question; or they enquire into the Cause of some determined Appearance, or endeavour to find out the certainty of something in question, as what is the cause of *Light*, of *Heat*, of *Gravity*, of a *Figure* propounded, and the like; or in what *Subiect* any propounded *Accident* is inhærent; or what may conduce most to the *generation* of some propounded *Effect* from many *Accidents*; or in what manner particular Causes ought to be compounded for the production of some certaine Effect. Now according to this variety of things in question, sometimes the *Analyticall Method* is to be used, and sometimes the *Syntheticall*.

4 But to those that search after Science indefinitely, which consists in the knowledge of the Causes of all things, as far forth as it may be attained, and the Causes of Singular things are compounded of the Causes of Universall or Simple things, it is necessary that they know the Causes of Universall things, or of such Accidents as are common to all Bodies, that is, to all Matter, before they can know the Causes of Singular things, that is, of those Accidents by which one thing is distinguished from another. And againe they must know what those Universall things are, before they can know their Causes. Moreover, seeing Universall things are contained in the Nature of Singular things, the knowledge of them is to be acquired by Reason, that is, by Re-

solution. For example, if there be propounded a Conception or *Idea* of some Singular thing, as of a *Square*, this Square is to be resolved into a *Plain, terminated with a certaine number of equall and straight lines and right angles.* For by this Resolution we have these things Universall or agreeable to all Matter, namely, *Line, Plain,* (which containes *Superficies,*) *Terminated, Angle, Straightness, Rectitude and Equality*; and if we can find out the Causes of these, we may compound them all together into the Cause of a Square. Againe, if any man propound to himselfe the Conception of *Gold*, he may by Resolving come to the Ideas of *Solid, Visible, Heavy,* (that is, tending to the Center of the Earth, or downwards,) and many other more Universall then Gold it selfe; and these he may Resolve againe, till he come to such things as are most Universall. And in this manner by Resolving continually, we may come to know what those things are, whose Causes being first known severally, and afterwards compounded, bring us to the Knowledge of Singular things. I conclude therefore, that the Method of attaining to the Universall Knowledge of Things, is purely *Analyticall.*

5 But the Causes of Universall things (of those at least that have any Cause) are manifest of themselues, or (as they say commonly) known to Nature; so that they need no Method at all; for they have all but one Universall Cause, which is Motion. For the variety of all Figures arises out of the variety of those Motions by which they are made; and Motion cannot be understood to have any other Cause besides Motion; nor has the Variety of those things we perceive by Sense, as of *Colours, Sounds, Savours, &c.* any other Cause then Motion, residing partly in the Objects that work upon our Senses, and partly in our selves, in such manner, as that it is manifestly some kind of Motion, though we cannot without Ratiocination come to know what kind. For though many cannot understand till it be in some sort demonstrated to them, that all Mutation consists in Motion; yet this happens not from any obscurity in the thing it selfe, (for it is not intelligible that any thing can depart either from Rest, or from the Motion it has, except by Motion;) but either by having their Naturall Discourse corrupted with former Opinions received from their Masters, or else for this, that they do not at all bend their mind to the enquiring out of Truth,

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6 By the Knowledge therefore of Universalls, and of their Causes (which are the first Principles by which we know the Διότι of things,) we have in the first place their Definitions, (which are nothing but the explication of our Simple Conceptions.) For example, he that has a true Conception of *Place*, cannot be ignorant of this Definition, *Place is that space which is possessed or filled adequately by some Body*; and so, he that conceives *Motion* aright, cannot but know, that *Motion is the privation of one Place, and the acquisition of another*. In the next place, we have their Generations or Descriptions; as, (for example,) that *a Line is made by the Motion of a Point, Superficies by the Motion of a Line*, and *one Motion by another Motion, &c.* It remains, that we enquire, what Motion begets such and such Effects; as, what Motion makes a Straight line, and what a Circular; what Motion thrusts, what drawes, and by what way; what makes a thing which is seen or heard, to be seen or heard sometimes in one manner, sometimes in another. Now the Method of this kind of Enquiry, is *Compositive*. For first we are to observe what Effect a Body moved produceth, when we consider nothing in it besides its Motion; and we see presently that this makes a Line, or length; next, what the Motion of a long Body produces, which we find to be Superficies; and so forwards, till we see what the Effects of Simple Motion are; and then in like manner, we are to observe what proceeds from the Addition, Multiplication, Substraction and Division of these Motions, and what Effects, what Figures, and what Properties they produce; from which kind of Contemplation sprung that part of Philosophy which is called *Geometry*.

From this consideration of what is produced by Simple Motion, we are to passe to the consideration of what Effects one Body moved worketh upon another; and because there may be Motion in all the severall parts of a Body, yet so as that the whole Body remain still in the same place, we must enquire, first, what Motion causeth such and such Motion in the whole, that is, when one Body invades another Body which is either at Rest, or in Motion, what way, and with what swiftnesse the invaded Body shall move; and again, what Motion this second Body will generate in a third, and so forwards. From which Contemplation shall be drawn that part of Philosophy which treats of Motion.

In the Third place we must proceed to the Enquiry of such Effects, as are made by the Motion of the Parts of any Body, as, how it comes to passe, that things when they are the same, yet seeme not to be the same, but changed. And here the things we search after are sensible Qualities, such as *Light*, *Colour*, *Transparency*, *opacity*, *Sound*, *Odour*, *Savour*, *Heat*, *Cold* and the like; which because they cannot be known till we know the Causes of Sense it selfe, therefore the consideration of the Causes of *Seeing*, *Hearing*, *Smelling*, *Tasting* and *Touching* belongs to this third place; and all those qualities and Changes above mentioned are to be referred to the fourth place; which two considerations comprehend that part of Philosophy which is called *Physiques*. And in these four parts is contained whatsoever in Naturall Philosophy may be explicated by Demonstration properly so called. For if a Cause were to be rendred of Natural Appearances in special, as, what are the Motions and Influences of the heavenly Bodies, and of their parts, the reason hereof must either be drawn from the parts of the Sciences above mentioned, or no reason at all will be given, but all left to uncertaine conjecture.

After *Physiques* we must come to *Morall Philosophy*, in which we are to consider the Motions of the Mind, namely *Appetite*, *Aversion*, *Love*, *Benevolence*, *Hope*, *Fear*, *Anger*, *Emulation*, *Envy*, *&c.* what Causes they have, and of what they be Causes. And the reason why these are to be considered after *Physiques*, is, that they have their Causes in Sense and Imagination, which are the Subject of *Physicall* Contemplation. Also the reason why all these Things are to be searched after in the order abovesaid, is, that Physiques cannot be understood, except we know first what Motions are in the smallest parts of Bodies; nor such Motion of Parts, till we know what it is that makes another Body move; nor this, till we know what Simple Motion will effect. And because all Appearance of things to sense is determined, and made to be of such and such Quality and Quantity by Compounded Motions, every one of which has a certaine degree of Velocity, and a certaine and determined way; therefore in the first place we are to search out the wayes of Motion simply, (in which Geometry consists;) next the wayes of such generated Motions as are manifest; and lastly the wayes of internal and invisible Motions, (which is the

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Enquiry of Naturall Philosophers.) And therefore they that study Naturall Philosophy, study in vaine, except they begin at Geometry; and such Writers or Disputers thereof, as are ignorant of Geometry, do but make their Readers and Hearers lose their time.

7 *Civill* and *Morall Philosophy* doe not so adhere to one another, but that they may be severed. For the Causes of the Motions of the Mind are known not onely by Ratiocination, but also by the Experience of every man that takes the paines to observe those Motions within himselfe. And therefore not only they that have attained the knowledge of the Passions and Perturbations of the Mind, by the *Syntheticall Method* and from the very first Principles of Philosophy, may by proceeding in the same way come to the Causes and Necessity of constituting Common-wealths, and to get the Knowledge of what is Naturall Right, and what are Civill Duties; and in every kind of Goverment, what are the Rights of the Commonwealth, and all other Knowledge appertaining to Civill Philosophy, for this reason, that the Principles of the Politiques consist in the Knowledge of the Motions of the Mind, and the Knowledge of these Motions from the knowledge of Sense and Imagination; but even they also that have not learned the first part of Philosophy, namely *Geometry* and *Physiques*, may notwithstanding attain the Principles of Civill Philosophy, by the *Analyticall Method*. For if a Question be propounded, as *Whether such an Action be Just or Uniust*; if that *Uniust* be resolved into *Fact against Law*, and that notion of *Law* into the *Command* of him or them that have *Coercive Power*; and that *Power* be derived from the *Wills* of Men that constitute such Power to the end they may live in Peace, they may at last come to this, that the Appetites of Men and the Passions of their Minds are such, that unlesse they be restrained by some Power, they will alwayes be making warre upon one another; which may be known to be so by any mans experience, that will but examine his owne Mind. And therefore from hence he may proceed by Compounding, to the determination of the Justice or Injustice of any propounded Action. So that it is manifest by what has been said, that the Method of Philosophy to such as seek Science simply, without propounding to themselves the Solution of any Particular question, is partly *Analyticall*,

nalyticall, and partly Syntheticall; namely, that which proceeds from Sense to the invention of Principles, *Analyticall*; and the rest *Syntheticall*.

8 To those that seek the Cause of some certaine and propounded Appearance or Effect, it happens sometimes, that they know not whether the thing whose Cause is sought after, be Matter or Body, or some Accident of a Body. For though in *Geometry*, when the Cause is sought of Magnitude, or Proportion, or Figure, it be certainly known that these things, namely Magnitude, Proportion and Figure are Accidents; yet in Naturall Philosophy, where all questions are concerning the Causes of the Phantasmes of sensible things, it is not so easie to discern between the things themselves from which those Phantasmes proceed, and the Appearances of those things to the sense; which have deceived many, especially when the Phantasmes have been made by Light. For Example, a Man that looks upon the Sunne, has a certaine shining *Idea* of the Magnitude of about a foot over; and this he calls the Sunne, though he know the Sunne to be truly a great deale bigger; and in like manner, the Phantasme of the same thing appears sometimes round, by being seen a farre off, and sometimes square, by being neerer. Whereupon it may well be doubted whether that Phantasme be Matter, or some Body Naturall, or onely some Accident of a Body; in the examination of which doubt we may use this Method. The Properties of Matter and Accidents already found out by Us by the *Syntheticall Method* from their Definitions, are to be compared with the *Idea* we have before us; and if it agree with the Properties of Matter or Body, then it is a Body; otherwise it is an Accident. Seeing therefore Matter cannot by any endeavour of ours be either Made or Destroyed, or Encreased, or Diminished, or Moved out of its place; whereas that *Idea* Appeares, Vanishes, is Encreased, and Diminished, and Moved hither and thither at pleasure; we may certainly conclude that it is not a Body, but an Accident onely. And this Method is *Syntheticall*.

9 But if there be a doubt made concerning the Subject of any known Accident, (for this may be doubted sometimes, as in the præcedent example doubt may be made in what Subject that Splendor and apparent Magnitude of the Sunne is) then our en-

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quiry must proceed in this manner. First, Matter in Generall must be divided into parts, as into Object, Medium, and the Sentient it selfe, or such other parts as seem most conformable to the thing propounded. Next, these parts are severally to be examined how they agree with the Definition of the Subject; and such of them as are not capable of that Accident are to be rejected. For example, If by any true Ratiocination the Sunne be found to be greater then its apparent Magnitude, then that Magnitude is not in the Sunne; If the Sunne be in one determined straight line, and one determined distance, and the Magnitude and Splendor be seen in more lines and distances then one, as it is in Reflection or Refraction, then neither that Splendor nor apparent Magnitude are in the Sun it self, and therefore the Body of the Sun cannot be the Subject of that Splendor and Magnitude. And for the same reasons the Aire and other parts will be rejected, till at last nothing remain which can be the Subject of that Splendor and Magnitude but the Sentient it selfe. And this Method, in regard the Subject is divided into parts is *Analitycall*; and in regard the Properties both of the Subject and Accident are compared with the Accident concerning whose Subject the enquiry is made, it is *Syntheticall*.

10 But when we seek after the Cause of any propounded Effect; we must in the first place get into our Mind an exact Notion or *Idea* of that which we call Cause, namely, that *A Cause is the Summe or Aggregate of all such Accidents both in the Agents and the Patient, as concurre to the producing of the Effect propounded; all which existing together, it cannot be understood but that the Effect existeth with them; or that it cannot possibly exist if any one of them be absent.* This being known, in the next place we must examine singly every Accident that accompanies or præcedes the Effect, as farre forth as it seemes to conduce in any manner to the production of the same, and see whether the propounded Effect may be conceived to exist, without the existence of any of those Accidents; and by this meanes separate such Accidents as do not concurre, from such as concurre to produce the said Effect; which being done, we are to put together the concurring Accidents, and consider whether we can possibly conceive that when these are all present, the Effect propounded will not follow; and if it be evident that the

the Effect will follow, then that Aggregate of Accidents is the entire Cause, otherwise not; but we must still search out and put together other Accidents. For example, if the Cause of Light be propounded to be sought out; first, we examine things without us, and find that whensoever Light appeares, there is some principall Object, as it were the fountaine of Light, without which we cannot have any perception of Light; and therefore the concurrence of that Object is necessary to the generation of Light. Next we consider the Medium, and find that unlesse it be disposed in a certaine manner, namely, that it be *transparent*, though the Object remain the same, yet the Effect will not follow; and therefore the concurrence of Transparency is also necessary to the generation of Light. Thirdly, we observe our own Body, and find that by the indisposition of the Eyes, the Brain, the Nerves, and the Heart, that is, by Obstructions, Stupidity and Debility we are deprived of Light, so that a fitting disposition of the Organs to receive impressions from without is likewise a necessary part of the Cause of Light. Again, of all the Accidents inhærent in the Object, there is none that can conduce to the effecting of Light, but onely Action, (or a certain Motion,) which cannot be conceived to be wanting, whensoever the Effect is present; for, that any thing may shine, it is not requisite that it be of such or such Magnitude or Figure, or that the whole Body of it be moved out of the place it is in, (unlesse it may perhaps be said, that in the Sun or other Body, that which causeth Light is the light it hath in it selfe; which yet is but a trifling exception, seeing nothing is meant thereby but the Cause of Light; as if any man should say that the Cause of Light is that in the Sunne which produceth it;) it remaines therefore that the Action by which Light is generated, is Motion only in the parts of the Object. Which being understood, we may easily conceive what it is the Medium contributes, namely, the continuation of that Motion to the Eye; and lastly what the Eye and the rest of the Organs of the Sentient contribute, namely, the continuation of the same Motion to the last Organ of Sense, the Heart. And in this manner the Cause of Light may be made up of Motion continued from the Original of the same Motion, to the Original of Vitall Motion, Light being nothing but the alteration of Vitall Motion, made by the impression upon it of Motion continued from the Ob-

ject. But I give this onely for an example, for I shall speak more at large of Light, and the generation of it in its proper place. In the mean time it is manifest, that in the searching out of Causes, there is need partly of the *Analyticall*, and partly of the *Syntheticall Method*; of the *Analyticall*, to conceive how circumstances conduce severally to the production of Effects; and of the *Syntheticall*, for the adding together and compounding of what they can effect singly by themselves. And thus much may serve for the Method of Invention. It remaines that I speake of the Method of Teaching, that is, of Demonstration, and of the Meanes by which we demonstrate.

11 In the Method of Invention the use of words consists in this, that they may serve for *Marks*, by which, whatsoever we have found out may be recalled to memory; for without this all our Inventions perish, nor will it be possible for us to go on from Principles beyond a Syllogisme or two, by reason of the weaknesse of Memory. For example, if any man by considering a Triangle set before him, should find that all its angles together taken are equall to two right angles, and that by thinking of the same tacitely, without any use of words either understood or expressed; and it should happen afterwards that another Triangle unlike the former, or the same in different scituation should be offered to his consideration, he would not know readily whether the same property were in this last or no; but would be forced as often as a different Triangle were brought before him (and the difference of Triangles is infinite) to begin his contemplation anew; which he would have no need to do if he had the use of Names; for every Universal Name denotes the conceptions we have of infinite Singular things.. Neverthelesse as I said above, they serve as *Markes* for the helpe of our Memory, whereby we register to our selves our own Inventions; but not as *Signes* by which we declare the same to others; so that a man may be a Philosopher alone by himselfe without any Master; Adam had this capacity; But to Teach, that is to Demonstrate, supposes two at the least, and Syllogisticall Speech.

12 And seeing Teaching is nothing but leading the Mind of him we teach, to the knowledge of our Inventions, in that Track by which we attained the same with our own Mind; therefore the

the ſame Method that ſerved for our Invention, will ſerve alſo for Demonſtration to others; ſaving that we omit the firſt part of Method which proceeded from the Senſe of Things to Univerſal Principles; which becauſe they are Principles, cannot be demonſtrated; and ſeeing they are known by Nature (as was ſaid above in the 5th. Article) they need no Demonſtration, though they need Explication. The whole Method therefore of Demonſtration is *Syntheticall*, conſiſting of that order of Speech, which begins from Primary or moſt Univerſall Propoſitions, which are manifeſt of themſelves, and proceeds by a perpetuall compoſition of Propoſitions into Syllogiſmes, till at laſt the Learner underſtand the truth of the Concluſion ſought after.

13 Now ſuch Principles are nothing but Definitions; whereof there are two ſorts; one, of Names that ſignifie ſuch things as have ſome conceiveable Cauſe, and another of ſuch Names as ſignifie things of which we can conceive no Cauſe at all. Names of the former kind are, *Body* or *Matter*, *Quantity* or *Extenſion*, *Motion*, and whatſoever is common to all Matter. Of the ſecond kind are, *ſuch a Body*, *ſuch and ſo great Motion*, *ſo great Magnitude*, *ſuch Figure*, and whatſoever we can diſtinguiſh one Body from another by. And Names of the former kind are well enough defined, when by Speech as ſhort as may be, we raiſe in the Mind of the Hearer perfect and cleer Ideas or Conceptions of the Things named, as when we Define Motion to be *the leaving of one place, and the acquiring of another continually*; for though no Thing Moved, nor any Cauſe of Motion be in that Definition, yet at the hearing of that Speech, there will come into the Mind of the Hearer an *Idea* of Motion cleer enough. But Definitions of things which may be underſtood to have ſome Cauſe, muſt conſiſt of ſuch Names as expreſſe the Cauſe or Manner of their Generation, as when we Define a Circle to be a Figure made by the circumduction of a ſtraight line in a plaine, &c. Beſides Definitions, there is no other Propoſition that ought to be called Primary, or (according to ſevere truth) be received into the number of Principles. For thoſe *Axiomes* of *Euclide*, ſeeing they may be demonſtrated are no Principles of Demonſtration, though they have by the conſent of all Men gotten the authority of Principles, becauſe they need not be Demonſtrated. Alſo thoſe *Petitions* or *Poſtulata* (as they call them) though they be Principles

yet they are not Principles of Demonstration, but of Construction onely; that is, not of Science, but of Power; or (which is all one) not of *Theoremes*, which are Speculations, but of *Problemes*, which belong to Practice, or the doing of something. But as for those common received Opinions, *Nature abhorres Vanity*, *Nature doth nothing in Vaine*, and the like, which are neither evident in themselves, nor at all to be demonstrated, and which are oftner false then true, they are much lesse to be ackowledged for Principles.

To returne therefore to Definitions, The reason why I say that the Cause and Generation of such things as have any Cause or Generation, ought to enter into their Definitions, is this. The End of Science, is the Demonstration of the Causes and Generations of Things; which if they be not in the Definitions, they cannot be found in the Conclusion of the first Syllogisme that is made from those Definitions; and if they be not in the first Conclusion, they will not be found in any further Conclusion deduced from that; and therefore by proceeding in this manner we shall never come to Science; which is against the scope and intention of Demonstration.

14 Now seeing Definitions (as I have said) are Principles or Primary Propositions, they are therefore Speeches; and seeing they are used for the raising of an *Idea* of some Thing in the mind of the Learner, whensoever that Thing has a Name, the Definition of it can be nothing but the Explication of that Name by Speech; and if that Name be given it for some compounded Conception, the Definition is nothing but a Resolution of that Name into its most Universall parts. As when we define Man, saying, *Man is a Body Animated, Sentient, Rationall*, those Names *Body Animated, &c.* are parts of that whole Name *Man*; so that Definitions of this kind alwayes consist of *Genus* and *Difference*; the former Names being all till the last, *Generall*; and the last of all, *Difference*. But if any Name be the most Universall in its kind, then the Definition of it cannot consist of *Genus* and *Difference*, but is to be made by such circumlocution as best explicateth the force of that Name. Again, it is possible, and happens often that the *Genus* and *Difference* are put together, and yet make no Definition; as these Words *a Straight Line* containe both

both the *Genus* and *Difference*; but are not a Definition, unlesse we should thinke a Straight Line may be thus defined, *A Straight Line is a Straight Line*; and yet if there were added another Name consisting of different Words, but signifying the same thing which these signifie, then these might be the Definition of that Name. From what has been said it may be understood how a Definition ought to be defined, namely, *That it is a Proposition, whose Prædicate Resolves the Subiect, when it may, and when it may not, it exemplifies the same.*

15 The Properties of a Definition are,

First, that it takes away Æquivocation, as also all that multitude of Distinctions, which are used by such as think they may learn Philosophy by Disputation. For the Nature of a Definition is to define, that is, to determine the signification of the defined Name, and to pare from it all other Signification besides what is contained in the Definition it selfe; and therefore one Definition does as much, as all the Distinctions (how many soever) that can be used about the Name defined.

Secondly, That it gives an Universall Notion of the thing defined, representing a certaine Universall Picture thereof, not to the Eye, but to the Mind. For as when one paints a Man, he paints the image of some Man; so he that defines the Name *Man*, makes a Representation of some Man to the mind.

Thirdly, That it is not necessary to dispute whether Definitions are to be admitted or no. For when a Master is instructing his Scholar, if the Scholar understand all the parts of the thing defined, which are Resolved in the Definition, and yet will not admit of the Definition, there needs no further Controversie betwixt them, it being all one as if he refused to be taught. But if he understand nothing, then certainely the Definition is faulty; for the nature of a Definition consists in this, that it exhibit a cleare Idea of the thing defined; and Principles are either known by themselves, or else they are not Principles.

Fourthly, That in Philosophy, Definitions are before defined Names. For in teaching Philosophy, the first beginning is from Definitions, and all progression in the same till we come to the Knowledge of the thing compounded, is Compositive. Seeing therefore Definition is the explication of a Compounded Name by

by Resolution, and the progression is from the parts to the compound, Definitions must be understood before Compounded Names; nay when the Names of the parts of any Speech be explicated, is it not necessary that the Definition should be a Name Compounded of them. For example, when these Names, *Æquilaterall*, *Quadrilaterall*, *Right-angled*, are sufficiently understood, it is not necessary in Geometry that there should be at all such a Name as *Square*; for defined Names are received in Philosophy for brevities sake onely.

Fiftly, That Compounded Names which are defined one way in some one part of Philosophy, may in another part of the same be otherwise defined; as a *Parabola* and an *Hyperbole* have one Definition in Geometry, and another in Rhetorique; for Definitions are instituted and serve for the understanding of the Doctrine which is treated of. And therefore as in one part of Philosophy, a Definition may have in it some one fit Name for the more briefe explanation of some proposition in Geometry; so it may have the same liberty in other parts of Philosophy; for the use of Names is particular (even where many agree to the setling of them) and arbitrary.

Sixtly, That no Name can be defined by any one Word; because no one Word is sufficient for the Resolving of one or more words.

Seventhly, That a defined Name ought not to be repeated in the Definition. For a defined Name, is the whole Compound, and a Definition is the Resolution of that Compound into parts; but no Totall can be part of it selfe.

16 Any two Definitions that may be compounded into a Syllogisme, produce a Conclusion; which because it is derived from Principles, that is, from Definitions, is said to be *Demonstrated*; and the Derivation or Composition it selfe is called a *Demonstration*. In like manner, if a Syllogisme be made of two Propositions, whereof one is a Definition, the other a Demonstrated Conclusion, or neither of them is a Definition, but both formerly demonstrated, that Syllogisme is also called a Demonstration, and so successively. The Definition therefore of a Demonstration is this, *A* DEMONSTRATION *is a Syllogism or Series of Syllogisms derived and continued from the Definitions of Names, to the last Conclusion.*

And

And from hence it may be understood, that all true Ratiocination, which taketh its beginning from true Principles, produceth Science, and is true Demonstration. For as for the Originall of the Name, although that which the Greeks called ἀπόδειξις, and the Latines *Demonstratio* was understood by them for that sort onely of Ratiocination, in which by the describing of certaine Lines and Figures, they placed the thing they were to prove, as it were before mens Eyes, which is properly ἀποδεικνύειν, or to *shew* by the Figure; yet they seem to have done it for this reason, that unlesse it were in Geometry (in which only there is place for such Figures) there was no Ratiocination certaine, and ending in Science, their Doctrines concerning all other things being nothing but Controversie and Clamour; which neverthelesse hapned not because the Truth to which they pretended could not be made evident without Figures, but because they wanted true Principles, from which they might derive their Ratiocination; and therefore there is no reason but that if true Definitions were præmised in all sorts of Doctrines, the Demonstrations also would be true.

17 It is proper to Methodical Demonstration,

First, That there be a true Succession of one Reason to another, according to the Rules of Syllogizing delivered above.

Secondly, That the Præmisses of all Syllogismes be demonstrated from the first Definitions.

Thirdly, That after Definitions, he that Teaches or Demonstrates any thing, proceed in the same Method by which he found it out; namely, that in the first place those things be demonstrated which immediately succeed to Universal Definitions (in which is contained that part of Philosophy which is called *Philosophia Prima.*) Next, those things which may be demonstrated by Simple Motion (in which Geometry consists.) After Geometry, such things as may be taught or shewed by manifest Action, that is, by Thrusting from, or Pulling towards. And after these, the Motion or Mutation of the invisible parts of Things, and the Doctrine of Sense & Imagination & of the internal Passions, especially those of Men, in which are comprehended the Grounds of Civil Duties, or Civil Philosophy; which takes up the last place. And that this Method ought to be kept in all sorts of Philosophy, is evident from hence, that such things as I have said are to be taught last,

last cannot be demonstrated, till such as are propounded to be first treated of, be fully understood. Of which Method no other Example can be given, but that Treatise of the Elements of Philosophy, which I shall begin in the next Chapter, and continue to the end of the worke.

18 Besides those *Paralogismes*, whose fault lies either in the Falsity of the Præmisses, or the want of true Composition, of which I have spoken in the præcedent Chapter, there are two more which are frequent in Demonstration; one whereof is commonly called *Petitio Principii*; the other is the supposing of a *False Cause*; and these do not onely deceive Unskilfull Learners but sometimes Masters themselves, by making them take that for well demonstrated which is not demonstrated at all. *Petitio Principii*, is when the Conclusion to be proved, is disguised in other Words, and put for the Definition or Principle from whence it is to be demonstrated; and thus by putting for the Cause of the Thing sought, either the Thing it selfe or some Effect of it, they make a Circle in their Demonstration. As for example, He that would Demonstrate that the Earth stands still in the Center of the World, and should suppose the Earths Gravity to be the Cause thereof, and define Gravity to be a quality by which every heavy Body tends towards the Center of the World, would lose his labour; for the question is, What is the Cause of that quality in the Earth; and therefore he that supposes Gravity to be the Cause, puts the Thing it selfe for its own Cause.

Of a *False Cause* I find this example in a certaine Treatise where the thing to be demonstrated is the Motion of the Earth. He begins therefore with this, that seeing the Earth and the Sun are not alwayes in the same scituation, it must needs be that one of them be locally moved; which is true; next he affirms that the Vapours which the Sun raises from the Earth and Sea are by reason of this Motion necessarily moved; which also is true; from whence he infers the Winds are made, and this may passe for granted; and by these Winds he sayes the Waters of the Sea are moved, and by their Motion the bottome of the Sea, as if it were beaten forwards, moves round; and let this also be granted; wherefore he concludes, the Earth is moved; which is neverthelesse is a Paralogisme. For if that wind were the

the Cause why the Earth was from the beginning moved round, and the Motion either of the Sunne or the Earth were the Cause of that Wind, then the Motion of the Sunne or the Earth was before the Wind it self; and if the Earth were Moved before the Wind was made, then the Wind could not be the cause of the Earths revolution; but if the Sunne were Moved, and the Earth stand still, then it is manifest the Earth might remain Unmoved notwithstanding that Wind; and therefore that motion was not made by the Cause which he alledgeth. But Paralogismes of this kind are very frequent among the Writers of *Physiques*, though none can be more elaborate then this in the Example given.

19 It may to some men seem pertinent to treat in this place of that Art of the Geometricians, which they call *Logistica*, that is, the Art, by which, from supposing the thing in question to be true, they proceed by Ratiocination, till either they come to something knowne, by which they may demonstrate the truth of the thing sought for; or to something which is impossible, from whence they collect that to be false which they supposed true. But this Art cannot be explicated here, for this reason, that the Method of it can neither be practised, nor understood unlesse by such as are well versed in Geometry; and among Geometricians themselves, they that have most Theoremes in readiness, are the most ready in the use of this *Logistica*; so that indeed it is not a distinct thing from Geometry it selfe; for there are in the Method of it three parts; the first whereof consists in the finding out of Equality betwixt known and unknown things, which they call Equation; and this Equation cannot be found out but by such as know perfectly the Nature, Properties and Transpositions of Proportion, as also the Addition, Substraction, Multiplication, and Division of Lines and Superficies, and the Extraction of Roots; which are the parts of no meane Geometrician. The Second is, when an Equation is found, to be able to judge whether the Truth or Falsity of the Question may be deduced from it or no; which yet requires greater Knowledge. And the third is, when such an Equation is found, as is fit for the solution of the Question, to know how to Resolve the same in such manner, that the Truth or Falsity may thereby

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by manifestly appeare; which in hard questions cannot be done without the Knowledge of the Nature of Crooked-lined Figures; but he that understands readily the Nature and Properties of these, is a Compleat Geometrician. It happens besides, that for the finding out of Equations there is no certaine Method, but he is best able to do it, that has the best Naturall Wit.

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THE FIRST GROUNDS OF PHILOSOPHY.

Chap. VII.
Of Place and Time.

1 Things that have no existence, may neverthelesse be understood and computed. 2 What is space. 3 Time. 4 Part. 5 Division. 6 One 7 Number. 8 Composition. 9 The Whole. 10 Spaces and Times Contiguous, and Continuall. 11 Beginning, End, Way, Finite, Infinite. 12 What is Infinite in Power. Nothing Infinite can be truly said to be either Whole, or One; Nor Infinite Spaces or Times, Many. 13 Division proceeds not to the Least.

1 IN the Teaching of Naturall Philosophy, I cannot begin better (as I have already shewn) then from *Privation*; that is, from feigning the World to be annihilated. But if such annihilation of all things be supposed, it may perhaps be asked, what would remain for any Man (whom onely I except from this Universal annihilation of things) to consider as the Subject of Philosophy, or at all to reason upon; or what to give Names unto for Ratiocinations sake.

I say therefore there would remain to that Man Ideas of the World, and of all such Bodies as he had, before their annihilation, seen with his eies, or perceived by any other Sense; that is to say, the Memory and Imagination of Magnitudes, Motions, Sounds, Colours, &c. as also of their order & parts. All wch things, though they be nothing but Ideas & Phantasms, hapning internally to him that imagineth; yet they will appear as if they were externall, and not at all depending upon any power of the Mind. And these are the

the things to which he would give Names, and substract them from, and compound them with one another. For seeing that after the destruction of all other things, I suppose Man still remaining, and namely that he thinkes, imagines, and remembers, there can be nothing for him to thinke of but what is Past; Nay, if we do but observe diligently what it is we doe when we consider and reason, we shall find, that though all things be still remaining in the world, yet we compute nothing but our own Phantasmes. For when we calculate the magnitude and motions of Heaven or Earth, we doe not ascend into Heaven that we may divide it into parts, or measure the motions thereof, but we doe it sitting still in our Closets or in the Darke. Now things may be considered, that is, be brought into Account, either as internal Accidents of our Mind, in which manner we consider them when the question is about some Faculty of the Mind; or as Species of external things, not as really existing, but appearing onely to exist, or to have a Being without Us. And in this manner we are now to consider them.

2 If therefore we remember, or have a Phantasme of any thing that was in the world before the supposed annihilation of the same; and consider, not that the thing was such or such, but onely that it had a Being without the Mind, we have presently a Conception of that we call *Space*: an Imaginary Space indeed, because a meere Phantasme, yet that very thing which all men call so. For no man calls it Space for being already filled, but because it may be filled; nor does any man think Bodies carry their Places away with them, but that the same Space contains sometimes one, sometimes another Body; which could not be if Space should alwayes accompany the Body which is once in it. And this is of it selfe so manifest, that I should not thinke it needed any explaining at all, but that I finde Space to be falsely defined by certaine Philosophers, who inferre from thence, One, that the world is Infinite; for taking *Space* to be the Extension of Bodies, and thinking Extension may encrease continually, he inferres that Bodies may be infinitely Extended; and Another from the same Definition concludes rashly, that it is impossible even to God himselfe to create more Worlds then one; for if another World were to be created, he sayes, that seeing there is nothing without this world, and therefore (according to his

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Definition) no Space , that new world must be placed in nothing ; but in nothing nothing can be placed ; which he affirms onely, without shewing any reason for the same ; whereas the contrary is the truth : for more cannot be put into a Place allready filled, so much is Empty Space fitter then that which is Full for the receiving of new Bodies. Having therefore spoken thus much for these mens sakes, and for theirs that assent to them, I return to my purpose, and define *Space* thus, SPACE *is the Phantasme of a Thing existing without the Mind simply*; that is to say, that Phantasme, in which we consider no other Accident, but onely that it appears without us.

3 As a Body leaves a Phantasme of its Magnitude in the mind, so also a Moved Body leaves a Phantasme of its Motion, namely an Idea of that Body passing out of one Space into another by continuall succession. And this Idea or Phantasme, is that which (without receding much from the common opinion, or from *Aristotles* Definition) I call *Time*. For seeing all men confesse a Yeare to be Time, and yet do not think a Year to be the Accident or Affection of any Body, they must needs confesse it to be, not in the things without Us, but only in the Thought of the Mind. So when they speake of the Times of their Predecessors, they do not think after their Predecessors are gone, that their Times can be any where else then in the Memory of those that remember them. And as for those that say, Dayes, Years and Moneths are the Motions of the Sunne and Moon, seeing it is all one to say, Motion *Past* and Motion *Destroyed*, and that *Future* Motion is the same with Motion which *Is not yet begun*, they say, that which they do not meane, that there neither is, nor has been, nor shall be any Time: for of whatsoever it may be said, *It has been* or *It shall be*, of the same also it might have been said heretofore, or may be said hereafter, *It is.* What then can Dayes, Moneths and Yeares be, but the Names of such Computations made in our Mind? *Time* therefore is a Phantasme, but a Phantasme of Motion, for if we would know by what Moments Time passes away, we make use of some Motion or other, as of the Sun, of a Clock, of the sand in an Hourglasse ; or we mark some Line upon which we imagine something to be Moved, there being no other means by which we can take notice of any Time at all. And yet when I say *Time* is a Phantasme

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of Motion, I doe not ſay this is ſufficient to define it by; for this word *Time* comprehends the notion of *Former* and *Later*, or of *Succeſſion* in the motion of a Body, in as much as it is firſt *Here* then *There*. Wherefore a compleat Definition of *Time* is ſuch as this, TIME *is the Phantaſme of Before and After in Motion*; which agrees with this Definition of *Ariſtotle*, *Time is the Number of Motion according to Former and Later*; for that Numbring is an act of the Mind; and therefore it is all one to ſay, *Time is the Number of Motion according to Former and Later*; and *Time is a Phantaſme of Motion Numbred*. But that other Definition, *Time is the Meaſure of Motion*, is not ſo exact; for we meaſure Time by Motion and not Motion by Time.

4 One Space is called *Part* of another Space, and one Time *Part* of another Time, when this containes that and ſomething beſides. From whence it may be collected, that nothing can rightly be called a PART, but that which is compared with ſomething that contains it.

5 And therefore to *make parts*, or to *Part* or DIVIDE Space or *Time*, is nothing elſe but to conſider One and Another within the ſame; ſo that if any Man *Divide* Space or Time, the diverſe Conceptions he has are more by one, then the Parts he makes; for his firſt Conception is of that which is to be divided, then of ſome Part of it, and again of ſome other Part of it, and ſo forwards as long as he goes on in Dividing.

But it is to be noted, that here by *Diviſion*, I doe not mean the ſevering or pulling aſunder of one Space or Time from another (for does any man think that one Hemiſphere may be ſeparated from the other Hemiſphere, or the firſt Hour from the ſecond?) but Diverſity of Conſideration; ſo that Diviſion is not made by the operation of the Hands but of the Mind.

6 When Space or Time is conſidered among other Spaces or Times, it is ſaid to be ONE, namely *One of them*; for except One Space might be added to another, and ſubſtracted from another Space, and ſo of Time, it would be ſufficient to ſay Space or Time ſimply, and ſuperfluous to ſay One Space or One Time, if it could not be conceived that there were another. The common Definition of *One*, namely, that *One is that which is Undivided*, is obnoxious to an abſurd Conſequence; for it may thence be inferred, that whatſoever is Divided, is many things, that is, that every Divided

vided thing, is Divided Things, which is Insignificant.

7 NUMBER is One and One, or One One and One, and so forwards; namely One and One make the Number *Two*, and One One and One, the Number *Three*; and so are all other Numbers made; which is all one as if we should say, *Number is Unities.*

8 To COMPOUND Space of Spaces, or Time of Times, is first to Consider them one after another, and then altogether as One; as if one should reckon first the Head, the Feet, the Armes and the Body severally, and then for the account of them all together put *Man.* And that which is so put for all the severalls of which it consists, is called the WHOLE; and those severalls, when by the Division of the Whole, they come again to be considered singly, are parts thereof; and therefore the *Whole*, and *all the Parts taken together*, are the same thing. And as I noted above, that in *Division* it is not necessary to pull the Parts asunder; so in *Composition* it is to be understood, that for the making up of a whole there is no need of putting the Parts together, so as to make them touch one another, but onely of collecting them into one summe in the Mind. For thus all Men being considered together, make up the Whole of Mankind, though never so much dispersed by Time and Place; and twelve Hours, though the hours of severall dayes, may be Compounded into one Number of Twelve.

9 This being well understood, it is manifest, that nothing can rightly be called a Whole, that is not conceived to be compounded of Parts, and that it may be divided into parts; so that if we deny that a thing has parts, we deny the same to be a Whole. For example, if we say the soul can have no Parts, we affirme that no soul can be a Whole soul. Also it is manifest, that Nothing has Parts till it be Divided; and when a Thing is Divided, the Parts are onely so many as the Division makes them. Againe, that a Part of a Part is a Part of the Whole; & thus any Part of the Number *Four*, as *Two*, is a Part of the Number *Eight*; for *Four* is made of *Two* and *Two*; but *Eight* is compounded of *Two*, *Two* and *Four*; and therefore *Two* which is a Part of the Part *Four*, is also a Part of the whole *Eight*.

10 Two Spaces are said to be CONTIGUOUS, when there is no other Space betwixt them. But two Times, betwixt which there

there is no other Time, are called IMMEDIATE, A B C, as AB, BC. And any two Spaces as well as Times are said to be CONTINUALL, when they have one common part, A B C D as AC, BD, where the part BC is common; and more Spaces and Times are Continual, when every two which are next one another are Continual.

11 That Part which is between two other Parts, is called a MEAN; & that which is not between two other parts, an EXTREME. And of Extremes, that which is first reckoned is the BEGINNING, and that which last, the END; and all the Means together taken, are the WAY. Also *Extreme Parts* and *Limits* are the same thing. And from hence it is manifest, that *Beginning* and *End* depend upon the order in which we number them; and that to *Terminate* or *Limit* Space and Time, is the same thing with *imagining their Beginning and End*; as also that every thing is FINITE or INFINITE, acording as we imagine or not imagine it *Limited* or *Terminated* every way; and that the *Limits* of any Number are *Unities*, and of these, that which is the first in our Numbering is the *Beginning*, and that which we number last, is the *End*. When we say Number is *Infinite*, we mean only that no Number is expressed; for when we speak of the Numbers *Two*, *Three*, *a Thousand*, *&c.* they are always *Finite*. But when no more is said but this, *Number is Infinite*, it is to be understood as if it were said, this Name *Number* is an *Indefinite* Name.

12 Space or Time is said to be *Finite in Power*, or *Terminable*, when there may be assigned a Number of finite Spaces or Times, as of Paces or Hours, than which there can be no greater Number of the same measure, in that Space or Time; and *Infinite in Power* is that Space or Time, in which a greater Number of the said Paces or Hours may be assigned, than any Number that can be given. But we must note, that although in that Space or Time which is Infinite in Power*, there may be numbered more Paces or Hours then any number that can be assigned, yet their number will alwayes be Finite; for every Number is Finite. And therefore his Ratiocination was not good, that undertaking to prove the World to be Finite, reasoned thus; *If the world be Infinite, then there may be taken in it some Part which is distant from us an Infinite number of Paces: But no such Part can be taken; wherefore the*

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world is not infinite; becauſe that Conſequence of the Major Propoſition is falſe; for in an Infinite ſpace, whatſoever we take, or deſign in our Mind, the diſtance of the ſame from us is a Finite ſpace; for in the very deſigning of the place thereof, we put an End to that ſpace, of whch we our ſelves are the Beginning; and whatſoever any man with his Mind cuts off both wayes from Infinite, he determines the ſame, that is, he makes it Finite.

Of Infinite Space or Time, it cannot be ſaid that it is a *Whole*, or *One*; not a *Whole*, becauſe not compounded of Parts; for ſeeing Parts, how many ſoever they be, are ſeverally Finite, they will alſo when they are all put together make a whole Finite; Nor *One*, becauſe nothing can be ſaid to be One, except there be Another to compare it with; but it cannot be conceived that there are two Spaces, or two Times Infinite. Laſtly, when we make queſtion whether the World be Finite or Infinite, we have nothing in our Minde anſwering to the Name *World*; for whatſoever we Imagine, is therefore Finite, though our Computation reach the fixed Stars, or the ninth or tenth, nay, the thouſanth Sphere. The meaning of the Queſtion is this onely, whether God has actually made ſo great an Addition of Body to Body, as we are able to make of Space to Space.

13 And therefore that which is commonly ſaid, that Space and Time may be divided Infinitely, is not to be ſo underſtood, as if there might be any Infinite or Eternal Diviſion; but rather to be taken in this ſenſe, *Whatſover is Divided, is divided into ſuch Parts as may again be Divided*; or thus, *The Leaſt Diviſible thing is not to be given*; or as Geometricians have it, *No Quantity is ſo ſmall, but a Leſs may be taken*; which may eaſily be demonſtrated in this manner. Let any Space or Time (that which was thought to be the Leaſt Diviſible) be divided into two equal Parts A and B. I ſay either of them, as A, may be divided again. For ſuppoſe the Part A to be contiguous to the Part B of one ſide, and of the other ſide to ſome other Space equal to B. This whole Space therefore (being greater then the Space given) is diviſible. Wherefore if it be divided into two equal Parts, the Part in the middle, which is A, will be alſo divided into two equal Parts; and therefore A was Diviſible.

Chap. VIII.

Of Body and Accident.

1 Body defined. 2 Accident defined. 3 How an Accident may be understood to be in its Subject. 4 Magnitude, what it is. 5 Place what it is, and that it is Immoveable. 6 What is Full and Empty. 7 Here, There, Somewhere, what they signifie. 8 Many Bodies cannot be in One place, nor One Body in Many places. 9 Contiguous and Continual what they are. 10 The definition of Motion. No Motion intelligible but with Time. 11 What it is to be at Rest*, to have been Moved, and to be Moved. No Motion to be conceived, without the conception of Past and* Future*. 12 A Point, a Line, Superficies and Solid, what they are. 13 Equal, Greater and Lesse in Bodies and Magnitudes, what they are. 14 One and the same Body has alwayes one and the the same Magnitude. 15 Velocity what it is. 16 Equal, Greater and Lesse in Times what they are. 17 Equal, Greater and Lesse in Velocity, what. 18* Equal*, Greater and Lesse in Motion, what. 19 That which is at Rest will alwayes be at Rest except it be Moved by some external thing; and that which is Moved will alwayes be Moved, unless it be hindered by some external thing. 20 Accidents are Generated and Destroyed, but Bodies not so. 21 An Accident cannot depart from its Subject. 22 Nor be Moved. 23 Essence, Form, and Matter, what they are. 24 First Matter, what. 25 That the whole is greater then any Part thereof, why demonstrated.*

1 HAving understood what Imaginary Space is, in which we supposed nothing remaining without us, but all those things to be destroyed that by existing heretofore left Images of themselves in our Minds; let us now suppose some one of those things to be placed again in the World, or created anew. It is necessary therefore that this new created or replaced thing do not onely fill some part of the Space above-mentioned, or be coincident and coextended with it, but also, that it have no dependance upon our thought. And this is that which for the

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Extension of it we commonly call *Body*; and because it depends not upon our Thought, we say is *a thing subsisting of it self*; as also *existing*, because without Us; and lastly, it is called the *Subject*, because it is so placed in and *subjected* to Imaginary Space, that it may be understood by Reason, as well as perceived by Sense. The Definition therefore of *Body* may be this, A BODY *is that which having no dependance upon our Thought is coincident or coextended with some part of Space.*

2 But what an *Accident* is, cannot so easily be explained by any Definition, as by Examples. Let us imagine therefore that a Body fills any Space, or is coextended with it, that Coextention is not the coextended Body; And in like manner, let us imagine that the same Body is removed out of its place, that Removing is not the removed Body; Or let us think the same not removed, that not-removing or Rest, is not the resting Body. What then are these things? They are *Accidents* of that Body. But the thing in question is *What is an Accident*; which is an Enquiry after that which we know already, and not that which we should enquire after. For who does not alwayes and in the same manner understand him that sayes any thing is Extended, or Moved, or not Moved? But most men will have it be said that *an Accident is something*, namely some part of a natural thing, when indeed it is no part of the same. To satisfie these men, as well as may be, they answer best that define an *Accident* to be *the Manner by which any Body is conceived*; which is all one as if they should say, *An Accident is that faculty of any Body by which it works in us a Conception of it self.* Which Definition though it be not an Answer to the Question propounded, yet it is an Answer to that Question which should have been propounded, namely, *Whence does it happen that one part of any Body appears here, another part there?* For this is well answered thus, *It happens from the Extension of that Body.* Or, *How comes it to pass that the whole Body by succession is seen now here now there?* and the answer will be, *By reason of its Motion.* Or lastly, *Whence is it that any Body possesseth the same Space for sometime?* And the answer will be, *because it is not moved.* For if concerning the Name of a Body, that is, concerning a Concrete Name, it be asked, *what is it?* the answer must be made by Definition; for the Question is concerning the signification of the Name. But if it be asked concerning an Abstract Name, *what is it?* the Cause is

is demanded why a thing appears ſo or ſo. As if it be asked, *what is Hard?* The Anſwer will be, Hard is that, whereof no Part gives place, but when the Whole gives place. But if it be demanded, *what is Hardneſs?* A Cauſe muſt be ſhewn why a Part does not give place except the Whole give place. Wherefore I define an ACCIDENT to be *the Manner of our conception of Body.*

3 When an *Accident* is ſaid *to be in a Body*, it is not ſo to be underſtood, as if any thing were conteined in that Body; as if, for example, Redneſs were in Blood, in the ſame manner, as Blood is in a Bloody cloth, that is, as a Part in the Whole; for ſo an Accident would be a Body alſo. But as Magnitude, or Reſt, or Motion, is in that which is Great, or which Reſteth, or which is Moved (which how it is to be underſtood, every man underſtands) ſo alſo it is to be underſtood that every other Accident *is in* its Subject. And this alſo is explicated by *Ariſtotle* no otherwiſe then negatively, namely, that *An Accident is in its Subject, not as any part thereof, but ſo as that it may be away, the Subject ſtill remaining*; which is right, ſaving that there are certain Accidents which can never periſh except the Body periſh alſo; for no Body can be conceived to be without Extenſion, or without Figure. All other Accidents, which are not common to all Bodies, but peculiar to ſome onely, as *To be at Reſt, to be Moved, Colour, Hardneſs*, and the like, do periſh continually, and are ſucceeded by others; yet ſo, as that the Body never periſheth. And as for the opinion that ſome may have, that all other Accidents are not in their Bodies in the ſame manner that Extenſion, Motion, Reſt, or Figure are in the ſame; for example, that Colour, Heat, Odour, Vertue, Vice and the like, are otherwiſe in them, and (as they ſay) *inherent*; I deſire they would ſuſpend their judgement for the preſent, and expect a little, till it be found out by Ratiocination, whether theſe very Accidents are not alſo certain Motions, either of the Mind of the perceiver, or of the Bodies themſelves which are perceived; for in the ſearch of this, a great part of Naturall Philoſophy conſiſts.

4 The Extenſion of a Body, is the ſame thing with the MAGNITUDE of it, or that which ſome call *Real Space*. But this *Magnitude* does not depend upon our Cogitation, as Imaginary Space doth; for this is an Effect of our Imagination, but *Magnitude* is the Cauſe of it; this is an Accident of the Mind, that of a Body exiſting out of the Mind.

5 That

5 That Space (by which word I here underſtand Imaginary Space) which is coincident with the Magnitude of any Body, is called the PLACE of that Body; and the Body it ſelf is that which we call the *Thing Placed.* Now *Place*, and the *Magnitude* of the Thing Placed differ: Firſt in this, that a Body keeps alwayes the ſame *Magnitude* both when it is at Reſt, and when it is Moved; but when it is Moved, it does not keep the ſame *Place.* Secondly, in this, that *Place* is a Phantaſme of any Body of ſuch and ſuch Quantity and Figure; but *Magnitude* is the peculiar Accident of every Body; for one Body may at ſeveral times have ſeveral Places, but has always one and the ſame Magnitude. Thirdly, in this, that *Place* is nothing out of the Mind, nor *Magnitude* any thing within it. And laſtly, *Place* is feigned Extenſion but *Magnitude* true Extenſion, and a Placed Body is not Extenſion, but a Thing Extended. Beſides, *Place is Immoveable*; for ſeeing that which is Moved, is underſtood to be carried from Place to Place, if Place were Moved, it would alſo be carried from Place to Place, ſo that one Place muſt have another Place, and that Place another Place, and ſo on infinitely, which is ridiculous. And as for thoſe, that by making *Place* to be of the ſame Nature with *Real Space*, would from thence maintain it to be Immoveable, they alſo make Place (though they do not perceive they make it ſo) to be a meer Phantaſme. For whileſt One affirms that Place is therefore ſaid to be Immoveable, becauſe Space in general is conſidered there; if he had remembred that nothing is General or Univerſal beſides Names or Signes, he would eaſily have ſeen that that Space which he ſayes is conſidered in general, is nothing but a Phantaſme in the Mind or the Memory, of a Body of ſuch Magnitude and ſuch Figure. And whileſt another ſayes, Real Space is made Immoveable by the Underſtanding; as when under the Superficies of running water, we imagine other and other water to come by continual ſucceſſion, that Superficies fixed there by the Underſtanding is the *Immoveable Place* of the River, what elſe does he make it to be but a Phantaſm, though he doe it obſcurely, and in perplexed words? Laſtly, the nature of *Place* does not conſiſt in the *Superficies of the Ambient*, but in *Solid Space*; for the whole Placed Body is coextended with its whole Place, and every part of it with every anſwering part of the ſame Place; but ſeeing every

Placed

Placed Body is a Solid thing, it cannot be understood to be coextended with Superficies. Besides, how can any whole Body be Moved, unless all its parts be moved together with it? Or how can the internall Parts of it be Moved, but by leaving their Place? But the internal Parts of a body cannot leave the Superficies of an external part contiguous to it; and therefore it followes, that if Place be the Superficies of the Ambient, then the parts of a Body Moved, that is Bodies moved, are not Moved.

6 Space (or Place) that is possessed by a Body, is called FULL, and that which is not so possessed is called EMPTY.

7 *Here, There, In the Country, In the City*, and other the like Names by which answer is made to the question *Where is it*, are not properly Names of Place, nor doe they of themselves bring into the mind the Place that is sought; for *Here* and *There* signifie nothing, unlesse the thing be shewn at the same time with the finger or something else, but when the Eye of him that seeks, is by pointing, or some other signe directed to the thing sought, the Place of it is not hereby defined by him that answers, but found out by him that askes the question. Now such Shewings as are made by words onely, as when we say, *In the Countrey*, or *In the City*, are some of greater latitude then others, as when we say *In the Countrey, In the City, In such a Street, In a House, In the Chamber, In Bed*, &c. For these do by little and little direct the Seeker neerer to the proper Place, & yet they do not determine the same, but onely restrain it to a lesser Space, & signifie no more then that the Place of the Thing is within a certain Space designed by those Words, as a Part is in the Whole. And all such Names (by which answer is made to the question *Where*) have for their highest *Genus* the Name *Somewhere*. From whence it may be understood, that whatsoever is Somewhere, is in some Place properly so called, which Place is part of that greater Space that is signified by some of these Names, *In the Countrey, In the City*, or the like.

8 A Body, and the Magnitude, and the Place thereof, are divided by one and the same act of the Mind; for, to divide an Extended Body, and the Extension thereof, and the Idea of that Extension, which is Place, is the same with dividing any one of them; because they are coincident, and it cannot be done but by the Mind, that is by the Division of Space. From whence it is manifest, that nei-

neither Two Bodies can be together in the ſame Place, nor One Body be in Two Places at the ſame Time. Not Two Bodies in the ſame Place; becauſe when a Body that fills its whole Place is divided into Two, the Place it ſelf is divided into Two alſo, ſo that there will be Two Places, Nor One Body in Two Places; for, the Place that a Body fills being divided into Two, the Placed Body will be alſo divided into Two, (for, as I ſaid, a Place and the Body that fills that Place are divided both together) and so there will be two Bodies.

9 Two Bodies are ſaid to be *Contiguous* to one another, and *Continual*, in the ſame manner as Spaces are; namely, *thoſe are Contiguous, between which there is no Space.* Now by Space I underſtand here as formerly an Idea or Phantaſme of a Body. Wherefore, though between two Bodies there be put no other Body, and conſequently no Magnitude, or (as they call it) Real Space, yet if another Body may be put between them, that is, if there intercede any imagined Space which may receive another Body, then thoſe Bodies are not Contiguous. And this is ſo eaſie to be underſtood, that I ſhould wonder at ſome men, who being otherwiſe skilful enough in Philoſophy, are of a different opinion, but that I finde that moſt of thoſe that affect Metaphyſical ſubtilties, wander from Truth, as if they were led out of their way by an *Ignis Fatuus*. For can any man that has his natural Senſes, think that two Bodies muſt therefore neceſſarily Touch one another, becauſe no other Body is between them? Or that there can be no *Vacuum*, becauſe *Vacuum* is nothing, or as they call it, *Non Ens*? Which is as childiſh, as if one ſhould reaſon thus; No Man can Faſt, becauſe to Faſt is to eat Nothing; but Nothing cannot be eaten. *Continual, are any two Bodies that have a common part; and more then two are Continual, when every two that are next to one another, are continual.*

10 MOTION, *is a continual relinquiſhing of one Place, and acquiring of another*; and that Place which is relinquiſhed is commonly called the *Terminus à quo*, as that which is acquired is called the *Terminus ad quem*; I ſay a continual Relinquiſhing, becauſe no Body, how little ſoever, can totally and at once go out of its former Place into another, ſo, but that ſome part of it will be in a part of Place which is common to both, namely, to the relinquiſhed and the acquired Places. For example, let any Body be in the Place

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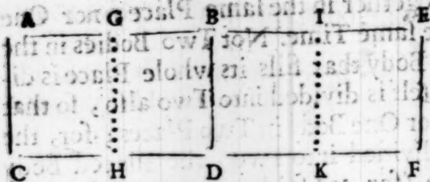

ACBD; the same Body cānot come into the Place BDEF, but it must first be in GHIK, whose part GHBD is common to both the Places ACBD, and GHIK; and whose part BDIK is common to both the Places GHIK, and BDEF. Now it cannot be conceived that any thing can be Moved without Time; for Time, is by the Definition of it, a Phantasme, that is a Conception of Motion; and therefore to conceive that any thing may be Moved without Time, were to conceive Motion without Motion, which is impossible.

11 *That is said to be at Rest, which during any time is in one place; and that to be Moved, or to have been Moved, which whether it be now at Rest, or Moved, was formerly in another Place then that which it is now in.* From which Definitions it may be inferred, First, that *Whatsoever is Moved, has been Moved*; for if it be still in the same Place in which it was formerly, it is at Rest, that is, it is not Moved, by the Definition of *Rest*; but if it be in another Place, it has been Moved, by the Definition of *Moved*. Secondly, that *what is Moved, will yet be Moved*; for that which is Moved, leaveth the Place where it is, and therefore will be in another Place, and consequently will be moved still. Thirdly, that *whatsoever is Moved, is not in One place during any time, how little soever that time be*; for by the Definition of Rest, that which is in one Place during any time, is at Rest.

There is a certain Sophisme against Motion, which seems to spring from the not understanding of this last Proposition. For they say, that, *If any Body be Moved, it is Moved either in the Place where it is, or in the Place where it is not; both which are false, and therefore nothing is Moved.* But the falsity lies in the Major Proposition; for that which is Moved, is neither Moved in the Place where it is, nor in the Place where it is not; but from the Place where it is, to the Place where it is not. Indeed it cannot be denied but that whatsoever is Moved, is Moved somewhere, that is, within some Space; but then the Place of that Body is not that whole Space, but a part of it, as is said above in the seventh Article. From what is above demonstrated, namely, that whatsoever is Moved, has also been

been Moved, and will be Moved, this alſo may be collected, that there can be no conception of Motion, without conceiving Paſt and Future time.

12 Though there be no Body which has not ſome Magnitude, yet if when any Body is moved, the Magnitude of it be not at all conſidered, the way it makes it called a LINE, or one ſingle Dimenſion; & the Space through which it paſſeth, is called LENGTH; and the Body it ſelf, a POINT; in which ſenſe the Earth is called a *Point*, and the Way of its yearly Revolution, the *Ecliptick Line*. But if a Body which is Moved, be conſidered as *long*, and be ſuppoſed to be ſo Moved, as that all the ſeveral parts of it be underſtood to make ſeveral Lines, then the Way of every part of that Body is called BREADTH, and the Space which is made is called SUPERFICIES, conſiſting of two Dimenſions, one whereof to every ſeveral part of the other is applyed whole. Again, if a Body be conſidered as having *Superficies*, and be underſtood to be ſo Moved, that all the ſeveral parts of it deſcribe ſeveral Lines, then the Way of every part of that Body is called THICKNESS, or DEPTH, and the Space which is Made is called SOLID, conſiſting of three Dimenſions, any two whereof are applyed whole to every ſeveral part of the third.

But if a Body be conſidered as *Solid*, then it is not poſſible that all the ſeveral parts of it ſhould deſcribe ſeveral lines; for what way ſoever it be Moved, the way of the following part will fall into the way of the part before it, ſo that the ſame Solid will ſtill be made which the formoſt Superficies would have made by it ſelf. And therefore there can be no other Dimenſion in any Body, as it is a Body, then the three which I have now deſcribed; though as it ſhall be ſhewed hereafter, *Velocity*, which is Motion according to *Length*, may by being applyed to all the parts of a *Solid*, make a Magnitude of Motion conſiſting of four Dimenſions; as the goodneſs of Gold computed in all the parts of it makes the Price and Value thereof.

13 *Bodies* (how many ſoever they be) that can fill every one the place of every one, are ſaid to be EQUAL every one to every other. Now one Body may fill the ſame Place which another Body filleth, though it be not of the ſame Figure with that other Body, if ſo be that it may be underſtood to be reducible to the ſame

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Figure, either by Flexion or Transposition of the parts. And *One Body is* GREATER *then another Body, when a part of that is equal to all this; and* LESSE, *when all that is equal to a part of this.* Also *Magnitudes* are *Equal*, or *Greater*, or *Lesser* then one another for the same consideration, namely, when the Bodies of which they are the Magnitudes, are either *Equal* or *Greater* or *Lesse*, &c.

14 One and the same Body, is alwayes of one and the same Magnitude. For seeing a Body and the Magnitude and Place thereof cannot be comprehended in the Minde, otherwise then as they are Coincident, if any Body be understood to be at Rest, that is, to remain in the same Place during some time, and the Magnitude thereof be in one part of that time Greater, and in another part Lesse, that Bodies Place, which is one and the same, will be coincident sometimes with Greater, sometimes with Lesse Magnitude, that is, the same Place will be greater and lesse then it self, which is impossible. But there would be no need at all of Demonstrating a thing that is in it self so manifest, if there were not some, whose opinion concerning Bodies and their Magnitudes is, that a Body may exist separated from its Magnitude, and have greater or lesse Magnitude bestowed upon it, making use of this Principle for the explication of the nature of *Raram* and *Densum*.

15 Motion, in as much as a certain Length may in a certain Time be transmitted by it, is called VELOCITY or *Swiftness*: &c. For though *Swift* be very often understood with relation to *Slower* or *less Swift*, as Great is in respect of Less, yet neverthelesse, as Magnitude is by Philosophers taken absolutely for Extension, so also *Velocity* or *Swiftness* may be put absolutely for Motion according to Length.

16 Many Motions are said to be made in Equal Times, when every one of them begins and ends together with some other Motion, or if it had begun together, would also have ended together with the same. For Time which is a Phantasme of Motion, cannot be reckoned but by some exposed Motion; as in Dials by the Motion of the Sun or of the Hand; and if two or more Motions begin and end with this Motion, they are said to be made in equal times; from whence also it is easie to understand what it is to be moved in Greater or Longer time, & in lesse time or not so long; namely, that that is longer Moved, which beginning with another

other, ends later; or ending together, began sooner.

17 Motions are said to be Equally Swift, when Equal lengths are transmitted in Equal times; and Greater Swiftness is that, wherein Greater length is passed in Equal time, or Equal length in less time. Also that Swiftness by which Equal lengths are passed in Equal parts of time, is called *Uniform* Swiftness or Motion; and of Motions *not Uniform*, such as become Swifter or Slower by equal Increasings or Decreasings in equal parts of time, are said to be *Accelerated* or *Retarded Uniformly*.

18 But Motion is said to be Greater, Lesse, and Equal, not onely in regard of the Length which is transmitted in a certain time, that is, in regard of Swiftness onely, but of Swiftness applyed to every smallest particle of Magnitude; For when any Body is Moved, every part of it is also Moved; and supposing the parts to be halves, the Motions of those halves have their Swiftness equal to one another, and severally equal to that of the Whole; but the Motion of the Whole is equal to those two Motions, either of which is of equal Swiftness with it; and therefore it is one thing for two Motions to be Equal to one another, & another thing for them to be Equally Swift. And this is manifest in two Horses that draw abreast, where the Motion of both the Horses together is of Equal Swiftness with the Motion of either of them singly; but the Motion of both is Greater then the Motion of one of them, namely Double. Wherefore *Motions are said to be simply Equal to one another when the Swiftness of one computed in every part of its Magnitude, is Equal to the Swiftnes of the other coputed also in every part of its Magnitude: & Greater then one another, when the Swiftness of one computed as above, is Greater then the Swiftness of the other so computed; and Lesse, when Lesse.* Besides, the Magnitude of Motion computed in this manner is that which is commonly called FORCE.

19 *Whatsoever is at Rest, will alwayes be at Rest, unless there be some other Body besides it, which by endeavouring to get into its Place by motion suffers it no longer to remain at Rest.* For suppose that some Finite Body exist, and be at Rest, and that all Space besides be Empty; if now this Body begin to be Moved, it will certainly be Moved some way; Seeing therefore there was nothing in that Body which did not dispose it to Rest, the reason why it is Moved this way is in something out of it; and in like manner, if it had been Moved any

other way, the reason of Motion that way had also been in something out of it; but seeing it was supposed that Nothing is out of it, the reason of its Motion one way would be the same with the reason of its Motion every other way; wherefore it would be Moved alike all wayes at once; which is impossible.

In like manner, *Whatsoever is Moved, will alwayes be Moved, except there be some other Body besides it, which causeth it to Rest.* For if we suppose Nothing to be without it, there will be no reason why it should Rest now, rather then at another time; wherefore its Motion would cease in every particle of time alike; which is not intelligible.

When we say a Living Creature, a Tree, or any other specified Body is *Generated*, or *Destroyed*, it is not to be so understood as if there were made a Body of that which is not-Body, or not a Body of a Body, but of a Living Creature not a Living Creature, of a Tree not a Tree, &c. that is, that those Accidents for which we call one thing a Living Creature, another thing a Tree, and another by some other Name, are Generated and Destroyed; and that therefore the same Names are not to be given to them now, which were given them before. But that Magnitude for which we give to any thing the Name of Body is neither Generated nor Destroyed. For though we may feign in our Mind that a Point may swell to a huge bulk, and that this may again contract it selfe to a Point; that is, though we may imagine something to arise where before was Nothing, and Nothing to be there where before was something, yet we cannot comprehend in our Minde how this may possibly be done in Nature. And therefore Philosophers, who tye themselves to Naturall Reason, Suppose that a Body can neither be Generated nor Destroyed, but onely that it may appear otherwise then it did to Us, that is under different *Species*, and consequently be called by other and other Names; so that that which is now called Man, may at another time have the Name of Not-Man; but that which is once called Body, can never be called Not-Body. But it is manifest, that all other Accidents besides Magnitude or Extension may be Generated and Destroyed; as when a White thing is made Black, the Whiteness that was in it Perisheth, and the Blackness that was not in it is now Generated; and therefore Bodies, and the Accidents under which they appear diversly, have

this

this difference, that Bodies are Things, and not Generated; Accidents are Generated, and not Things.

21 And therefore when any thing appears otherwise then it did, by reason of other and other Accidents, it is not to be thought that an Accident goes out of one Subject into another (for they are not, as I said above, in their Subjects as a Part in the Whole, or as a Conteined thing in that which Conteins it, or as a Master of a Family in his House,) but that one Accident Perisheth, and another is Generated. For example, when the Hand being Moved, Moves the Pen, Motion does not go out of the Hand into the Pen, for so the Writing might be continued though the Hand stood still, but a new Motion is Generated in the Pen, and is the Pens Motion.

22 And therefore also it is improper to say an Accident is Moved; as when in stead of saying, *Figure is an Accident of a Body carried away*, we say, *A Body carries away its Figure*.

23 Now that Accident for which we give a certain Name to any Body, or the Accident which denominates its Subject, is commonly called the ESSENCE thereof; as Rationality is the Essence of a Man, Whiteness; of any White Thing and Extension the Essence of a Body. And the same Essence in as much as it is Generated, is called the FORM. Again, a Body, in respect of any accident is called the SUBJECT, and in respect of the Form it is called the MATTER.

Also, the Production or Perishing of any Accident, makes its Subject be said *to be Changed*; onely the Production or Perishing of Form, make it be said it is *Generated* or *Destroyed*; but in all Generation and Mutation, the name of *Matter* still remains. For a Table made of Wood, is not onely Wooden, but Wood; and a Statue of Brass is Brass as well as Brazen; though *Aristotle* in his *Metaphysiques* say, that whatsoever is made of any thing ought not to be called ἐκεινὸ, but ἐκέινινον, as that which is made of Wood not ξύλον, but ξύλινον, that is, not Wood, but Wooden.

24 And as for that Matter which is common to all things, and which Philosophers following *Aristotle*, usually call *Materia Prima*, that is, *First Matter*, it is not any Body distinct from all other Bodies, nor is it one of them. What then is it? A mere Name; yet a Name which is not of vain Use; for it signifies a conception of

Body

Body without the consideration of any Form or other Accident except onely Magnitude or Extension, & aptness to receive Form & other Accidents; so that whensoever we have use of the Name *Body in general*, if we use that of *Materia Prima*, we do well. For as, when a Man not knowing which was first, Water or Ice, would finde out which of the two were the Matter of both, he would be fain to suppose some third Matter which were neither of these two; so he that would finde out what is the Matter of all things, ought to suppose such as is not the Matter of any thing that exists. Wherefore *Materia Prima* is no Thing; and therefore they do not attribute to it either Form or any other Accident besides Quantity; whereas all singular things have their Forms and Accidents certain.

Materia Prima therefore is Body in general, that is Body considered Universally, not as having neither Form nor any Accident, but in which no Form nor any other Accident but Quantity are at all considered, that is, they are not drawn into Argumentation.

25 From what has been said, those Axiomes may be demonstrated which are assumed by *Euclide* in the beginning of his first Element about the Equality and Inequality of Magnitudes; of (which omitting the rest) I will here demonstrate onely this one, *The Whole is greater then any Part thereof*; to the end that the Reader may know that those Axioms are not indemonstrable, & therefore not Principles of Demonstration; and from hence learn to be wary how he admits any thing for a Principle, which is not at least as evident as these are. *Greater* is defined to be that, whose Part is Equal to the Whole of another. Now if we suppose any Whole to be A, and a Part of it to be B; seeing the Whole B is Equal to it self, and the same B is a Part of A; therefore a Part of A will be Equal to the Whole B. Wherefore by the Definition above, A is Greater then B, which was to be proved.

CHAP.

Chap. IX.
Of Cauſe and Effect.

1 Action and Paſſion what they are. 2 Action and Paſſion Mediate and Immediate. 3 Cauſe ſimply taken. Cauſe without which no Effect follows, or Cauſe Neceſſary by Suppoſition. 4 Cauſe Efficient and Material. 5 An Entire Cauſe is alwayes ſufficient to produce its Effect. At the ſame inſtant that the Cauſe is Entire, the Effect is produced. Every Effect has a Neceſſary Cauſe. 6 The Generation of Effects is Continual. What is the Beginning in Cauſation. 7 No Cauſe of Motion but in a Body Contiguous and Moved. 8 The ſame Agents and Patients if alike diſpoſed, produce like Effects, though at different times. 9 All Mutation is Motion. 10 Contingent Accidents what they are.

1 A Body is ſaid to Work upon or *Act*, that is to ſay, *Do* ſome thing to another Body, when it either generates or deſtroys ſome Accident in it; and the Body in which an Accident is generated or deſtroyed is ſaid to *Suffer*, that is, to have ſomething *Done* to it by another Body, As when one Body by putting forwards another Body generates Motion in it, it is called the AGENT; and the Body in which Motion is ſo generated, is called the PATIENT; ſo Fire that warms the Hand is the *Agent*, and the Hand which is warmed is the *Patient*. That Accident which is generated in the Patient is called the EFFECT.

2 When an Agent and Patient are Contiguous to one another, their Action and Reaſon are then ſaid to be *Immediate*, otherwiſe *Mediate*; and when another Body lying betwixt the Agent and Patient is Contiguous to them both, it is then it ſelf both an Agent and a Patient, an Agent in reſpect of the Body next after it, upon which it Works, and a Patient in reſpect of the Body next before it, from which it ſuffers. Alſo if many Bodies be ſo ordered that every two which are next to one another be contiguous, then all thoſe that are betwixt the firſt and the laſt are both Agents and Patients, and the firſt is an Agent onely, and the laſt a Patient onely.

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3 An Agent is underſtood to *produce* its determined or certain Effect in the Patient, according to ſome certain Accident, or Accidents, with which both it and the Patient are affected; that is to ſay, the Agent hath its Effect preciſely ſuch, not becauſe it is a Body, but becauſe ſuch a Body, or ſo Moved; For otherwiſe all Agents, ſeeing they are all Bodies alike, would produce like Effects in all Patients; and therefore the Fire (for example) does not warm, becauſe it is a Body, but becauſe it is Hot; nor does one Body put forward another Body becauſe it is a Body, but becauſe it is moved into the place of that other Body. The Cauſe therefore of all Effects conſiſts in certain Accidents both in the Agents and in the Patient; which when they are all preſent, the Effect is produced; but if any one of them be wanting it is not produced; and that Accident either of the Agent or Patient, without which the Effect cannot be produced, is called *Cauſa ſine qua non*, or *Cauſe Neceſſary by Suppoſition*, as alſo the *Cauſe Requiſite for the Production of the Effect*. But a CAUSE. ſimply, or *An Entire Cauſe, is the Aggregate of all the Accidents both of the Agents how many ſoever they be, and of the Patient, put together; which when they are all ſuppoſed to be preſent, it cannot be underſtood but that the Effect is produced at the ſame inſtant; and if any one of them be wanting, it cannot be underſtood but that the Effect is not produced.*

4 The Aggregate of Accidents in the Agent or Agents, requiſite for the production of the Effect, the Effect being produced, is called the *Efficient Cauſe* thereof; and the Aggregate of Accidents in the Patient, the Effect being produced, is uſually called the *Material Cauſe*; I ſay the Effect being produced; for where there is no Effect, there can be no Cauſe; for nothing can be called a Cauſe where there is nothing that can be called an Effect. But the Efficient and Material Cauſes, are both but Partial Cauſes, or Parts of that Cauſe which in the next precedent article I called an Entire Cauſe. And from hence it is manifeſt, that the Effect we expect, though the Agents be not defective on their part, may nevertheleſs be fruſtrated by a defect in the Patient; and when the Patient is ſufficient, by a defect in the Agents.

5 An Entire Cauſe is alwayes ſufficient for the production of its Effect, if the Effect be at all poſſible. For let any Effect whatſoever be propounded to be produced; if the ſame

ſame be produced, it is manifeſt that the Cauſe which produced it was a ſufficient Cauſe; but if it be not produced, and yet be poſſible, it is evident that ſomething was wanting either in ſome Agent, or in the Patient, without which it could not be produced; that is, that ſome Accident was wanting which was requiſite for its Production; and therefore that Cauſe was not *Entire*, which is contrary to what was ſuppoſed.

It follows alſo from hence, that in whatſoever inſtant the Cauſe is Entire, in the ſame inſtant the Effect is produced. For if it be not produced, ſomething is ſtill wanting, which is requiſite for the production of it; and therefore the Cauſe was not Entire, as was ſuppoſed.

And ſeeing a Neceſſary Cauſe is defined to be that, which being ſuppoſed, the Effect cannot but follow; this alſo may be collected, that whatſoever Effect is produced at any time, the ſame is produced by a Neceſſary Cauſe. For whatſoever is produced, in as much as it is produced, had an Entire Cauſe, that is, had all thoſe things, which being ſuppoſed, it cannot be underſtood but that the Effect follows; that is, it had a Neceſſary Cauſe. And in the ſame manner it may be ſhewn, that whatſoever Effects are hereafter to be produced, ſhall have a Neceſſary Cauſe; ſo that all the Effects, that have been or ſhall be produced, have their Neceſſity in things antecedent.

6 And from this, that whenſoever the Cauſe is Entire, the Effect is produced in the ſame inſtant, it is manifeſt, that Cauſation and the Production of Effects conſiſt in a certain continual Progreſs; ſo that as there is a continual Mutation in the Agent or Agents by the working of other Agents upon them, ſo alſo the Patient upon which they work is continually altered and changed. For example, as the Heat of the Fire encreaſes more and more, ſo alſo the Effects thereof, namely the Heat of ſuch Bodies as are next to it, & again of ſuch other Bodies as are next to them, encreaſes more & more accordingly; which is already no litle argument that all Mutation conſiſts in Motion onely; the truth whereof ſhall be further demonſtrated in the ninth Article. But in this Progreſs of Cauſation, that is, of Action and Paſſion, if any man comprehend in his imagination a part thereof, and divide the ſame into parts, the firſt part or Beginning of it cannot be conſidered otherwiſe then

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as Action or Cause; for if it should be considered as Effect or Passion, then it would be necessary to consider something before it for its Cause or Action; which cannot be; for nothing can be before the Beginning. And in like manner, the last part is considered onely as Effect; for it cannot be called Cause if nothing follow it; but after the last nothing follows. And from hence it is, that in all Action the Beginning and Cause are taken for the same thing. But every one of the intermediate parts are both Action and Passion, and Cause and Effect, according as they are compared with the antecedent or subsequent part.

7 There can be no Cause of Motion, except in a Body Contiguous, and Moved. For let there be any two Bodies which are not contiguous, and betwixt which the intermediate Space is empty, or if filled, filled with another Body which is at Rest; and let one of the propounded Bodies be supposed to be at Rest, I say it shall always be at Rest. For if it shall be Moved, the Cause of that Motion (by the 8th. Chapter 19th. Article) will be in some external Body; and therefore if between it and that external Body there be nothing but empty Space, then whatsoever the disposition be of that external Body, or of the Patient it self, yet if it be supposed to be now at Rest, we may conceive it wil continue so til it be touched by some other Body; but seeing Cause (by the Definition) is the Aggregate of all such Accidents, which being supposed to be present it cannot be conceived but that the Effect will follow, those Accidents which are either in external Bodies, or in the Patient it self, cannot be the Cause of future Motion; and in like manner, seeing we may conceive, that whatsoever is at Rest, will still be at Rest, though it be touched by some other Body, except that other Body be moved, therefore in a contiguous Body which is at Rest, there can be no Cause of Motion. Wherefore there is no Cause of Motion in any Body, except it be Contiguous and Moved.

The same reason may serve to prove, that whatsoever is Moved, will alwayes be Moved on in the same way and with the same Velocity, except it be hindered by some other Contiguous and Moved Body; and consequently that no Bodies either when they are at Rest, or when there is an interposition of *Vacuum*, can generate or extinguish or lessen Motion in other Bodies. There is one that has writ-

written, that things Moved are more resisted by things at Rest, then by things contrarily Moved, for this reason, that he conceived Motion not to be so contrary to Motion as Rest. That which deceived him was, that the words *Rest* and *Motion* are but contradictory Names; whereas Motion indeed is not resisted by Rest, but by contrary Motion.

8 But if a Body work upon another Body at one time, and afterwards the same Body work upon the same Body at another time so, that both the Agent and Patient, and all their parts, be in all things as they were; and there be no difference except onely in time, that is, that one Action be former the other later in time; it is manifest of it self, that the Effects will be Equal and Like; as not differing in any thing besides time. And as Effects themselves proceed from their Causes; so the diversity of them depends upon the diversity of their Causes also.

9 This being true, it is necessary that *Mutation* can be nothing else, but Motion of the Parts of that Body which is Changed. For First, we do not say any thing is Changed, but that which appears to our Senses otherwise then it appeared formerly. Secondly, both those Appearances are Effects produced in the Sentient; & therefore if they be differẽt, it is necessary (by the preceding article) that either some part of the Agent which was formerly at Rest, is now Moved, and so the Mutation consists in this Motion; or some part which was formerly Moved, is now otherwise Moved, and so also the Mutation consists in this new Motion; or which being formerly Moved, is now at Rest, which (as I have shewn above) cannot come to pass without Motion, and so again Mutation is Motion; or, lastly, it happens in some of these manners to the Patient or some of its parts; so that Mutation, howsoever it be made, will consist in the Motion of the parts either of the Body which is perceived, or of the Sentient Body, or of both. Mutation therefore is Motion, (namely of the parts either of the Agent or of the Patient;) which was to be demonstrated. And to this it is consequent, that Rest cannot be the Cause of any thing; nor can any Action proceed from it, seeing neither Motion nor Mutation can be caused by it.

10 Accidents, in reſpect of other Accidents which precede them, or are before them in time, & upon which they do not depend as upon their Cauſes, are called *Contingent* Accidents; I ſay in reſpect of thoſe Accidents by which they are not generated; for in reſpect of their Cauſes all things come to paſs with equal neceſſity, for otherwiſe, they would have no Cauſes at all; which of things generated is not intelligible.

CHAP.

Chap. X.
Of Power and Act.

1 Power and Cauſe are the ſame thing. 2 An Act is produced at the ſame inſtant in which the Power is Plenary. 3 Active and Paſſive Power are parts onely of Plenary Power. 4 An Act when ſaid to be Poſſible. 5 An Act Neceſſary and Contingent, what. 6 Active Power conſiſts in Motion. 7 Cauſe Formal and Final, what they are.

1 COrreſpondent to *Cauſe* and *Effect* are POWER and ACT; Nay, thoſe and theſe are the ſame things, though for divers conſiderations they have divers names. For whenſoever any Agent has all thoſe Accidents which are neceſſarily requiſite for the production of ſome Effect in the Patient, then we ſay that Agent has *Power* to produce that Effect, if it be applyed to a Patient. But (as I have ſhewn in the precedent Chapter,) thoſe Accidents conſtitute the Efficient Cauſe; and therefore the ſame Accidents which conſtitute the Efficient Cauſe, conſtitute alſo the *Power* of the Agent. Wherefore the *Power of the Agent*, and the *Efficient Cauſe* are the ſame thing. But they are conſidered with this difference, that *Cauſe* is ſo called in reſpect of the Effect already produced, and *Power* in reſpect of the ſame Effect to be produced hereafter, ſo that *Cauſe* reſpects the Paſt, *Power* the Future time. Alſo the *Power of the Agent*, is that which is commonly called *Active Power*.

In like manner, whenſoever any Patient has all thoſe Accidents which it is requiſite it ſhould have for the production of ſome Effect in it, we ſay it is in the *Power* of that Patient to produce that Effect, if it be applyed to a fitting Agent. But thoſe Accidents (as is defined in the precedent Chapter) conſtitute the Material

Cauſes

Cause; and therefore the *Power of the Patient*, (commonly called *Passive Power)* and *Material Cause* are the same thing; but with this different consideration, that in Cause the Past time, and in Power the Future is respected. Wherefore the Power of the Agent and Patient together, which may be called Entire or *Plenary Power*, is the same thing with *Entire Cause*; for they both consist in the Sum or Aggregate of all the Accidents as well in the Agent as in the Patient, which are requisite for the production of the Effect. Lastly, as the Accident produced is in respect of the Cause called an *Effect*; so in respect of the Power it is called an *Act*.

2 As therefore the Effect is produced in the same instant in which the Cause is Entire; so also every Act that may be produced, is produced in the same instant, in which the Power is Plenary. And as there can be no Effect, but from a Sufficient and Necessary Cause; so also no Act can be produced, but by Sufficient Power, or that Power by which it could not but be produced.

3 And as it is manifest, (as I have shewn) that the Efficient and Material Causes are severally and by themselves parts onely of an Entire Cause, and cannot produce any Effect but by being joyned together; so also Power Active and Passive, are parts onely of Plenary and Entire Power; nor, except they be joyned, can any Act proceed from them; and therefore these Powers (as I said in the first Article) are but conditionall, namely, *the Agent has Power, if it be applyed to a Patient; and the Patient has Power, if it be applyed to an Agent*; otherwise neither of them have Power, nor can the Accidents which are in them severally be properly called Powers; nor any Action be said to be Possible, for the Power of the Agent alone, or of the Patient alone.

4 For that is an Impossible Act, for the production of which there is no Power Plenary. For seeing Plenary Power is that in which all things concurre which are requisite for the production of an Act, if the Power shall never be Plenary, there will always be wanting some of those things, without which the Act cannot be produced; wherefore that Act shall never be produced, that is, that Act is IMPOSSIBLE: And every Act which is not Impossible, is POSSIBLE. Every Act therefore which is Possible shall at some time be produced; for if it shall never be produced, then those things shall never concurre which are requisite for the production of

of it; wherefore that Act is *Impossible* by the Definition; which is contrary to what was supposed.

5 A *Necessary Act* is that, the production whereof it is Impossible to hinder; and therefore every Act that shall be produced, shall necessarily be produced; for that it shall not be produced is Impossible, because (as is already demonstrated) every Possible Act shall at some time be produced; Nay, this Proposition, *What shall be, shall be*, is as necessary a Proposition, as this, *A Man is a Man.*

But here perhaps some man may ask, whether those Future things, which are commonly called *Contingents*, are Necessary. I say therefore that generally all Contingents, have their Necessary Causes, (as is shewn in the preceding Chapter,) but are called Contingents in respect of other Events upon which they do not depend; as the Rain which shall be to morrow, shall be Necessarily, (that is from necessary Causes;) but we think and say it happens by chance, because we doe not yet perceive the Causes thereof, though they exist now; for men commonly call that *Casuall* or *Contingent*, whereof they do not perceive the necessary Cause; and in the same manner they use to speake of things past, when not knowing whether a thing be done or no, they say it is possible it never was done.

Wherefore all Propositions concerning Future things contingent or not contingent, as this, *It will rayne to morrow*, or this, *To morrow the Sun will rise*, are either necessarily true, or necessarily false; but we call them Contingent because we doe not yet know whether they be true or false; whereas their Verity depends not upon our Knowledge, but upon the foregoing of their Causes. But there are some who though they confess this whole Proposition, *To morrow it will either rain or not rain*, to be true, yet they will not acknowledge the parts of it, as *To morrow it will rain*, or *To morrow it will not rain*, to be either of them true by it self, because they say neither this nor that is true *determinately*. But what is this *determinately true*, but true upon our knowledge, or evidently true? and therefore they say no more but that it is not yet known whether it be true or no; but they say it more obscurely, and darken the Evidence of the truth with the same words with which they endevour to hide their own ignorance.

6. In the 9th. Article of the precedent Chapter I have shewn, that

that the Efficient Cause of all Motion and Mutation consists in the Motion of the Agent or Agents; And in the first Article of this Chapter, that the Power of the Agent is the same thing with the Efficient Cause. From whence it may be understood, that all Active Power consists in Motion also; and that Power is not a certain Accident which differs from all Acts, but is indeed an Act, namely Motion, which is therefore called Power, because another Act shall be produced by it afterwards. For example, if of three Bodies the first put forwards the second, and this the third, the Motion of the second in respect of the first which produceth it, is the Act of the second Body, but in respect of the third it is the Active Power of the same second Body.

7 The Writers of *Metaphysiques* reckon up two other Causes besides the *Efficient* and *Material*, namely the ESSENCE, which some call the *Formal Cause*; and the *End*, or *Final Cause*; both which are neverthelesse Efficient Causes. For when it is said, the Essence of a thing is the Cause thereof, *as to be Rational, is the Cause of Man*, it is not intelligible; for it is all one as if it were said, *To be a Man is the Cause of Man*, which is not well said. And yet the knowledge of the *Essence* of any thing, is the Cause of the knowledge of the thing it selfe; for if I first know that a thing is *Rational*, I know from thence that the same is *Man*; but this is no other then an Efficient Cause. A *Final Cause* has no place but in such things as have Sense and Will; and this also I shall prove hereafter to be an Efficient Cause.

CHAP.

Chap. XI.

Of Identity and Difference.

1 *What it is for one thing to Differ from another.* 2 *To Differ in Number, Magnitude, Species and Genus, what.* 3 *What is Relation, Proportion, and Relatives.* 4 *Proportionals what.* 5 *The Proportion of Magnitudes to one another, wherein it conſiſts.* 6 *Relation is no new Accident, but one of thoſe that were in the Relative before the Relation or Compariſon was made. Alſo the Cauſes of Accidents in the Correlatives are the Cauſe of Relation.* 7 *Of the Beginning of Individuation.*

1 HItherto I have ſpoken of Body ſimply, and of Accidents common to all Bodies as *Magnitude, Motion, Reſt, Action, Paſſion, Power, Poſſible, &c.* And I ſhould now deſcend to thoſe Accidents by which one Body is diſtinguiſhed from another, but that it is firſt to be declared what it is to be *Diſtinct*, and *not Diſtinct*, namely what are the SAME and DIFFERENT; for this alſo is common to all Bodies, that they may be diſtinguiſhed and differenced from one another. Now two Bodies are ſaid to *Differ* from one another, when ſomething may be ſaid of one of them, which cannot be ſaid of the other at the ſame time.

2 And firſt of all, it is manifeſt that no Two Bodies are the *Same*; for ſeeing they are Two, they are in two places at the ſame time; as that which is the *Same*, is at the ſame time in one and the ſame place. All Bodies therefore differ from one another in *Number*, namely, as One and Another; ſo that the *Same* and *different in Number* are Names oppoſed to one another by Contradiction.

In *Magnitude* Bodies differ when One is greater then Another, as *a Cubit long*, and *two Cubits long*, *of two pound weight*, and of *three pound weight*. And to theſe, *Equals* are oppoſed.

Bodies which differ more then in Magnitude are called *Unlike*, and those which differ onely in Magnitude, *Like*. Also of Unlike Bodies some are said to differ in the *Species*, other in the *Genus*; in the *Species* when their difference is perceived by one and the same Sense, as *White* and *Black*; and in the *Genus*, when their differenceis not perceived but by divers Senses, as *White* and *Hot*.

3 And the *Likeness*, or *Unlikeness*, *Equality* or *Inequality* of one Body to another, is called their RELATION; and the Bodies themselves *Relatives* or *Correlatives*; *Aristotle* calls them τὰ πρός τι; the first whereof is usually named the *Antecedent*, and the second the *Consequent*; and the *Relation* of the Antecedent to the Consequent according to Magnitude, namely, the Equality, the Excess or Defect thereof, is called the PROPORTION of the Antecedent to the Consequent, so that *Proportion* is nothing but the Equality or Inequality of the Magnitude of the Antecedent compared to the Magnitude of the Consequent by their difference only, or compared also with their difference. For Example, the *Proportion* of Three to Two consists only in this, that Three *exceeds* Two by Unity; and the Proportion of Two to Five in this, that Two compared with Five is *deficient* of it by Three, either simply, or compared with the numbers different; and therefore in the Proportion of Unequals, the Proportion of the Lesse to the Greater is called DEFECT, and that of the Greater to the Lesse EXCESS.

4 Besides, of Unequals, some are more, some lesse, and some equally unequall; so that there is *Proportion of Proportions*, as well as of *Magnitudes*, namely, where two Unequals have relation to two other Unequals; as when the Inequality which is between 2 and 3 is compared with the Inequality which is between 4 and 5. In which Comparison there are alwayes four Magnitudes, or (which is all one) if there be but three, the midlemost is twice numbred; and if the Proportion of the first to the second be equal to the Proportion of the third to the fourth, then the four are said to be *Proportionals*; otherwise they are not Proportionals.

5 The Proportion of the Antecedent to the Consequent, consists in their Difference, not onely simply taken, but also as compared with one of the Relatives; that is, either in that part of the greater by which it exceeds the lesse, or in the Remainder after the

the lesse is taken out of the greater; as the Proportion of Two to Five, consists in the Three by which Five exceeds Two, not in Three, simply onely, but also as compared with Five or Two. For though there be the same difference between Two & Five, which is between Nine and Twelve, namely Three, yet there is not the same Inequality; and therefore the Proportion of Two to Five, is not in all Relation the same with that of Nine to Twelve, but onely in that which is called Arithmetical.

6 But we must not so think of Relation, as if it were an Accident differing from all the other Accidents of the Relative; but one of them; namely that, by which the Comparison is made. For example, the likeness of one *White* to another *White*, or its Unlikeness to *Black*, is the same Accident with its *Whiteness*; and *Equality* and *Inequality*, the same Accident with the *Magnitude* of the thing compared, though under another Name; for that which is called *White* or *Great*, when it is not compared with something else, the same when it is compared is called *Like* or *Unlike*, *Equal* or *Unequal*. And from this it follows, that the Causes of the Accidents which are in Relatives, are the Causes also of *Likeness*, *Unlikeness*, *Equality* and *Inequality*, namely, that he that makes two Unequal Bodies, makes also their Inequality; and he that makes a Rule and an Action, makes also, if the Action be congruous to the Rule, their Congruity; if Incongruous, their Incongruity. And thus much concerning *Comparison* of one Body with another.

7 But the same Body may at different times be Compared with it self. And from hence springs a great controversie among Philosophers about the *Beginning of Individuation*; namely, in what sense it may be conceived that a Body is at one time the same, at another time not the same it was formerly. For example, whether a Man grown old be the same Man he was whilest he was young, or another Man; or whether a City be in different Ages the same, or another City. Some place *Individuity* in the Unity of *Matter*; others in the Unity of *Form*; and one sayes it consists in the Unity of the *Aggregate of all the Accidents together*. For *Matter*, it is pleaded, that a lump of Wax, whether it be Spherical or Cubical, is the same Wax, because the same Matter. For *Form*, that when a Man is grown from an Infant to be an Old Man, though his Matter be changed, yet he is still the same Numerical Man; for that

Identity which cannot be attributed to the Matter, ought probably to be aſcribed to the Form. For the *Aggregate of Accidents* no Inſtance can be made; but becauſe when any new Accident is generated, a new Name is commonly impoſed on the Thing, therefore he that aſſigned this cauſe of *Individuity*, thought the thing it ſelf alſo was become another thing. According to the firſt Opinion, He that ſins, and he that is puniſhed ſhould not be the ſame Man, by reaſon of the perpetual flux and change of Mans Body; nor ſhould the City which makes Lawes in one Age, and abrogates them in another, be the ſame City; which were to confound all Civil Rights. According to the ſecond Opinion, two Bodies exiſting both at once, would be one and the ſame Numerical Body; for if (for example) that Ship of *Theſeus* (concerning the Difference whereof, made by continual reparation, in taking out the old Planks, and putting in new, the Sophiſters of *Athens* were wont to diſpute) were, after all the Planks were changed, the ſame Numerical Ship it was at the beginning; and if ſome Man had kept the Old Planks as they were taken out, and by putting them afterwards together in the ſame order, had again made a Ship of them, this without doubt had alſo been the ſame Numerical Ship with that which was at the beginning; and ſo there would have been two Ships Numerically the ſame, which is abſurd. But according to the third Opinion, Nothing would be the ſame it was; ſo that a Man ſtanding, would not be the ſame he was ſitting; nor the Water which is in the Veſſel, the ſame with that which is poured out of it. Wherefore the beginning of *Individuation* is not alwayes to be taken either from Matter alone, or from Form alone.

But we muſt conſider by what name any thing is called, when we enquire concerning the *Identity* of it; for it is one thing to ask concerning *Socrates* whether he be the ſame Man, and another to ask whether he be the ſame Body; for his Body when he is Old, cannot be the ſame it was when he was an Infant, by reaſon of the difference of Magnitude; for One Body has alwayes One and the ſame Magnitude; yet neverthele ſs he may be the ſame Man. And therefore whenſoever the Name by which it is asked whether a thing be the ſame it was, is given it for the Matter onely, then if the Matter be the ſame, the thing alſo is *Individually* the ſame; as the Water which was in the Sea, is the ſame which is afterwards in

in the Cloud; and any Body is the ſame, whether the parts of it be put together, or diſperſed, or whether it be congealed or diſſolved. Alſo if the Name be given for ſuch Form as is the beginning of Motion, then as long as that Motion remains it will be the ſame *Individual* thing; as that Man will be alwayes the ſame, whoſe Actions and Thoughts proceed all from the ſame beginning of Motion, namely, that which was in his generation; and that will be the ſame River, which flows from one and the ſame Fountain, whether the ſame Water, or other Water, or ſomething elſe then Water flow from thence; and that the ſame City, whoſe Acts proceed continually from the ſame Inſtitution, whether the Men be the ſame or no. Laſtly, if the Name be given for ſome Accident, then the *Identity* of the thing will depend upon the Matter; for by the taking away and ſupplying of Matter, the Accidents that were are deſtroyed, and other new ones are generated, which cannot be the ſame Numerically; ſo that a Ship, which ſignifies Matter ſo figured, will be the ſame, as long as the Matter remains the ſame; but if no part of the Matter be the ſame, then it is Numerically another Ship; and if part of the Matter remain, and part be changed, then the Ship will be partly the ſame, and partly not the ſame.

CHAP.

CHAP. XII.
Of Quantity.

1 *The Definition of Quantity.* 2 *The Expoſition of Quantity what it is.* 3 *How Line, Superficies and Solid are expoſed.* 4 *How Time is expoſed.* 5 *How Number is expoſed.* 6 *How Velocity is expoſed.* 7 *How Weight is expoſed* 8 *How the Proportion of Magnitudes is expoſed.* 9 *How the Proportion of Times and Velocities is expoſed.*

1 WHat, and how manifold *Dimenſion* is, has been ſaid in the 8th. Chapter, namely, that there are three Dimenſions, Line (or Length) Superficies and Solid; Every one of which, if it be determined, that is, if the limits of it be made known is commonly called *Quantity*; For by *Quantity* all men underſtand that which is ſignified by that word, by which anſwer is made to the queſtion *How much is it.* Whenſoever therefore it is asked (for example) *How long is the Journey*, it is not anſwered indefinitely *Length*; nor when it is asked, *How big is the Field* is it anſwered indefinitely *Superficies*; nor if a man ask *How great is the bulk*, indefinitely *Solid*; but it is anſwered determinately, The Journey is a hundred Miles; the Field is a hundred Acres; the Bulk is a hundred Cubical Feet; or at leaſt in ſome ſuch manner, that the Magnitude of the thing enquired after may by certain Limits be comprehended in the Mind. QUANTITY therefore cannot otherwiſe be defined, then to be *a Dimenſion determined* or *a Dimenſion, whoſe Limits are ſet out, either by their Place, or by ſome Compariſon.*

2 And *Quantity* is determined two wayes; One, by the Senſe, when ſome ſenſible Object is ſet before it; as when a Line, a Superficies or Solid, of a Foot or Cubit, marked out in ſome Matter, is objected to the Eyes; which way of Determining is called *Expoſition*, and the Quantity ſo known is called *Expoſed Quantity*; The Other, by Memory, that is, by Compariſon with ſome Expoſed Quantity. In the firſt manner, when it is asked of what Quantity a thing is, it is anſwered, of ſuch Quantity as you ſee

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Exposed. In the second manner, answer cannot be made but by Comparison with some Exposed Quantity; for if it be asked, How long is the Way, the answer is, so many thousand Paces; that is, by Comparing the Way with a Pace, or some other Measure determined and known by Exposition; or the Quantity of it is to some other Quantity known by Exposition, as the Diameter of a Square is to the Side of the same, or by some other the like means. But it is to be understood that the Quantity Exposed must be some standing or permanent thing, such as is marked out in consistent or durable matter; or at least something which is revocable to sense; for otherwise no Comparison can be made by it. Seeing therefore (by what has been said in the next preceding Chapter) Comparison of one Magnitude with another, is the same thing with Proportion; it is manifest, that Quantity determined in the second manner, is nothing else but the Proportion of a Dimension not Exposed to another which is Exposed; that is, the Comparison of the Equality or Inequality thereof with an Exposed Quantity.

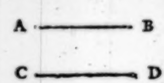
3 *Lines, Superficies* and *Solids* are Exposed, First, by *Motion*, in such manner, as (in the 8th Chapter) I have said they are generated; but so, as that the Marks of such Motion be permanent; as when they are designed upon some Matter, as a Line upon Paper; or graven in some durable Matter. Secondly, by *Apposition*, as when one Line or Length is applyed to another Line or Length, one Breadth to another Breadth, and one Thickness to another Thickness; which is as much as to describe a Line by Points, a Superficies by Lines, and a Solid by Superficies, saving that by Points in this place are to be understood very short Lines, and by Superficies very Thin Solids. Thirdly, Lines and Superficies may be Exposed by *Section*; namely a Line may be made by Cutting an Exposed Superficies, and a Superficies by the Cutting of an Exposed Solid.

4 *Time* is Exposed not onely by the Exposition of a Line, but also of some Moveable thing, which is moved Uniformly upon that Line, or at least is supposed so to be Moved. For seeing Time is an Idea of Motion in which we consider Former and Later, that is, Succession, it is not sufficient for the Exposition of Time that a Line be described, but we must also have in our Minde an Imagination

nation of ſome Moveable thing paſſing over that Line, and the Motion of it muſt be Uniform, that Time may be divided and compounded as often as there ſhall be need. And therefore when Philoſophers in their Demonſtrations draw a Line, and ſay, *Let that Line be Time*, it is to be underſtood as if they ſaid, *Let the Conception of Uniform Motion upon that Line, be Time.* For though the Circles in Dials be Lines, yet they are not of themſelves ſufficient to note Time by, except alſo there be, or be ſuppoſed to be a Motion of the Shadow or the Hand.

5 *Number* is Expoſed either by the Expoſition of Points, or of the Names of Number *One, Two, Three, &c.* and thoſe Points muſt not be contiguous, ſo as that they cannot be diſtinguiſhed by Notes, but they muſt be ſo placed that they may be *diſcerned* one from another; for from this it is that Number is called *Diſcrete Quantity*, whereas all Quantity which is deſigned by Motion, is called *Continual Quantity*. But that Number may be Expoſed by the Names of Number, it is neceſſary that they be recited by heart and in order, as One, Two, Three, &c. for by ſaying One, One, One and ſo forward, we know not what Number we are at beyond Two or Three, which alſo appear to Us in this manner not as Number, but as Figure.

6 For the Expoſition of *Velocity* (which by the Definition therof is a Motion which in a certain Time paſſeth over a certain Space) it is requiſite not onely that Time be Expoſed, but that there be alſo Expoſed that Space which is tranſmitted by the Body whoſe Velocity we would determine, and that a Body be underſtood to be Moved in that Space alſo; ſo that there muſt be Expoſed two Lines, upon One of which Uniform Motion muſt be underſtood to be made, that the Time may be determined, and upon the other the Velocity is to be computed. As if we would Expoſe the Velocity of the Body A, we draw two lines A B and C D, and place a Body in C alſo, which done, we ſay, the Velocity of the Body A is ſo great, that it paſſeth over the Line A B in the ſame Time, in which the Body C paſſeth over the Line C D with Uniform Motion.



7 *Weight* is Expoſed by any Heavy Body, of what Matter ſoever, ſo it be alwayes alike Heavy.

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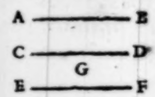
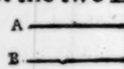
8 The *Proportion* of two Magnitudes is then Expoſed when the Magnitudes themſelves are Expoſed; namely, the Proportion of Equality, when the Magnitudes are Equal; and of Inequality, when they are Unequal. For ſeeing (by the 5th Article of the precedent Chapter) the Proportion of two Unequal Magnitudes conſiſts in their Difference compared with either of them; and when two Unequal Magnitudes are Expoſed, their Difference is alſo Expoſed; it follows, that when Magnitudes which have Proportion to one another are Expoſed, their Proportion alſo is Expoſed with them; and in like manner, the Proportion of Equals (which conſiſts in this, that there is no Difference of Magnitude betwixt them) is Expoſed at the ſame time when the Equal Magnitudes themſelves are Expoſed. For Example, if the Expoſed Lines A B and C D be Equal, the Proportion of Equality is Expoſed in them; and if the Expoſed Lines E F & E G be Unequal, the Proportion which E F has to E G, and that which E G has to E F are alſo Expoſed in them; for not onely the Lines themſelves, but alſo their Difference G F is Expoſed. The Proportion of Unequals is Quantity; for the Difference G F, in which it conſiſts, is Quantity. But the Proportion of Equality is not Quantity, becauſe between Equals there is no Difference; nor is one Equality greater then another, as one Inequality is greater then another Inequality.



9 The Proportion of two *Times*, or of two *Uniform Velocities*, is then Expoſed, when two lines are expoſed, by which two Bodies are underſtood to be Moved Uniformly; and therefore the ſame two Lines ſerve to exhibit both their own Proportion, and that of the Times and Velocities, according as they are conſidered to be Expoſed for the Magnitudes themſelves, or for the Times or Velocities. For let the two Lines A and B be Expoſed; their Proportion therefore (by the laſt foregoing Article) is Expoſed; and if they be conſidered as drawn with equal and Uniform Velocity, then ſeeing their Times are greater, or equal, or leſſe, according as the ſame Spaces are tranſmitted in greater, or equal, or leſſe Time, the Lines A and B will exhibit the Equality or Inequality, that is, the Proportion of the Times. To conclude, If the



ſame Lines A and B be conſidered as drawn in the ſame Time, then ſeeing their Velocities are greater, or equal, or leſſe, according as they paſs over in the ſame time longer, or equal, or ſhorter Lines, the ſame Lines A and B will exhibit the Equality or Inequality, that is, the Proportion of their Velocities.

CHAP.

CHAP. XIII.
Of Analogisme *or* the Same Proportion.

1, 2, 3, 4. The Nature and Definition of Proportion Arithmetical and Geometrical. 5 The Definition and some properties of the Same Arithmetical Proportion. 6, 7. The Definition and Transmutations of Analogisme, or The Same Geometrical Proportion. 8, 9. The Definitions of Hyperlogisme and Hypologisme, that is, of Greater and Lesse Proportion, and their Transmutations. 10, 11, 12. Comparison of Analogical quantities according to Magnitude. 13, 14, 15. Composition of Proportions. 16, 17, 18, 19, 20, 21, 22, 23, 24, 25. The Definition and Properties of Continual Proportion. 26, 27, 28, 29. Comparison of Arithmetical and Geometrical Proportions.

Note that in this Chapter, the signe +, signifies that the quantities betwixt which it is put, are added together; and this signe —, the Remainder, after the later quantity is taken out of the former. So that A+B *is equal to both* A *and* B *together; and where you see* A—B, *there* A *is the Whole,* B *the part taken out of it, and* A—B *the Remainder. Also two letters set together without any sign, signifies (unlesse they belong to a Figure) that one of the quantities is multiplyed by the other; as* AB *signifies the Product of* A *multiplyed by* B.

1 GReat & *Little* are not intelligible but by Comparison. Now that to which they are compared is something Exposed, that is, some Magnitude either perceived by Sense, or so defined by Words, that it may be comprehended by the Mind. Also that to which any Magnitude is compared, is either Greater, or Less, or Equal to it. And therefore Proportion (which as I have shewn, is the Estimation, or Comprehension of Magnitudes by Comparison) is threefold; namely Proportion of *Equality*, that is, of Equal to Equal; or of *Excesse*, which is of the Greater to the Lesse; or of *Defect*, which is the Proportion of the Lesse to the Greater.

Again, every one of these Proportions is twofold; For if it be asked concerning any Magnitude given, how Great it is, the answer may be made by Comparing it two wayes; First, by saying it is greater or lesse then another Magnitude by so much; as Seven is lesse then Ten by three Unities; and this is called *Arithmetical Proportion*. Secondly, by saying it is greater or lesse then another Magnitude by such a part or parts thereof; as Seven is less then Ten by three tenth parts of the same Ten. And though this Proportion be not alwayes explicable by Number, yet it is a determinate Proportion, and of a different kind from the former, and called *Geometrical Proportion*, and most commonly *Proportion simply*.

2 Proportion, whether it be Arithmetical or Geometrical, cannot be Exposed but in two Magnitudes (of which the former is cōmonly called the *Antecedent*, & the later the *Consequent* of the Proportion) as I have shewn in the 8th. Article of the precedent Chapter. And therefore if two Proportions be to be compared; there must be four Magnitudes Exposed, namely two Antecedents and two Consequents; for though it happen sometimes, that the Consequent of the former Proportion be the same with the Antecedent of the later, yet in that double Comparison it must of necessity be twice numbred; so that there will be alwayes four Terms.

3 Of two Proportions, whether they be Arithmetical or Geometrical, when the Magnitudes compared in both (which *Euclide* in the fifth Definition of his sixth Book calls the *Quantities of Proportions*) are equal, then one of the Proportions cannot be either greater or lesse then the other; For one Equality is neither greater nor lesse then another Equality. But of two Proportions of Inequality, whether they be Proportions of Excesse or of Defect, one of them may be either greater or lesse then the other, or they may both be equal; for though there be propounded two Magnitudes that are unequal to one another, yet there may be other two more unequal, and other two equally unequall, and other two less unequal then the two which were propounded. And from hence it may be understood that the Proportions of Excess and Defect are Quantity, being capable of More & Less; but the Proportion of Equality is not Quantity, because not capable, neither of *More* nor of *Less*. And therefore Proportions of Inequality may be

be added together or substracted from one another, or be multiplyed or divided by one another, or by Number; but Proportions of Equality not so.

4 Two Equal Proportions are commonly called *The Same Proportion*; and it is said, that the Proportion of the first Antecedent to the first Consequent is the *same* with that of the second Antecedent to the second Consequent. And when four Magnitudes are thus to one another in Geometrical Proportion, they are called *Proportionals*, and by some more briefly *Analogisme*. And *Greater Proportion*, is the Proportion of a Greater Antecedent to the same Consequent, or of the same Antecedent to a Less Consequent; and when the Proportion of the first Antecedent to the first Consequent, is greater then that of the second Antecedent to the second Consequent, the four Magnitudes which are so to one another may be called *Hyperlogisme*.

Less Proportion is the Proportion of a Less Antecedent to the same Consequent, or of the same Antecedent to a Greater Consequent; and when the Proportion of the first Antecedent to the first Consequent, is less then that of the second to the second, the four Magnitudes may be called *Hypologisme*.

5 One Arithmetical Proportion is the *Same* with another Arithmetical Proportion, when one of the Antecedents exceeds its Consequent, or is exceeded by it, as much as the other Antecedent exceeds its Consequent, or is exceeded by it. And therefore in four Magnitudes Arithmetically Proportional, the sum of the Extremes is equal to the sum of the Means. For if A. B :: C. D be Arithmetically Proportional, and the Difference on both sides be the same Excess or the same Defect E; then B+C (if A be greater then B) will be equal to A−E+C; and A+D will be equal to A+C−E; But A−E+C and A+C−E are equal. Or if A be less then B, then B+C will be equal to A+E+C; and A+D will be equal to A+C+E; But A+E+C, and A+C+E are equal.

Also if there be never so many Magnitudes Arithmetically Proportional, the Sum of them all will be equal to the Product of half the number of the Terms multiplyed by the Sum of the Extremes. For if A. B :: C. D :: E. F be Arithmetically Proportional, the Couples A+F, B+E, C+D will be equal to one another; and

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their Sum will be equal to A+F multiplyed by the number of their Combinations, that is, by half the number of the Terms.

If of four Unequal Magnitudes, any two together taken be equal to the other two together taken, then the greatest and the least of them will be in the same Combination. Let the Unequal Magnitudes be A, B, C, D; and let A+B be equal to C+D; & let A be the greatest of them all; I say B will be the least. For if it may be, let any of the rest, as D, be the least. Seeing therefore A is greater then C, and B then D, A+B will be greater then C+D; which is contrary to what was supposed.

If there be any four Magnitudes, the Sum of the greatest and least, the Sum of the Means, the difference of the two greatest, and the difference of the two least will be Arithmetically Proportional. For let there be four Magnitudes, whereof A is the greatest, D the least, and B and C the Means; I say A+D. B+C :: A—B. C—D are Arithmetically Proportional. For the difference between the first Antecedent and its Consequent is this, A+D—B—C; and the difference between the second Antecedent and its Consequent this, A—B—C+D; but these two Differences are equal, and therefore (by this 5th. Article) A+D. B+C :: A—B. C—D are Arithmetically Proportional.

If of four Magnitudes, two be equal to the other two, they will be in reciprocal Arithmetical Proportion. For let A+B be equal to C+D; I say A. C :: D. B are Arithmetically Proportional. For if they be not, let A. C :: D. E (supposing E to be greater or less then B) be Arithmetically Proportional, and then A+E will be equal to C+D; wherefore A+B and C+D are not equal; which is contrary to what was supposed.

6 One Geometrical Proportion is the same with another Geometrical Proportion, when the same Cause producing equal Effects in equal Times, determines both the Proportions.

If a Point Uniformly moved, describe two Lines either with the same, or different Velocity, all the parts of them which are contemporary, that is, which are described in the same time, will be Two to Two in Geometrical Proportion, whether the Antecedents be taken in the same Line, or not. For, from the point A (in the 10 Figure at the end of the 14 Chapter) let the two Lines A D, A G, be described with Uniform Motion; and let there be taken

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in them two parts A B, A E, and again two other parts A C, A F, in ſuch manner, that A B, A E, be contemporary, and likewiſe A C, A F contemporary. I ſay firſt (taking the Antecedents A B, A C in the Line A D, and the Conſequents A E, A F in the Line A G) that A B. A C : : A E. A F are Proportionals. For ſeeing (by the 8th. Chapter and the 15 Article) Velocity is Motion conſidered as determined by a certain Length or Line, in a certain Time tranſmitted by it, the quantity of the Line A B will be determined by the Velocity and Time by which the ſame A B is deſcribed; and for the ſame reaſon, the quantity of the Line A C will be determined by the Velocity and Time, by which the ſame A C is deſcribed; and therefore the proportion of A B to A C, whether it be Proportion of Equality, or of Exceſs or Defect, is determined by the Velocities and Times by which A B, A C are deſcribed; But ſeeing the Motion of the Point A upon A B and A C is Uniform, they are both deſribed with equal Velocity; and therefore whether one of them have to the other the Proportion of Majority or of Minority, the ſole cauſe of that Proportion is the difference of their Times; and by the ſame reaſon it is evident, that the proportion of A E to A F is determined by the difference of their Times onely. Seeing therefore A B, A E, as alſo A C, A F are contemporary, the difference of the Times in which A B and A C are deſcribed, is the ſame with that in which A E and A F are deſcribed. Wherfore the proportion of A B to A C, and the proportion of A E to A F are both determined by the ſame Cauſe. But the Cauſe which ſo determines the proportion of both, works equally in equal Times, for it is Uniform Motion; and therefore (by the laſt precedent Definition) the proportion of A B to A C is the ſame with that of A E to A F; and conſequently A B. A C : : A E. A F are Proportionals; which is the firſt.

Secondly (taking the Antecedents in different Lines,) I ſay, A B. A E : : A C. A F are Proportionals; For ſeeing A B, A E are deſcribed in the ſame Time, the difference of the Velocities in which they are deſcribed are the ſole Cauſe of the proportion they have to one another. And the ſame may be ſaid of the proportion of A C to A F. But ſeeing both the Lines A D and A G are paſſed over by Uniform Motion, the difference of the Velocities in which A B, A E are deſcribed, will be the ſame with the

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difference of the Velocities, in which A C, A F are described. Wherefore the Cause which determines the proportion of A B to A E, is the same with that which determines the proportion of A C to A F; and therefore A B. A E : : A C. A F, are Proportionals; which remained to be proved.

1 *Corollary.* If four Magnitudes be in Geometrical Proportion, they will also be Proportionals by *Permutation*, (that is, by transposing the Middle Terms.) For I have shewn, that not onely A B. A C : : A E. A F, but also that (by *Permutation*) A B. A E : : A C. A F are Proportionals.

2 *Coroll.* If there be four Proportionals, they will also be Proportionals by *Inversion* or *Conversion*, that is, by turning the Antecedents into Consequents. For if in the last *Analogisme*, I had for A B, A C, put by Inversion A C, A B, and in like manner converted A E, A F into A F, A E, yet the same Demonstration had served. For as well A C, A B, as A B, A C are of equal Velocity; and A C, A F, as well as A F, A C are Contemporary.

3 *Coroll.* If Proportionals be added to Proportionals, or taken from them, the Aggregates, or Remainders will be Proportionals. For Contemporaries whether they be added to Contemporaries, or taken from them; make the Aggregates or Remainders Contemporary, though the Addition or Substraction be of all the Terms, or of the Antecedents alone, or of the Consequents alone.

4 *Coroll.* If both the Antecedents of four Proportionals, or both the Consequents, or all the Terms, be multiplyed or divided by the same Number or Quantity, the Products or Quotients will be Proportionals. For the Multiplication and Division of Proportionals, is the same with the Addition and Substraction of them.

5 *Coroll.* If there be four Proportionals, they will also be Proportionals by *Composition* (that is, by compounding an Antecedent of the Antecedent and Consequent put together, and by taking for Consequent either the Consequent singly, or the Antecedent singly). For this Composition is nothing but Addition of Proportionals, namely, of Consequents to their own Antecedents, which by Supposition are Proportionals.

6 *Coroll.* In like manner, if the Antecedent singly, or Consequent singly be put for Antecedent, and the Consequent be made of both put together; these also will be Proportionals. For it is the *Inversion of Porportion by Composition.*

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7 *Coroll.* If there be four Proportionals, they will also be Proportionals by *Division* (that is, by taking the Remainder after the Consequent is substracted from the Antecedent, or the Difference between the Antecedent and Consequent for Antecedent, and either the Whole or the Remainder for Consequent;) As if A.B :: C.D be Proportionals, they will by Division be A—B. B :: C—D. D, and A—B. A :: C—D. C; and when the Consequent is greater then the Antecedent, B—A. A :: D—C. C, and B—A B :: D—C. D. For in all these Divisions, Proportionals are (by the very supposition of the Analogisme A. B :: C. D) taken from A and B, and from C and D.

8 *Coroll.* If there be four Proportionals, they will also be Proportionals by the *Conversion of Proportion*, (that is, by Inverting the Divided Proportion, or by taking the Whole for Antecedent, & the Difference or Remainder for Consequent.) As, if A. B :: C. D be Proportionals, then A. A—B :: C. C—D, as also B. A—B :: D. C—D will be Proportionals. For seeing these Inverted be Proportionals, they are also themselves Proportionals.

9 *Coroll.* If there be two Analogismes which have their quantities equal, the second to the second, and the fourth to the fourth, then either the Sum or Difference of the first quantities will be to the second, as the Sum or Difference of the third quantities is to the fourth. Let A.B :: C.D and E. B :: F.D be Analogismes; I say C+E.B :: F.D are Proportionals. For the said Analogismes will by Permutation be A.C :: B.D, and E.F :: B.D; and therefore A.C :: E.F will be Proportionals (for they have both the proportion of B to D common.) Wherefore if in the Permutation of the first Analogisme, there be added E and F to A and C, which E and F are proportional to A and C, then (by the 3d *Coroll.*) A+E.B :: C+F.D will be Proportionals, which was to be proved.

Also in the same manner it may be shewn, that A—E.B :: C+F.D are Proportionals.

7 If there be two Analogismes, where foure Antecedents make an Analogisme, their Consequents also shall make an Analogisme; as also the Sums of their Antecedents will be proportionall to the Sums of their Consequents. For if A.B :: C.D and E. F :: G.H be two Analogismes; and A. E :: C. G be Proportionals, then by Permutation A.C :: E.G, and E.G :: F.H, and A.C :: B.D will be Proportionals; Wherefore B.D :: E.G, that is, B.D :: F.H, and by

Permutation B.F : : D.H are Proportionals, which is the first. Secondly, I say A+E.B+F : : C+G.D+H will be Proportionalls. For seeing A.E : : C.G are Proportionals, A+E.E : : C+G. G will also by Composition be Proportionals, and by Permutation A+E. C+G : : E.G will be Proportionals; Wherefore also A+E.C+G : : F.H will be Proportionals. Again, seeing (as is shewn above) B.F : : D.H are Proportionals, B+F. F : : D+H. H will also by Composition be Proportionals; and by Permutation B+F. D+H : : F. H will also be Proportionals; Wherefore A+E. C+G : : B+F. D+H are Proportionals; which remained to be proved.

Coroll. By the same reason, if there be never so many Analogismes, and the Antecedents be proportional to the Antecedents, it may be demonstrated also that the Consequents will be proportional to the Consequents, as also the Sum of the Antecedents to the Sum of the Consequents.

8 In an Hyperlogisme, that is, where the Proportion of the first Antecedent to its Consequent, is greater then the proportion of the second Antecedent to its Consequent, the Permutation of the Proportionals, and the Addition of Proportionals to Proportionals, & Substraction of them from one another; as also their Composition & Division, & their Multiplication & Division by the same Number, produce always an Hyperlogisme. For suppose A.B : : C.D & A.C : : E.F be Analogismes, A+E.B : : C+F.D will be also an Analogisme; But A+E.B : : C. D will be an Hyperlogisme; Wherefore by Permutation, A+E. C : : B. D is an Hyperlogisme, because A.B : : C.D is an Analogisme. Secondly, if to the Hyperlogisme A+E. B : : C.D the Proportionals G and H be added, A+E+G. B : : C+H. D will be an Hyperlogisme, by reason A+E+G. B : : C+F+H. D is an Analogisme. Also if G and H be taken away, A+E−G. B : : C−H.D will be an Hyperlogisme; for A+E−G. B : : C+F—H. D are an Analogisme. Thirdly, by Composition A+E+B. B : : C+D. D will be an Hyperlogisme, because A+E+B.B : : C+F+D.D is an Analogisme, & so it will be in all the varieties of Composition. Fourthly, by Division A+E—B.B : : C—D.D will be an Hyperlogisme, by reason A+E—B. B : : C+F—D. D is an Analogisme. Also A+E—B. A+E : : C—D. C is an Hyperlogisme; for A+E—B. A+E : : C+F—D. C is an Analogisme. Fifthly, by Multiplication 4 A+E.B : : 4 C.D is an Hyperlogisme, because 4 A. B : : 4 C. D is an Analogisme;

gisme; and by Division ¾A+¾E. B :: ¾ C. D is an Hyperlogisme, because ¾ A. B :: ¾ C. D is an Analogisme.

9 But if A+E. B :: C. D be an Hyperlogisme, then by Inversion B. A+E :: D. C will be an Hypologisme, because B. A :: D. C being an Analogisme, the first Consequent will be too great. Also by Conversion of Proportion A+E. A+E−B :: C. C−D is an Hypologisme, because the Inversion of it, namely A+E−B. A+E :: C−D. C is an Hyperlogisme (as I have shewn but now). So also B. A+E−B :: D. C−D is an Hypologisme, because (as I have newly shewn) the Inversion of it, namely A+E−B. B :: C−D. D is an Hyperlogisme. Note that this Hypologisme A+E. A+E−B :: C. C−D is commonly thus expressed; If the proportion of the Whole (A+E) to that which is taken out of it (B), be greater then the proportion of the Whole (C) to that which is taken out of it (D,) then the proportion of the whole (A+E) to the Remainder (A+E−B) will be less then the proportion of the whole (C) to the Remainder (C−D.)

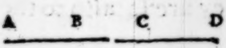
10 If there be four Proportionals, the Difference of the two first, to the Difference of the two last will be as the first Antecedent is to the second Antecedent, or as the first Consequent to the second Consequent. For if A. B :: C. D be Proportionals, then by Division A−B. B :: C−D. D will be Proportionals; and by Permutation A−B. C−D :: B. D; that is, the Differences are proportional to the Consequents, and therefore they are so also to the Antecedents.

11 Of four Proportionals, if the first be greater then the second, the third also shall be greater then the fourth. For seeing the first is greater then the second, the proportion of the first to the second is the proportion of Excess; But the proportion of the third to the fourth is the same with that of the first to the second; and therefore also the proportion of the third to the fourth is the Proportion of Excess; Wherefore the third is greater then the fourth. In the same manner it may be proved, that whensoever the first is less then the second, the third also is less then the fourth, and when those are equal, that these also are equal.

12 If there be four Proportionals whatsoever A. B :: C. D, and the first and third be multiplyed by any one number, as by 2; and again the second and fourth be multiplyed by any one number, as

by 3; and the product of the firſt 2 A, be greater then the product of the ſecond 3 B; the product alſo of the third 2 C, will be greater then the product of the fourth 3 D. But if the product of the firſt be leſs then the product of the ſecond, then the product of the third will be leſs then that of the fourth. And laſtly, if the products of the firſt and ſecond be equal, the products of the third and fourth ſhall alſo be equal. Now this Theoreme is all one with *Euclides* Definition of *The Same Proportion*; and it may be demonſtrated thus. Seeing A. B : : C. D are Proportionals, by Permutation alſo (Art. 6. Corol. 1.) A. C : : B. D will be Proportionals; Wherefore (by the 4 Corol. of the ſame 6 Article) 2 A. 2 C : : 3 B. 3 D will be Proportionals; and again by Permutation 2 A. 3 B : : 2 C. 3 D will be Proportionals; and therefore (by the laſt Article) If 2 A be greater then 3 B, then 2 C will be greater then 3 D; if leſs, leſs; and if equal, equal; which was to be demonſtrated.

13 If any three Magnitudes be propounded, or three things whatſoever that have any proportion one to another, as three Nūbers, three Times, three Degrees, &c; the proportions of the firſt to the ſecond, and of the ſecond to the third together taken; are equal to the proportion of the firſt to the third. Let there be three Lines (for any proportion may be reduced to the proportion of Lines) A B, A C, A D; And in the firſt place, let the proportion as well of the firſt A B to the ſecond A C, as of the ſecond A C to the third A D be the proportion of Defect, or of Leſs to Greater; I ſay the proportions together taken of A B to A C, and of A C to A D, are equal to the proportion of A B to A D. Suppoſe the Point A to be moved over the whole Line A D with Uniform Motion; then the proportions as well of A B to A C, as of A C to A D are determined by the difference of the Times in which they are deſcribed; that is, A B has to A C ſuch proportion as is determined by the different Times of their deſcription; and A C to A D ſuch proportion as is determined by their Times. But the proporrion of A B to A D is ſuch as is determined by the difference of the Times in which A B and A D are deſcribed; and the difference of the Times in which A B and A C are deſcribed, together with the difference of the Times in which A C and A D are deſcribed, is the ſame with the difference of the Times in which A B and A D are de-



described. And therefore the same Cause which determines the two proportions of A B to A C and of A C to A D, determines also the proportion of A B to A D. Wherefore (by the Definition of the Same Proportion delivered above in the *6* Art.) the proportion of A B to A C together with the proportion of A C to A D, is the same with the proportion of A B to A D.

In the second place, let A D be the first, A C the second, & A B the third, and let their proportion be the Porportion of Excess, or of Greater to Less; then, as before, the proportions of A D to A C, and of A C to A B, and of A D to A B will be determined by the difference of their Times; which in the description of A D and A C, and of A C and A B together taken, is the same with the difference of the Times in the description of A D and A B. Wherefore the proportion of A D to A B is equal to the two proportions of A D to A C and of A C to A B.

In the last place. If one of the proportions, namely of A D to A B be the Proportion of Excess; and another of them, as of A B to A C be the Proportion of Defect, thus also the proportion of A D to A C will be equal to the two proportions together taken of A D to A B, and of A B to A C. For the difference of the Times in which A D and A B are described is Excess of Time; for there goes more time to the description of A D then of A B; and the difference of the Times in which A B and A C are described is Defect of Time, for less Time goes to the description of A B then of A C; but this Excess, and Defect being added together make D B— B C, which is equal to D C, by which the first A D exceeds the third A C; and therefore the proportions of the first A D to the second A B, and of the second A B to the third A C, are determined by the same Cause which determines the Proportion of the first A D to the third A C. Wherefore, If any three Magnitudes, &c.

1 *Coroll.* If there be never so many Magnitudes having proportion to one another, the proportion of the first to the last is compounded of the proportions of the first to the second, of the second to the third, & so on till you come to the last; or, the proportion of the first to the last, is the same with the Sum of all the intermediate proportions. For any Number of Magnitudes having proportion to one another, as A, B, C, D, E being propounded, the proportion

of

of A to E (as is newly shewn) is compounded of the Proportions of A to D and of D to E; and again the proportion of A to D, of the proportions of A to C, and of C to D; and lastly, the proportion of A to C, of the proportions of A to B, and of B to C.

2 *Coroll.* From hence it may be understood how any two proportions may be compounded. For if the proportions of A to B, and of C to D be propounded to be added together, let B have to something else, as to E, the same proportion which C has to D, and let them be set in this order A, B, E; for so the proportion of A to E will evidently be the Sum of the two Proportions of A to B, and of B to E, that is, of C to D. Or let it be as D to C, so A to something else, as to E, and let them be ordered thus E, A, B; for the proportion of E to B will be compounded of the proportions of E to A (that is, of C to D), and of A to B. Also it may be understood how one Proportion may be taken out of another. For if the proportion of C to D be to be substracted out of the proportion of A to B, let it be as C to D, so A to something else, as E, and setting them in this order, A, E, B, and taking away the proportion of A to E, that is, of C to D, there will remain the proportion of E to B.

3 *Coroll.* If there be two Orders of Magnitudes which have proportion to one another beginning and ending with the same Magnitudes, and the several proportions of the first Order be the same and equal in number with the proportions of the second Order; then, whether the proportions in both Orders be successively answerable to one another, which is called *Ordinate Proportion*, or not successively answerable, which is called *Perturbed Proportion*, the first and the last in both will be Proportionals. For the Proportion of the first to the last is equal to all the intermediate proporons; which being in both Orders the same, and equal in number, the Aggregates of those proportions will also be equal to one another; but to their Aggregates the proportions of the first to the last are Equal; and therefore the proportion of the first to the last in one Order, is the same with the proportion of the first to the last in the other Order. Wherefore the first and the last in both are Proportionals.

14 If any two quantities be made of the mutual Multiplication of many quantities which have proportion to one another, and the

the Efficient quantities on both sides be equal in number, the proportion of the Products will be compounded of the several proportions which the Efficient quantities have to one another.

First, let the two Products be A B, and C D, whereof one is made of the Multiplication of A into B, and the other of the Multiplication of C into D. I say the porportion of A B to C D is componnded of the proportions of the Efficient A to the Efficient C, and of the Efficient B to the Efficient D. For let A B, C B and C D be set in Order; and as B is to D, so let C be to another quantity as E; and let A, C, E, be set also in Order. Then by (the 4 *Coroll.* of the 6 Art.) it will be as A B the first quantity, to C B the second quantity in the first Order, so A to C in the second Order; and again, as C B to C D in the first Order, so B to D, that is, (by Construction) so C to E in the second Order; and therefore (by the last *Corollary*) A B. C D : : A. E will be Proportionals. But the proportion of A to E is compounded of the Proportions of A to C, and of B to D; Wherefore also the proportion of A B to C D is compounded of the same.

A B	A
C B	C
C D	E

Secondly, let the two Products be A B F, and C D G, each of them made of three Efficients, the first of A, B and F, and the second of C, D and G; I say, the proportion of A B F to C D G is compounded of the proportions of A to C, of B to D and of F to G. For let them be set in Order as before; and as B is to D, so let C be to another quantity E; and again, as F is to G, so let E be to another, H; and let the first Order stand thus, A B F, C B F, C D F and C D G; and the second Order thus, A, C, E, H, Then the proportion of A B F to C B F in the first Order, will be as A to C in the second; and the Proportion of C B F to C D F in the first Order, as B to D, that is, as C to E (by Construction) in the second Order; and the Proportion of C D F to C D G in the first, as F to G, that is, as E to H (by Construction) in the second Order; and therefore A B F. C D G : : A. H will be Proportionals. But the Proportion of A to H is compounded of the Proportions of A to C, B to D, and F to G. Wherefore the Proportion of the Product A B F to C D G is also

A B F	A
C B F	C
C D F	E
C D G	H

com-

compounded of the ſame. And this operation ſerves, how many ſoever the Efficients be that make the Quantities given.

From hence ariſeth another way of Compounding many Proportions into One, namely, that which is ſuppoſed in the 5 Definition of the 6 Book of *Euclide*; which is, by multiplying all the Antecedents of the Proportions into one another, and in like manner all the Conſequents into one another. And from hence alſo it is evident, in the firſt place, That the Cauſe why Parallelograms, which are made by the Duction of two ſtraight Lines into one another, and all Solids which are equal to Figures ſo made, have their proportions compounded of the proportions of the Efficients; And in the ſecond place, why the Multiplication of two or more Fractions into one another, is the ſame thing with the Compoſition of the proportions of their ſeveral Numerators to their ſeveral Denominators. For example, if theſe Fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ be to be multiplyed into one another, the Numerators 1, 2, 3 are firſt to be multiplyed into one another, which make 6; and next the Denominators 2, 3, 4, which make 24; and theſe two Products make the Fraction $\frac{6}{24}$. In like manner, if the proportions of 1 to 2, of 2 to 3, and of 3 to 4 be to be compounded, by working as I have ſhewn above, the ſame proportion, of 6 to 24 will be produced.

15 If any Proportion be compounded with it ſelf inverted, the Compound will be the Proportion of Equality. For let any Proportion be given, as of A to B, and let the Inverſe of it be that of C to D; and as C to D, ſo let B be to another quantity; for thus they will be compounded (by the 2 *Coroll.* of the 12 Art.) Now ſeeing the proportion of C to D is the Inverſe of the proportion of A to B, it will be as C to D, ſo B to A; and therefore if they be placed in Order A, B, A, the proportion compounded of the proportions of A to B, and of C to D will be the proportion of A to A, that is, the proportion of Equality. And from hence the cauſe is evident, why two equal products have their Efficients reciprocally proportional. For, for the making of two products equal, the proportions of their Efficients muſt be ſuch, as being compounded may make the proportion of Equality, which cannot be, except one be the Inverſe of the other; for if betwixt A and A any other quantity as C be interpoſed, their order will be A, C, A, and the later proportion of C to A will be the Inverſe of the former proportion of A to C.

16 A

16 A Proportion is said to be multiplied by a Number when it is so often taken as there be Unities in that Number; and if the Proportion be of the Greater to the Less, then shall also the quantity of the Proportion be increased by the Multiplication; but when the Proportion is of the Less to the Greater, then as the Number increaseth, the quantity of the Proportion diminisheth; as in these three Numbers 4, 2, 1, the Proportion of 4 to 1, is not onely the Duplicate of 4 to 2, but also twice as great; but inverting the order of those Numbers thus 1, 2, 4, the Proportion of 1 to 2 is greater then that of 1 to 4; and therefore though the proportion of 1 to 4 be the Duplicate of 1 to 2, yet it is not twice so great as that of 1 to 2, but contrarily the half of it. In like manner, a Proportion is said to be Divided, when between two quantities are interposed one or more Means in continual Proportion, and then the Proportion of the first to the second is said to be Subduplicate of that of the first to the third, and Subtriplicate of that of the first to the fourth, &c.

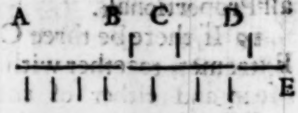
This mixture of Proportions, where some are Proportions of Excess, others of Defect (as in a Merchants accompt of *Debitor* and *Creditor*) is not so easily reckoned as some think; but maketh the Composition of Proportions sometimes to be Addition, sometimes Substraction; which soundeth absurdly to such as have alwayes by Composition understood Addition, and by Diminution Substraction. Therefore to make this account a little clearer, we are to consider (that which is commonly assumed, and truly) that if there be never so many Quantities, the Proportion of the first to the last is compounded of the Proportions of the first to the second, and of the second to the third, and so on to the last, without regarding their Equality, Excess or Defect; So that if two Proportions, one of Inequality, the other of Equality be added together, the Proportion is not thereby made Greater nor Less; as for example, if the Proportions of A to B and of B to B be compounded, the Proportion of the first to the second is as much as the Sum of both, because Proportion of Equality (being not quantity) neither augmenteth quantity nor lesseneth it. But if there be three quantities A, B, C, unequal, and the first be greatest, the last least, then the Proportion of B to C is an addition to that of A to B, and

makes it greater; and on the contrary, if A be the least, and C the greatest quantity, then doth the addition of the Proportion of B to C make the cōpounded Proportion of A to C less then the Proportion of A to B, that is, the Whole less then the Part. The Composition therefore of Proportions is not in this case the Augmentation of them, but the Diminution; for the same quantity (*Euclide* the 5, 8.) compared with two other quantities hath a greater Proportion to the lesser of them then to the greater. Likewise, when the Proportions compounded are one of Excess, the other of Defect, if the first be of Excess, as in these numbers 8,6,9, the Proportion compounded, namely, of 8 to 9, is less then the Proportion of one of the parts of it, namely of 8 to 6; but if the Proportion of the first to the second be of Defect, and that of the second to the third be of Excess, as in these Numbers 6,8,4, then shall the Proportion of the first to the third be greater then that of the first to the second, as 6 hath a greater Proportion to 4 then to 8; the reason whereof is manifestly this, that the less any quantity is deficient of another, or the more one exceedeth another, the proportion of it to that other is the greater.

Suppose now three quantities in continual Proportion A B 4, A C 6, A D 9. Because therefore A D is greater then A C, but not greater then A D, the proportion of A D to A C will be (by *Euclide* 5, 8.) greater then that of A D to A D; and likewise, because the Proportions of A D to A C, and of A C to A B are the same, the proportions of A D to A C and of A C to A B (being both Proportions of Excess) make the whole Proportion of A D to A B (or of 9 to 4) not onely the Duplicate of A D to A C (that is, of 9 to 6) but also the Double, or twice so great. On the other side, because the proportion of A D to A D (or 9 to 9) being Proportion of Equality, is no quantity, & yet greater then that of A C to A D (or 6 to 9) it will be as 0—9 to 0—6, so A C to A D, and again, as 0—9 to 0—6, so 0—6 to 0—4; but 0—4, 0—6, 0—9 are in continual proportion; and because 0—4 is greater then 0—6, the protion of 0—4 to 0—6 will be Double to the proportion of 0—4 to 0—9, Double I say, and yet not Duplicate, but Subduplicate.

If any be unsatisfied with this ratiocination, let him first consider that (by *Euclide* 5, 8) the proportion of A B to A C is greater then that

that of A B to A D wheresoever D be placed in the line A C prolonged; and the further off the point D is from C, so much the greater is the proportion of A B to A C then that of A B to A D.



There is therefore some point (which suppose be E) in such distance from C, as that the proportion of A B to A C will be twice as great as that of A B to A E. That considered, let him determine the length of the line A E, and demonstrate (if he can) that A E is greater or less then A D.

By the same method, if there be more quantities then three, as A, B, C, D in continual Proportion, and A be the least; it may be made appear, that the Proportion of A to B is Triple Magnitude (though Subtriple in Multitude) to the Proportion of A to D.

17 If there be never so many quantities, the number whereof is odd, and their Order such, that from the middlemost quantity both wayes they proceed in continual Proportion, the proportion of the two which are next on either side to the middlemost, is Subduplicate to the proportion of the two which are next to these on both sides, and Subtriplicate of the proportion of the two which are yet one place more remote, &c. For let the Magnitudes be C, B, A, D, E, and let A, B, C, as also A, D, E be in Continual Proportion; I say the proportion of D to B is Subduplicate of the proportion of E to C. For the proportion of D to B is compounded of the proportions of D to A, and of A to B once taken; But the proportion of E to C is compounded of the same twice taken; and therefore the proportion of D to B is Subduplicate of the proportion of E to C. And in the same manner, if there were three Terms on either side, it might be demonstrated that the proportion of D to B would be Subtriplicate of that of the Extremes, &c.

18 If there be never so many continual Proportionals, as the first, second, third, &c. their Differences will be Proportional to them. For the second, third, &c. are severally Consequents of the preceding, and Antecedents of the following Proportion. But (by the 10 *Art.*) the Difference of the first Antecedent and Consequent, to Difference of the second Antecedent and Consequent, is as the first Antecedent to the second Antecedent, that is, as the first

Term to the ſecond, or as the ſecond to the third, &c. in continuall Proportionals.

19 If there be three Continual Proportionals, The Sum of the Extremes, together with the Mean twice taken; The Sum of the Mean and either of the Extremes; and the ſame Extreme, are Continual Proportionals. For let A. B. C ÷ be Continual Proportionals. Seeing therefore A. B : : B. C are Proportionals; by Compoſition alſo A+B.B : : B+C. C will be Proportionals; and by Permutation A+B. B+C : : B. C will alſo be Proportionals; and again by Compoſition A+2 B+C. B+C : : B +C. C; which was to be proved.

20 In four Continual Proportionals, the greateſt and the leaſt put together, is a greater quantity then the other two put together. Let A.B : : C.D be Continual Proportionals, whereof let the greateſt be A, and the leaſt be D; I ſay A+D is greater then B+C. For (by the 10 *Art.*) A—B.C—D : : A.C are Proportionals; and therefore A—B is (by the 11 *Art.*) greater then C—D. Add B on both ſides, and A will be greater then C+B—D. And again, add D on both ſides, and A+D will be greater then B+C; which was to be proved.

21 If there be four Proportionals, the Extremes multiplyed into one another, & the Means multiplied into one another wil make equal Products. Let A.B : : C.D be Proportionals, I ſay A D is equal to B C. For the Proportion of A D to B C is compounded (by the 13 *Art.*) of the Proportions of A to B, and D to C, that is, its Inverſe B to A; and therefore (by the 14 *Art*) this Compounded Proportion is the Proportion of Equality; and therefore alſo the Proportion of A D to B C is the Proportion of Equality. Wherefore they are equal.

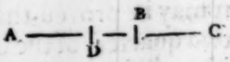
22 If there be four quantities, and the Proportion of the firſt to the ſecond be Duplicate of the Proportion of the third to the fourth, the Product of the Extremes to the Product of the Means will be as the third to the fourth. Let the four Quantities be A, B C and D; and let the Proportion of A to B be Duplicate of the Proportion of C to D, I ſay A D, that is, the Product of A into D, is to BC, that is, to the Product of the Means, as C to D. For ſeeing the Proportion of A to B is Duplicate of the proportion of C to D, if it be as C to D, ſo D to another E, then A.B : : C.E will be Proportionals;

portionals; for the proportion of A to B is by supposition duplicate of the Proportion of C to D; and C to E duplicate also of that of C to D by the Definition (15 *Art.*) Wherefore (by the last *Article*) A E or A into E, is equal to B C or B into C; But (by the 4 *Coroll.* of the 6 *Art.*) A D is to A E as D to E, that is, as C to D; and therefore A D is to B C (which as I have shewn, is equal to A E) as C to D; which was to be proved.

Moreover, If the proportion of the first A, to the second B be triplicate of the proportion of the third C to the fourth D, the Product of the Extremes to the product of the Means will be duplicate of the Proportion of the third to the fourth. For if it be as C to D so D to E, and again, as D to E so E to another F, then the proportion of C to F will be triplicate of the proportion of C to D; and consequently A.B : : C.F will be proportionals, and A F equal to B C. But as A D to A F, so is D to F; and therefore also as A D to B C, so D to F, that is, so C to E; But the proportion of C to E is duplicate of the proportion of C to D; Wherefore also the proportion of A D to B C is duplicate of that of C to D, as was propounded.

23 If there be four Proportionals, and a Mean be interposed betwixt the first and second, and another betwixt the third and fourth, the first of these Means will be to the second, as the first of the Proportionals is to the third, or as the second of them is to the fourth. For let A.B : : C.D be Proportionals, and let E be a Mean betwixt A and B, & F a Mean betwixt C and D; I say A.C : : E.F are Proportionals. For the proportion of A to E is Subduplicate of the proportion of A to B, or of C to D. Also the proportion of C to F is Subduplicate of that of C to D; and therefore A.E : : C.F are Proportionals; and by Permutation A.C : : E.F are also Proportionals; which was to be proved.

24 Any thing is said to be divided into *Extreme* and *Mean Proportion*, when the Whole and the Parts are in Continual Proportion. As (for example) when A + B. A. B ∺ are Continual Proportionals; or when the straight line A C is so divided in B, that A C. A B. B C ∺ are in Continual Proportion. And if the same Line A C be again divided in D, so as that A C. C D. A D ∺ be continual Proportionals; then also A C. A B. A D ∺ will be continual Proportionals; and in like manner, though in



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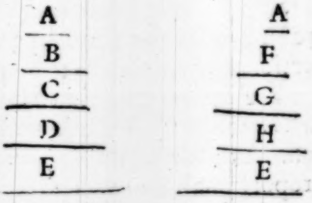
contrary order, CA.C D.CB ∺ will be continual Proportionals; which cannot happen in any Line otherwise divided.

25 If there be three continual Proportionals, and again, three other continual Proportionals which have the same Middle Term, their Extremes will be in reciprocal Proportion. For let A. B.C ∺ and D. B. E ∺ be continal Proportionals, I say A.D : : E. C shall be Proportionals. For the Proportion of A to D is compounded of the Proportions of A to B, and of B to D; and the Proportion of E to C is compounded of those of E to B, that is, of B to A, and of B to C, that is, of A to B. Wherefore (by Equality) A.D : : E.C. are Proportionals.

26 If any two unequal quantities be made Extremes, and there be interposed betwixt them any Number of Means in Geometrical Proportion, and the same Number of Means in Arithmetical Proportiō, the several Means in Geometrical Proportion wil be less then the several Means in Arithmetical proportion. For betwixt A the lesser, & E the greater Extreme, let there be interposed three Means B, A, D in Geometrical Proportion, & as many more F, G, H in Arithmetical Proportion; I say B will be less then F, A then G, & D then H. For first, the difference between A & F is the same with that between F & G, & with that between G & H (by the Definition of Arithmetical Proportion;) & therefore the difference of the proportionals which stand next to one another, to the difference of the Extremes, is, when there is but one Mean, half their difference; when two, a 3d part of it; when three, a quarter, &c. so that in this example it is a quarter. But the difference between D and E (by the 17. *Art.*) is more then a quarter of the difference between the Extremes, because the Proportion is Geometrical; & therefore the difference between A and D is less then 3 quarters of the same difference of the Extremes. In like manner, if the difference between A & D be understood to be divided into three equall parts,



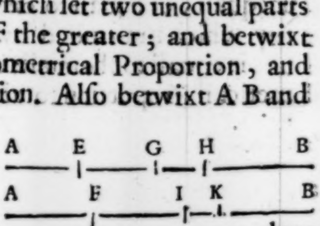
it may be proved, that the difference between A and C is less then two quarters of the difference of the Extremes A & E. And lastly, if the difference between A and C be divided into two equal parts, that the difference between A and B is less then a quarter of the difference of the Extremes A and E.

From

From the conſideration hereof it is manifeſt, that B, that is, A together with ſomething elſe which is leſs then a fourth part of the difference of the Extremes A and E, is leſs then F, that is, then the ſame A with ſomething elſe which is equal to the ſaid fourth part. Alſo, that C, that is A with ſomething elſe which is leſs then two fourth parts of the ſaid difference, is leſs then G, that is, then A together with the ſaid two fourths. And laſtly, that D which exceeds A by leſs then three fourths of the ſaid difference, is leſs then H, which exceeds the ſame A by three entire fourths of the ſaid difference. And in the ſame manner it would be if there were four Means, ſaving that in ſtead of fourths of the difference of the Extremes, we are to take fifth parts; and ſo on.

27 *Lemma.* If a quantity being given, firſt one quantity be both added to it and ſubſtracted from it, and then another greater or leſſe, the proportion of the Remainder to the Aggregate, is greater where the leſs quantity is added and ſubſtracted, then where the greater quantity is added and ſubſtracted. Let B be added to, and ſubſtracted from the quantity A, ſo that A—B be the Remainder, and A+B the Aggregate; And again let C, a greater quantity then B be added to, and ſubſtracted from the ſame A, ſo that A—C be the Remainder and A+C the Aggregate; I ſay A—B.A+B : : A—C. A+C will be an Hyperlogiſme. For A—B. A : : A—C.A is an Hyperlogiſme of a leſs Antecedent to the ſame Conſequent; and therefore A—B.A+B : : A—C. A+C is a much greater Hyperlogiſme, being made of a leſs Antecedent to a greater Conſequent.

28 If unequal parts be taken from two equal quantities, and betwixt the whole and the part of each there be interpoſed two Means, One in Geometrical, the other in Arithmetical proportion; The Difference betwixt the two Means will be greateſt, where the Difference betwixt the Whole and its part is greateſt. For let A B & A B be two equal quantities; from which let two unequal parts be taken, namely, A E the leſs, and A F the greater; and betwixt A B and A E let A G be a Mean in Geometrical Proportion, and A H, a Mean in Arithmetical Proportion. Alſo betwixt A B and A F let A I be a Mean in Geometrical Porportion, and A K a Mean in Arithmetical Proportion; I ſay H G is greater then K I. For in the firſt



place

place we have this Analogisme $AB . AG :: BG . GE$ (by the 18 *Art.*)

Then by Composition we have this $AB + AG . AB :: BG + GE$ (that is BE). BG.

And by taking the halves of the Antecedents this third $\frac{1}{2}AB + \frac{1}{2}AG . AB :: \frac{1}{2}BG + \frac{1}{2}GE$ (that is BH). BG.

And by Conversion a fourth $AB . \frac{1}{2}AB + \frac{1}{2}AG :: BG . BH$

And by Division this fifth $\frac{1}{2}AB - \frac{1}{2}AG . \frac{1}{2}AB + \frac{1}{2}AG :: HG . BH$.

And by doubling the first Antecedent and the first Consequent $\left.\right\}$ $AB - AG . AB + AG :: HG . BH$.

Also by the same method may be found out this Analogisme $\left.\right\}$ $AB - AI . AB + AI :: KI . BK$.

Now seeing the proportion of AB to AE is greater then that of AB to AF, the proportion of AB to AG, which is half the greater proportion, is greater then the proportion of AB to AI the half of the less Proportion; and therefore AI is greater then AG. Wherefore the proportion of $AB - AG$ to $AB + AG$ (by the precedent *Lemma*) will be greater then the proportion of $AB - AI$ to $AB + AI$; & therefore also the proportion of HG to BH will be greater then that of KI to BK, and much greater then the proportion of KI to BH, which is greater then BK, (for BH is the half of BE, as BK is the half of BF, which (by supposition) is less less then BE). Wherefore HG is greater then KI; which was to be proved.

Coroll. It is manifest from hence, that if any quantity be supposed to be divided into equal parts infinite in number, the difference between the Arithmetical and Geometrical Means will be infinitely little, that is, none at all. And upon this foundation chiefly, the Art of making those Numbers which are called *Logarithmes* seems to have been built.

29 If any number of quantities be propounded, whether they be unequall, or equall to one another; and there be another quantity which multiplied by the number of the propounded quantities is equall to them all; that other Quantity is a mean in Arithmeticall Proportion to all those propounded quantities.

CHAP.

CHAP. XIV.
Of Straight *and* Crooked, Angle *and* Figure.

1 The Definition and Properties of a Straight Line 2 The Definition and Properties of a Plain Superficies. 3 Several sorts of Crooked Lines. 4 The Definition and Properties of a Circular Line. 5 The Properties of a Straight Line taken in a Plain. 6 The Definition of Tangent Lines. 7 The Definition of an Angle, and the kindes thereof. 8 In Concentrick Circles, Arches of the same Angle are to one another as the whole Circumferences are. 9 The Quantity of an Angle in what it consists. 10 The Distinction of Angles simply so called. 11 Of Straight Lines from the Center of a Circle to a Tangent of the same. 12 The general Definition of Parallels; and the Properties of Straight Parallels. 13 The Circumferences of Circles are to one another as their Diameters are. 14 In Triangles, Straight Lines parallel to the Bases, are to one another, as the parts of the Sides which they cut off from the Vertex. 15 By what Fraction of a Straight Line the Circumference of a Circle is made. 16 That an Angle of Contingence is Quantity, but of a Different kinde from that of an Angle simply so called; and that it can neither add nor take away any thing from the same. 17 That the Inclination of Plains is Angle simply so called. 18 A Solid Angle what it is. 19 What is the Nature of Asymptotes. 20 Situation, by what it is determined. 21 What is like Situation; What is Figure; and what are like Figures.

1 BEtween two points given, the shortest Line is that, whose extreme points cannot be drawn further asunder, without altering the quantity, that is, without altering the proportion of that line to any other line given. For the Magnitude of a Line is computed by the greatest distance which may be between its extreme points; So that any one Line, whether it be extended, or bowed, has alwayes one and the same Length, because it can have but one greatest distance between its extreme points.

And seeing the action by which a Straight Line is made Crooked, or contrarily a Crooked Line is made Straight, is nothing but the bringing of its extreme points neerer to one another, or the setting of them further asunder, a CROOKED *Line* may rightly be defined to be *That, whose extreme points may be understood to be drawn further asunder*; and a STRAIGHT *Line* to be *That, whose extreme points cannot be drawn further asunder*; *and comparatively*, *A more Crooked, to be That line whose extreme points are neerer to one another then those of the other, (supposing both the Lines to be of equal Length.)* Now howsoever a Line be bowed, it makes alwayes a *Sinus* or Cavity, sometimes on one side, sometimes on another; So that the same Crooked Line may either have its whole Cavity on one Side onely, or it may have it part on one side and part on other sides. Which being well understood, it will be easie to understand the following Comparisons of Straight and Crooked *Lines*.

First, If a Straight & a Crooked Line have their Extreme points common, the Crooked Line is longer then the Straight Line. For if the extreme points of the Crooked Line be drawn out to their greatest distance, it wil be made a straight line, of which that which was a Straight Line from the beginning will be but a part; and therefore the Straight Line was shorter then the Crooked Line which had the same extreme points. And for the same reason, if two Crooked Lines have their extreme points common, and both of them have all their cavity on one and the same side, the outermost of the two will be the longest Line.

Secondly, A Straight Line and a perpetually Crooked Line cānot be coincident, no not in the least part. For if they should, then not onely some Straight Line would have its extreme points common with some Crooked Line, but also they would by reason of their coincidence, be equal to one another; which, as I have newly shewn, cannot be.

Thirdly, Between two points given there can be understood but one straight Line; because there cannot be more then one least Interval or Length between the same points. For if there may be two, they will either be coincident, and so both of them will be one Straight Line; or if they be not coincident, then the application of one to the other by extension, will make the extended Line

have

have its extreme points at greater distance then the other; and consequently it was Crooked from the beginning.

Fourthly, From this last it follows, that two Straight Lines cannot include a Superficies. For if they have both their extreme points common, they are coincident; and if they have but one, or neither of them common, then at one, or both ends, the extreme points will be disjoyned, and include no Superficies, but leave all open and undetermined.

Fifthly, Every part of a Straight Line is a Straight Line. For seeing every part of a Straight Line is the least that can be drawn between its own extreme points, if all the parts should not constitute a Straight Line, they would all together be longer then the whole Line.

2 A PLAIN, or a *Plain Superficies, is that which is described by a Straight Line so moved that all the several points thereof describe several Straight Lines.* A straight line therefore is necessarily all of it in the same Plain which it describes. Also the Straight Lines which are made by the points that describe a Plain, are all of them in the Same Plain. Moreover, if any Line whatsoever be moved in a Plain, the Lines which are described by it are all of them in the same Plain.

All other *Superficies* which are not Plain, are Crooked, that is, are either *Concave* or *Convex*. And the same Comparisons which were made of Straight and Crooked Lines: may also be made of Plain and Crooked Superficies. For

First, If a Plain and a Crooked Superficies be terminated with the same Lines, the Crooked Superficies is greater then the Plain Superficies. For if the Lines of which the Crooked Superficies consists be extended, they will be found to be longer then those of which the Plain Superficies consists, which cannot be extended because they are Straight.

Secondly, Two Superficies, whereof the one is Plain, and the other continually Crooked, cannot be coincident, no not in the least part. For if they were coincident they would be equal; nay, the same Superficies would be both Plain and Crooked, which is impossible.

Thirdly, Within the same terminating Lines, there can be no more then one Plain Superficies; because there can be but one least Superficies within the same.

Fourthly, No number of Plain Superficies can include a Solid, unleſs more then two of them end in a Common *Vertex*. For if two Plains have both the ſame terminating Lines, they are coincident, that is, they are but one Superficies; and if their terminating Lines be not the ſame, they leave one or more ſides open.

Fifthly, Every part of a Plain Superficies is a Plain Superficies. For ſeeing the whole Plain Superficies is the leaſt of all thoſe that have the ſame terminating Lines; and alſo every part of the ſame Superficies is the leaſt of all thoſe that are terminated with the ſame Lines; if every part ſhould not conſtitute a Plain Superficies, all the parts put together would not be equal to the whole.

3 Of *Straightneſs*, whether it be in Lines, or in Superficies there is but one kinde; but of *Crookedneſs* there are many kindes; for of Crooked Magnitudes, ſome are *Congruous*, that is, are coincident when they are applyed to one another; others are *Incongruous*. Again, ſome are ὁμοιομερεῖς, or *Uniform*, that is, have their parts howſoever taken, congruous to one another; others are ἀνομοιομερεῖς or of *ſeveral Forms*. Moreover, of ſuch as are Crooked, ſome are *Continually Crooked*, others *have parts which are not Crooked*.

4 If a Straight Line be moved in a Plain, in ſuch manner, that while one end of it ſtands ſtill, the whole Line be carried round about, til it come again into the ſame place from whence it was firſt moved, it will deſcribe a plain Superficies, which will be terminated every way by that Crooked Line which is made by that end of the Straight Line which was carried round. Now this Superficies is called a CIRCLE; and of this Circle, the Unmoved Point, is the the *Center*; the Crooked Line which terminates it, the *Perimeter*; and every part of that Crooked Line, a *Circumference* or *Arch*; the ſtraight Line which generated the *Circle*, is the *Semidiameter* or *Radius*; and any ſtraight Line which paſſeth through the Center, and is terminated on both ſides in the Circumference is called the *Diameter*. Moreover, every point of the *Radius* which deſcribes the Circle, deſcribes in the ſame time its own Perimeter, terminating its own Circle, which is ſaid to be *Concentrick* to all the other Circles, becauſe this and all thoſe have one common Center.

Wherefore in every Circle, all Straight Lines from the Center-

ter to the Circumference are equal. For they are all coincident with the Radius which generates the Circle.

Alſo the Diameter divides both the Perimeter and the Circle it ſelf into two equal parts. For if thoſe two parts be applyed to one another, and the Semiperimeters be coincident, then ſeeing they have one common Diameter, they will be equal; and the Semicircles will be equal alſo; for theſe alſo will be coincident. But if the Semiperimeters be not coincident, then ſome one ſtraight Line which paſſes through the Center (which Center is in the Diameter) will be cut by them in two points. Wherefore, ſeeing all the ſtraight Lines from the Center to the Circumference are equal, a part of the ſame ſtraight Line will be equal to the whole; which is impoſſible.

For the ſame reaſon the Perimeter of a Circle will be Uniform, that is, any one part of it will be coincident with any other equal part of the ſame.

5 From hence may be collected this property of a Straight Line, namely, that it is all conteined in that Plain which conteins both its extreme points. For ſeeing both its extreme points are in the Plain, that Straight Line which deſcribes the Plain will paſs through them both; and if one of them be made a Center, and at the diſtance between both, a Circumference be deſcribed, whoſe Radius is the Straight Line which deſcribes the Plain, that Circumference will paſs through the other point. Wherefore between the two propounded points, there is one ſtraight line (by the Definition of a Circle) conteined wholly in the propounded Plain; and therefore if another ſtraight Line might be drawn between the ſame points, and yet not be conteined in the ſame Plain, it would follow, that between two points two ſtraight lines may be drawn; which has been demonſtrated to be impoſſible.

It may alſo be collected, That if two Plains cut one another, their common ſection will be a ſtraight Line. For the two extreme points of the interſection are in both the interſecting Plains; and between thoſe points a ſtraight Line may be drawn; but a ſtraight Line between any two points is in the ſame Plain in which the Points are; and ſeeing theſe are in both the Plains, the ſtraight line which connects them will alſo be in both the ſame Plains, and therefore it is the cōmon ſection of both. And every other Line that

can

can be drawn between those points will be either coincident with that Line; that is, it will be the Same Line; or it will not be coincident; and then it wil be in neither, or but in one of those Plains.

As a straight Line may be understood to be moved round about whilest one end thereof remains fixed, as the Center; so in like manner it is easie to understand, that a Plain may be circumduced about a straight line, whilest the straight line remaines still in one and the same place, as the *Axis* of that motion. Now from hence it is manifest, that any three Points are in some one Plain. For as any two Points, if they be connected by a straight Line, are understood to be in the same Plaine in which the straight Line is; so, if that Plaine be circumduced about the same straight Line, it will in its revolution take in any third Point, howsoever it be situate; and then the three Points will be all in that Plaine; and consequently the three straight Lines which connect those Points, will also be in the same Plain.

6 Two Lines are said to *Touch* one another, which being both drawne to one and the same point, will not cut one another, though they be produced, produced I say in the same manner in which they were generated. And therefore if two straight Lines touch one another in any one point, they wil be contiguous through their whole length. Also two Lines continually crooked wil do the same, if they be congruous, and be applyed to one another according to their congruity; otherwise, if they be incongruously applyed, they will, as all other crooked Lines, touch one another (where they touch) but in one point onely. Which is manifest from this, that there can be no congruity between a straight line and a line that is continually crooked; for otherwise the same line might be both straight and crooked. Besides when a straight line touches a crooked line, if the straight line be never so little moved about upon the point of contact, it will cut the crooked line; for seeing it touches it but in one point, if it incline any way, it will do more then touch it; that is, it will either be congruous to it, or it will cut it; but it cannot be congruous to it; and therefore it will cut it.

7 An Angle, according to the most general acception of the word, may be thus defined; *When two Lines, or many Superficies concurre in one sole point, and diverge every where else, the quantity of*

that

that divergence is an ANGLE. And an Angle is of two sorts; for first it may be made by the concurrence of Lines, and then it is a *Superficiall Angle*; or by the concurrence of Superficies, and then it is called a *Solid Angle*.

Again, from the two wayes by which two lines may diverge from one another, Superficial Angles are divided into two kindes. For two straight lines which are applyed to one another, and are contiguous in their whole length, may be separated or pulled open in such manner, that their concurrence in one point will still remain; And this Separation or Opening may be either by Circular Motion, the Center whereof is their point of concurrence, and the Lines will still retein their straightness, the quantity of which Separation, or Divergence is an *Angle* simply so called; Or they may be separated by continual Flexion or Curvation in every imaginable point; and the quantity of this Separation is that which is called an *Angle of Contingence*.

Besides of Superficial Angles simply so called, those which are in a plain Superficies are Plain; and those which are not plain, are denominated from the Superficies in which they are.

Lastly, those are *Straight-lined Angles* which are made by straight lines; as those which are made by crooked lines are *Crooked-lined*; and those which are made both of straight and crooked lines, are *Mixed Angles*.

8 Two Arches intercepted between two *Radii* of Concentrick Circles, have the same proportion to one another, which their whole Perimeters have to one another. For let the point A (in the first Figure) be the Center of the two Circles B C D and E F G; in which the *Radii* A E B and A F C intercept the Arches B C and E F; I say the proportion of the Arch B C to the Arch E F, is the same with that of the Perimeter B C D to the Perimeter E F G. For if the Radius A F C be understood to be moved about the Center A with Circular & Uniform Motion, that is, with equal Swiftness every where, the point C will in a certain time describe the Perimeter B C D, and in a part of that time the Arch B C; and because the Velocities are equal by which both the Arch and the whole Perimeter are described, the proportion of the magnitude of the Perimeter B C D to the magnitude of the Arch B C, is determined by nothing

thing but the difference of the times in which the Perimeter and the Arch are described. But both the Perimeters are described in one and the same time, and both the Arches in one and the same time; and therefore the proportions of the Perimeter B C D to the Arch B C, and of the Perimeter E F G to the Arch E F, are both determined by the same cause. Wherefore B C D, B C :: E F G. E F are Proportionals (by the *6 Art.* of the last Chapter), and by Permutation B C D. E F G :: B C. E F will also be Proportionals, which was to be demonstrated.

9 Nothing is contributed towards the Quantity of an Angle, neither by the Length, nor by the Equality, nor by the Inequality of the lines which comprehend it. For the lines AB and AC comprehend the same Angle which is comprehended by the lines A E and A F, or A B and A F. Nor is an Angle either increased or diminished by the absolute quantity of the Arch which subtends the same; for both the greater Arch B C, and the lesser Arch E F are subtended to the same Angle. But the quantity of an Angle is estimated by the quantity of the subtending Arch compared with the quantity of the whole Perimeter. And therefore the Quantity of an Angle simply so called may be thus defined.

The Quantity of an Angle, is an Arch or Circumference of a Circle, determined by its proportion to the whole Perimeter. So that when an Arch is intercepted between two straight lines drawn from the Center, look how great a portion that Arch is of the whole Perimeter, so great is the Angle. From whence it may be understood, that when the lines which contein an Angle are straight lines, the quantity of that Angle may be taken at any distance from the Center. But if one, or both of the conteining lines be crooked, then the quantity of the Angle is to be taken in the least distance from the Center, or from their concurrence; for the least distance is to be considered as a straight line, seeing no crooked line can be imagined so little, but that there may be a less straight line. And although the least straight line cannot be given, because the least given line may still be divided, yet we may come to a part so small, as is not at all considerable; which we call a Point. And this point may be understood to be in a straight line which touches a crooked line; for an Angle is generated by separating by circular motion one straight line from another which touches it, (as has been said above in the

7th

7th *Article).* Wherefore an Angle which two crooked lines make is the same with that which is made by two straight lines which touch them.

10 From hence it follows, that *Vertical Angles*, such as are ABC, D B F (in the second Figure) are equal to one another. For if from the two Semiperimeters D A C, F D A which are equal to one another, the common Arch D A be taken away, the remayning Arches A C, D F will be equal to one another.

Another distinction of Angles is, into *Right* and *Oblique. A Right Angle is that, whose quantity is the fourth part of the Perimeter.* And the lines which make a Right Angle are said to be *Perpendicular* to one another. Also of Oblique Angles, that which is greater then a Right, is called an *Obtuse Angle*; and that which is less, an *Acute Angle.* From whence it follows, that all the Angles that can possibly be made at one and the same Point, together taken, are equal to four Right Angles; because the quantities of them all put together make the whole Perimeter. Also, that all the Angles which are made on one side of a straight line from any one point taken in the same, are equal to two Right Angles, for if that point be made the Center, that straight line will be the Diameter of a Circle, by whose Circumference the quantity of an Angle is determined; and that Diameter will divide the Perimeter into two equal parts.

11 If a Tangent be made the Diameter of a Circle whose Center is the point of Contact, a straight line drawn from the Center of the former Circle to the Center of the later Circle, will make two Angles with the Tangent (that is, with the Diameter of the later Circle) equal to two Right Angles (by the last *Article*). And because (by the 6th *Article*) the Tangent has on both sides equal inclination to the Circle, each of them will be a Right Angle; as also the Semidiameter will be perpendicular to the same Tangent. Moreover, the Semidiameter, in as much as it is the Semidiameter, is the least straight line which can be drawn frō the Center to the Tangent; and every other straight line that reaches the Tangent will pass out of the Circle, and will therefore be greater then the Semidiameter. In like manner, of all the straight lines which may be drawn from the Center to the Tangent, that is the greatest which makes the greatest Angle with the Perpenpicular; which

will be manifest, if about the same Center another Circle be described, whose Semidiameter is a straight line taken neerer to the Perpendicular, and there be drawn a Perpendicular (that is, a Tangent) to the same.

From whence it is also manifest, that if two straight lines which make equal Angles on either side of the Perpendicular, be produced to the Tangent, they will be equal.

12 There is in *Euclide* a Definition of Straight-lined Parallels; but I do not find that Parallels in general, are any where defined; and therefore for an Universal definition of them, I say, that *Any two lines whatsoever (Straight or Crooked) as also any two Superficies*, *are* PARALLEL, *when two equal straight lines wheresoever they fall upon them, make always equal Angles with each of them.* From which Definition it follows.

First, that any two straight lines not enclined opposite wayes, falling upon two other straight lines which are Parallel, and intercepting equal parts in both of them, are themselves also equal and Parallel. As if A B and C D (in the third Figure) enclined both the same way (fall upon the Parallels A C and B D, and A C and B D be equal, A B and C D will also be equal and Parallel. For the Perpendiculars B E and D F being drawn, the Right Angles E B D and F D H will be equal. Wherefore seeing E F and B D are parallel, the angles E B A and F D C will be equal. Now if D C be not equal to B A, let any other straight line equal to B A be drawn from the point D; which seeing it cannot fall upon the point C, let it fall upon G. Whereore AG will be either greater or less then B D; and therefore the angles E B A and F D G are not equal, as was supposed. Wherefore A B and C D are equal; which is the first.

Again, because they make equal Angles with the Perpendiculars B E and D F; therefore the Angle C D H will be equal to the Angle A B D, and (by the Definition of Parallels) A B and C D will be parallel; which is the second.

That Plain which is included both wayes within parallel lines is called a PARALLELOGRAM.

1 *Corollary.* From this last it follows, That the Angles A B D and C D H are equal; that is, that a straight line (as B H) falling upon two Parallels (as A B and C D) makes the internal Angle A B D equal to the external and opposite Angle C D H. 2 *Co-*

2 *Coroll.* And from hence again it follows, that a straight line falling upon two Parallels, makes the alternate Angles equal; that is, the Angle A G F (in the fourth figure) equal to the Angle G F D. For seeing G F D is equal to the external opposite Angle E G B, it will be also equal to its vertical Angle A G F, which is alternate to G F D.

3 *Coroll.* That the internal Angles on the same side of the line F G are equal to two Right Angles. For the Angles at F, namely G F C and G F D are equal to two Right Angles. But G F D is equal to its alternate Angle A G F. Wherefore both the Angles G F C and A G F which are internal on the same side of the line F G, are equal to two Right Angles.

4 *Coroll.* That the three Angles of a straight-lined plain Triangle are equal to two Right Angles; and any side being produced, the external Angle will be equal to the two opposite internal Angles. For if there be drawn by the Vertex of the plain Triangle A B C (figure 5.) a Parallel to any of the sides, as to A B, the Angles A and B will be equal to their alternate Angles E & F, & the Angle C is common. But (by the 10th *Article*) the three Angles E, C and F are equal to two Right Angles; and therefore the three Angles of the Triangle are equal to the same; which is the first. Again, the two Angles B and D are equal to two Right Angles (by the 10th *Article*). Wherefore taking away B, there will remain the Angles A and C equal to the Angle D; which is the second.

5 *Coroll.* If the Angles A and B be equal, the sides A C and C B will also be equal, because A B and E F are parallel; And on the contrary, if the sides A C and C B be equal, the Angles A and B will also be equal. For if they be not equal, let the Angles B and G be equal. Wherefore seeing G B and E F are parallels, and the Angles G and B equal, the sides G C and C B will also be equal; and because C B and A C are equal by supposition, C G and C A will also be equal; which cannot be (by the 11th *Article*.)

6 *Coroll.* From hence it is manifest, that if two *Radii* of a Circle be connected by a straight Line, the Angles they make with that connecting Line will be equal to one another; and if there be added that segment of the Circle which is subtended by the same line which connects the *Radii*, then the Angles which those *Radii* make with the circumference wil also be equal to one another. For

a ſtraight-line which ſubtends any Arch, makes equal Angles with the ſame, becauſe if the Arch and the Subtenſe be divided in the middle, the two halves of the ſegment wil be congruous to one another, by reaſon of the Uniformity both of the Circumference of the Circle and of the ſtraight Line.

13 Perimeters of Circles are to one another as their Semidiameters are. For let there be any two Circles, as (in the firſt figure) B C D the greater, and E F G the leſſer, having their common Center at A; and let their Semidiameters be A C and A E. I ſay A C has the ſame proportion to A E which the Perimeter B C D has to the Perimeter E F G. For the magnitude of the Semidiameters A C and A E is determined by the diſtances of the points C and E from the Center A; and the ſame diſtances are acquired by the uniform motion of a point from A to C in ſuch manner that in equal times the diſtances acquired be equal. But the Perimeters B C D and E F G are alſo determined by the ſame diſtances of the points C and E from the Center A; and therefore the Perimeters B C D and E F G, as well as the Semidiameters A C and A E have their magnitudes determined by the ſame cauſe, which cauſe makes in equal times equal ſpaces. Wherefore (by the 13 Chapter and 6th *Article*) the Perimeters of Circles and their Semidiameters are Proportionals; which was to be proved.

14 If two ſtraight Lines wch cōſtitute an Angle, be cut by ſtraight-lined Parallels, the intercepted Parallels will be to one another, as the parts wch they cut off frō the Vertex. Let the ſtraight lines AB and A C (in the 6 figure) make an Angle at A, & be cut by the two ſtraight-lined Parallels B C and D E, ſo that the parts cut off from the Vertex in either of thoſe Lines (as in A B) may be A B and A D. I ſay the Parallels B C and D E are to one another as the parts A B and A D. For let A B be divided into any number of equal parts, as into AF, F D, D B; and by the points F and D, let FG and D E be drawn Parallel to the baſe B C, and cut A C in G and E; and again by the points G and E let other ſtraight lines be drawn Parallel to A B, and cut B C in H and I. If now the point A be underſtood to be moved uniformly over A B, and in the ſame time B be moved to C, and all the points F, D and B be moved uniformly and with equal Swiftneſs over F G, D E, and B C; then ſhall B paſs over B H (equal to F G) in the ſame time that A paſſes over A F; and A F and F G will be to one another as their Velocities

locities are; and when A is in F, D will be in K; when A is in D, D will be in E; and in what manner the point A passes by the points F, D and B, in the same manner the point B will pass by the points H, I and C; & the straight lines F G, D K, K E, B H, H I & I C are equal by reason of their Parallelisme; and therefore, as the velocitie in A B is to the velocity in B C, so is A D to DE; but as the velocity in A B is to the velocity in B C, so is A B to B C; that is to say, all the Parallels will be severally to all the parts cut off from the Vertex, as A F is to F G. Wherefore, AF.GF :: A D.D E :: A B, B C are Proportionals.

The Subtenses of equal Angles in different Circles (as the straight lines B C and F E (in the 1 figure) are to one another as the Arches which they subtend. For (by the 8th *Article*) the Arches of equal Angles are to one another as their Perimeters are; and (by the 13th *Art.*) the Perimeters as their Semidiameters; But the the Subtenses B C and F E are parallel to one another by reason of the equality of the Angles which they make with the Semidiameters; and therefore the same Subtenses (by the last precedent *Article*) will be proportional to the Semidiameters, that is, to the Perimeters, that is, to the Arches which they subtend.

15 If in a Circle any number of equal Subtenses be placed immediatly after one another, and straight lines be drawn from the extreme point of the first Subtense to the extreme points of all the rest, The first Subtense being produced will make with the second Subtense an external Angle double to that which is made by the same first Subtense and a Tangent to the Circle touching it in the extreme point thereof; and if a straight line which subtends two of those Arches be produced, it will make an external Angle with the third Subtense, triple to the Angle which is made by the Tangent with the first Subtense; and so continually. For with the Radius AB (in the 7th figure) let a circle be described, & in it let any number of equal Subtenses BC, CD & D E be placed; also let B D & BE be drawn; & by producing B C, BD & BE to any distance in G, H and I, let them make Angles with the Subtenses which succeed one another, namely the external Angles G C D, and H D E. Lastly, let the Tangent K B be drawn, making with the first Subtense the Angle K B C. I say the Angle G C D is double to the Angle K B C, and the Angle H D E triple to the same Angle K B C. For if A C be drawn cutting B D in M, and from the point C

there

there be drawn L C perpendicular to the same A C, then C L and M D will be parallel by reason of the right Angles at C and M; and therefore the alterne Angles L C D and B D C wil be equal; as also the Angles B D C and C B D will be equal because of the equality of the straight lines B C and C D. Wherefore the Angle G C D is double to either of the Angles C B D or C D B; and therefore also the Angle G C D is double to the Angle L C D, that is, to the Angle K B C. Again, C D is parallel to B E by reason of the equality of the Angles C B E and D E B, and of the straight lines C B and D E; and therefore the Angles G C D and G B E are equal; and consequently G B E, as also D E B is double to the Angle K B C. But the external Angle H D E is equal to the two internal D E B and D B E; and therefore the Angle H D E is triple to the Angle K B C, &c. which was to be proved.

1 *Corollary*. From hence it is manifest, that the Angles K B C and C B D, as also, that all the Angles that are comprehended by two straight lines meeting in the circumference of a Circle and insisting upon equal Arches, are equal to one another.

2 *Coroll*. If the Tangent B K be moved in the Circumference with Uniform motion about the Center B, it will in equal times cut off equal Arches; and will pass over the whole Perimeter in the same time in which it self describes a semiperimeter about the Center B.

3 *Coroll*. From hence also we may understand what it is that determines the bending or *Curvation* of a straight line into the circumference of a Circle; namely, that it is *Fraction* continually encreasing in the same manner as Nūbers from *One* upwards encrease by the continual addition of *Unity*. For the indefinite straight Line K B being *broken* in B according to any Angle, as that of KBC, & again in C according to a double Angle, and in D according to an Angle which is triple, and in E according to an Angle which is quadruple to the first Angle, and so continually, there will be described a figure which will indeed be *rectilineal* if the broken parts be considered as having magnitude; but if they be understood to be the least that can be, that is, as so many Points; then the figure described will not be rectilineal, but a Circle, whose Circumference will be the broken line.

4 *Coroll*. From what has been said in this present *Article* it may also be demonstrated, that *An Angle in the center is double to an Angle*

in

in the Circumference of the same Circle, if the intercepted Arches be equal. For seeing that straight Line by whose motion an Angle is determined, passes over equal Arches in equal times, as well from the Center as from the Circumference; and while that which is from the Circumference is passing over half its own Perimeter, it passes in the same time over the whole Perimeter of that which is from the Center, the Arches wch it cuts off in the Perimeter whose Center is A, wil be double to those which it makes in its own Semiperimeter whose Center is B. But in equal Circles, as Arches are to one another, so also are Angles.

It may also be demonstrated that the external Angle made by a Subtense produced and the next equal Subtense, is equal to an Angle from the Center insisting upon the same Arch; As in the last Diagram, the Angle G C D is equal to the Angle C A D; For the external Angle G C D is double to the Angle C B D; and the Angle C A D insisting upon the same Arch C D, is also double to the same Angle C B D or K B C.

16 An Angle of Contingence, if it be compared with an Angle simply so called how little soever, has such proportion to it as a Point has to a Line; that is, no proportion at all, nor any quantity. For first, an Angle of cōtingence is made by cōtinual flexion, so that in the generation of it there is no circular motion at all, in which consists the nature of an Angle simply so called; and therefore it cannot be compared with it according to Quantity. Secondly, seeing the external Angle made by a Subtense produced and the next Subtense, is equal to an Angle from the Center insisting upon the same Arch (as in the last figure the Angle G C D is equal to the Angle C A D) the Angle of Contingence wil be equal to that Angle from the Center which is made by A B and the same A B; for no part of a Tangent can subtend any Arch; but as the point of Contact is to be taken for the Subtense, so the Angle of Contingence is to be accounted for the external Angle, and equal to that Angle whose Arch is the same point B.

Now seeing an Angle in general is defined to be the Opening or Divergence of two lines which concurre in one sole point; & seeing one Opening is greater then another, it cānot be denied but that by the very generation of it an Angle of Contingence is Quantity; for wheresoever there is Greater and Less, there is also Quantity;

but

but this quantity consists in greater and less Flexion; for how much the greater a Circle is, so much the neerer comes the Circumference of it to the nature of a straight Line; for the Circumference of a Circle being made by the curvation of a straight line, the less that straight line is, the greater is the curvation; & therfore when one straight line is a Tangent to many Circles, the Angle of Contingence which it makes with a less Circle is greater then that which it makes with a greater Circle.

Nothing therefore is added to, or taken from an Angle simply so called, by the addition to it or taking from it of never so many Angles of Contingence. And as an Angle of one sort can never be equal to an Angle of the other sort, so they cannot be either greater or less then one another.

From whence it follows, that an Angle of a Segment, that is, the Angle which any straight line makes with any Arch, is equal to the Angle which is made by the same straight line, & another which touches the Circle in the point of their Concurrence; as in the last figure, the Angle which is made between G B and B K is equal to that which is made between G B and the Arch B C.

17 An Angle which is made by two Plains, is commonly called the *Inclination* of those Plains; And because Plains have equal Inclination in all their parts, instead of their Inclination an Angle is taken which is made by two straight lines, one of which is in one, the other in the other of those Plains, but both perpendicular to the common Section.

18 A Solid Angle may be conceived two wayes. First, for the aggregate of all the Angles which are made by the motion of a straight line, while one extreme point thereof remayning fixed, it is carried about any plain figure in which the fixed point of the straight line is not conteined. And in this sense it seems to be understood by *Euclide*. Now it is manifest that the quantity of a Solid Angle so conceived is no other then the aggregate of all the Angles in a Superficies so described, that is, in the Superficies of a Pyramidal Solid. Secondly, when a Pyramis or Cone has its Vertex in the Center of a Sphere, a Solid Angle may be understood to be the proportion of a Spherical Superficies subtending that Vertex, to the whole Superficies of the Sphere. In which sense solid Angles are to one another as the

the Spherical Bases of Solids which have their Vertex in the Center of the same Sphere.

19 All the wayes by which two lines respect one another, or all the variety of their position may be comprehended under four kindes; For any two lines whatsoever are either *Parallels* or being produced (if need be) or moved one of them to the other parallelly to it self, they make an *Angle*; or else (by the like production and motion) they *Touch* one another; or lastly, they are *Asymptotes*. The nature of Parallels, Angles and Tangents has been already declared. It remains that I speak briefly of the nature of *Asymptotes*.

Asymptosy depends upon this, that Quantity is infinitly divisible. And from hence it follows, that any line being given, and a Body supposed to be moved from one extreme thereof towards the other, it is possible (by taking degrees of Velocity alwayes lesse and lesse in such proportion as the parts of the Line are made lesse by continual division) that the same Body may be alwayes moved forwards in that Line, and yet never reach the end of it. For it is manifest that if any straight Line, as AF (in the 8th figure) be cut any where in B, and again BF be cut in C, and CF in D, and DF in E, and so eternally, and there be drawn from the point F the straight Line FF at any Angle AFF; and lastly, if the straight Lines AF, BF, CF, DF, EF, &c. having the same proportion to one another with the Segments of the Line AF, be set in order and parallel to the same AF, the crooked Line ABCDE and the straight Line FF will be *Asymptotes*, that is, they will alwayes come neerer and neerer together, but never touch one another. Now because any Line may be cut eternally according to the proportions which the Segments have to one another, therefore the divers kindes of *Asymptotes* are infinite in number, and not necessary to be further spoken of in this place. In the nature of Asymptotes in general there is no more, then that they come still neerer and neerer but never touch. But in special in the Asymptosie of Hyperbolique Lines, it is understood they should approach to a distance lesse then any given quantity.

20 SITUATION *is the relation of one place to another*; & where there are many places, their *Situation* is determined by four things; By

their Distances from one another; By *several Distances from a place assigned*; By *the order of straight lines drawn from a place assigned to the places of them all*; and by *the Angles which are made by the lines so drawn*. For if their Distances, Order, and Angles be given, that is, be certainly known, their several places will also be so certainly known, as that they can be no other.

31 Points, how many soever they be, have *Like Situation* with an equal number of other Points, when all the straight lines that are drawn from some one point to all these, have severally the same proportion to those that are drawn in the same order and at equal Angles from some one point to all those. For let there be any number of Points as A, B and C, (in the 9 figure), to which from some one point D let the straight Lines D A, D B and D C be drawn; and let there be an equal number of other Points as E, F and G, and from some point H let the straight Lines H E, H F and H G be drawn, so that the Angles A D B and B D C be severally and in the same order equal to the Angles E H F and F H G, and the straight Lines D A, D B and D C proportional to the straight Lines H E, H F and H G; I say the three points A, B and C, have Like Situation with the three points E, F & G, or are placed Alike. For if H E be understood to be layed upon D A, so that the point H be in D, the point F will be in the straight Line D B by reason of the equality of the Angles A D B and E H F; and the point G will be in the straight Line D C by reason of the equality of the Angles B D C aud F H G; and the strright Lines A B and E F, as also B C and F G will be parallel, because A D. E D : : B H. F H : : C D. G H are Proportionals by construction; and therefore the distances between the points A and B, and the points B and C, will be proportional to the distances between the points E and F, and the points F and G. Wherefore in the situation of the points A, B and C, and the situation of the points E, F and G the Angles in the same order are equal, so that their situations differ in nothing but the inequality of their distances from one another, and of their distances from the points D and H. Now in both the orders of Points those inequalities are equal; for A B. B C : : E F. F G, which are their distances from one another, as also D A. D B. D C : : H E. H F. H G which are their distances from the assumed points D and H, are Proportionals. Their difference therefore con-

consists solely in the magnitude of their distances. But by the definition of *Like* (*Chap.*11.*Art*,1) those things which differ onely in Magnitude are Like. Wherefore the points A,B and C have to one another Like Situation with the points E,F and G, or are placed Alike; which was to be proved.

FIGURE, *is quantity determined by the Situation, or placing of all its extreme Points.* Now I call those points Extreme which are contiguous to the place which is without the figure. In Lines therefore and Superficies all Points may be called Extreme, but in Solids onely those which are in the Superficies that includes them.

Like Figures, are those, whose extreme points in one of them, are all placed like all the extreme points in the other; for such Figures differ in nothing but Magnitude.

And like Figures are *alike placed*, when in both of them the homologal straight lines, that is, the straight lines which connect the points which answer one another are parallel, and have their proportional sides enclined the same way.

And seeing every Straight Line is like every other Straight Line, and every Plain like every other Plain when nothing but Plainness is considered; if the Lines which include Plains, or the Superficies which include Solids have their proportions known, it will not be hard to know whether any Figure be like or unlike to another propounded Figure.

And thus much concerning the *First Grounds of Philosophy*. The next place belongs to *Geometry*; in which the Quantities of Figures are sought out from the Proportions of Lines and Angles. Wherefore it is necessary for him that would study Geometry to know first what is the nature of Quantity, Proportion, Angle and Figure. Having therefore explained these in the three last Chapters, I thought fit to add them to this Part; and so passe to the next.

consists solely in the magnitude of their distances. But by the definition of Like (Chap. 1. Art. 5.) those things which differ only in Magnitude are Like. Wherefore the [illegible] A E and C [illegible] one another [illegible] A [illegible] [illegible] centre.

[illegible]

[illegible] or plain figures [illegible] I call [illegible] points Extremes, which are contiguous [illegible] and [illegible] [illegible] Lines therefore are one [illegible] [illegible] in Solids [illegible]

[illegible] are those, which [illegible] points [illegible] of them are all placed like all the extreme points in the others; for such figures differ in nothing but Magnitude.

And like Figures are alike placed, when in both of them the homological straight lines, that is, the straight lines which connect the points which answer one another, are parallel; and have their proportional sides enclined the same way.

And seeing every Straight Line is like every other Straight Line; and every Plain like every other Plain when nothing but Plainness is considered; if the Lines which include Plains, or the Superficies which include Solids have their proportions known, it will not be hard to know whether any Figure be like or unlike to another propounded Figure.

And thus much concerning the first Grounds of Philosophy. The next place belongs to Geometry, in which the Quantities of Figures are sought out from the proportions of Lines and Angles. Wherefore it is necessary [illegible] what is the nature of Quantity, Proportion, Angle and Figure. Having therefore explained these in the three last Chapters, I thought fit to add [illegible] and so passe to the next.

OF

OF THE PROPORTIONS OF MOTIONS AND MAGNITUDES.

CHAP. XV.

Of the Nature, Properties, and diverse Considerations of Motion and Endeavour.

1 *Repetition of some Principles of the doctrine of Motion formerly set down. 2. Other Principles added to them. 3 Certain Theoremes concerning the nature of Motion. 4 Diverse Considerations of Motion. 5 The way by which the first Endeavour of Bodies Moved tendeth. 6 In Motion which is made by Concourse, one of the Movents ceasing, the Endeavour is made by the way by which the rest tend. 7 The Endeavour of any Moved Body, which having its Motion in the Circumference of a Circle, parts from the same, proceeds afterwards in a straight line which toucheth the Circle. 8 How much greater the Velocity or Magnitude is of a Movent, so much greater is the Efficacy thereof upon any other Body in its way.*

1 THe next things in order to be treated of are MOTION and MAGNITUDE, which are the most common Accidents of all Bodies. This place therefore most properly belongs to the Elements of Geometry. But because this part of Philosophy having been improved by the best Wits of all Ages has afforded greater plenty of matter then can well be thrust together within the narrow limits of this discourse; I thought fit to admonish the Reader, that before he pro-

proceed further, he take into his hands the Works of *Euclide*, *Archimedes*, *Apollonius* and other as well Ancient as Modern Writers. For to what end is it to do over again that which is already done? The little therefore that I shall say concerning Geometry in some of the following Chapters, shall be such onely as is new, and conducing to Natural Philosophy.

I have already delivered some of the Principles of this doctrine in the 8 & 9 Chapters, which I shall briefly put together here, that the Reader in going on may have their light neerer at hand.

First therefore in the 8th Chap. and 10 *Article*, *Motion* is defined to be *the continual privation of one place, and acquisition of another.*

Secondly, it is there shewn, that *Whatsoever is Moved is Moved in Time.*

Thirdly, in the same Chap. 11. *Article*, I have defined *Rest to be when a Body remains for some time in one place.*

Fourthly, it is there shewn, that *Whatsoever is Moved is not in any determined place*; as also that the same *has been Moved, is still Moved, and will yet be Moved*; So that in every part of that Space in which Motion is made, we may consider three Times, namely the *Past*, the *Present*, and the *Future Time.*

Fiftly, in the 15 *Article* of the same Chapter, I have defined *Velocity* or *Swiftness to be Motion considered as Power, namely, that Power by which a Body Moved may in a certain Time transmit a certain Length*; which also may more briefly be enunciated thus, *Velocity is the quantity of Motion determined by Time and Line.*

Sixthly, in the same Chap. 16. *Article*, I have shewn that *Motion is the Measure of Time.*

Seventhly, in the same Chap. 17th *Art.* I have defined Motions to be Equally Swift, when in Equal Times Equal Lengths are transmitted by them.

Eighthly, in the 18 *Article* of the same Chapter, *Motions* are defined to be *Equal, when the Swiftness of one Moved Body computed in every part of its magnitude, is equal to the Swiftness of another computed also in every part of its magnitude.* From whence it is to be noted, that *Motions Equal to one another, and Motions Equally Swift*, do not signifie the same thing; for when two horses draw abrest, the Motion of both is greater then the Motion of either of them singly; but the Swiftness of both together is but Equal to that of either.

Ninthly, in the 19 *Article* of the same Chapter, I have shewn, that

that *Whatsoever is at Rest will alwayes be at Rest, unless there be some other Body besides it, which by getting into its place, suffers it no longer to remain at Rest.* And that *Whatsoever is Moved, will alwayes be Moved, unless there be some other Body besides it, which hinders its Motion.*

Tenthly, In the 9 Chapter and 7 *Article*, I have demonstrated, that *When any Body is moved which was formerly at Rest, the immediate efficient cause of that Motion is in some other Moved and Contiguous Body.*

Eleventhly, I have shewn in the same place, that *Whatsoever is Moved, will always be Moved in the same way, and with the same Swiftness, if it be not hindered by some other Moved and Contiguous Body.*

2 To which Principles I shall here add these that follow. First, I define ENDEAVOUR *to be Motion made in less Space and Time then can be given*; that is, *less then can be determined or assigned by Exposition or Number*; that is, *Motion made through the length of a Point, and in an Instant or Point of Time.* For the explayning of which Definition it must be remembred, that by a Point is not to be understood that which has no quantity, or which cannot by any means be divided (for there is no such thing in Nature); but that whose quantity is not at all considered, that is, whereof neither quantity nor any part is computed in demonstration; so that a Point is not to be taken for an Indivisible, but for an Undivided thing; as also an Instant is to be taken for an Undivided, and not for an Indivisible Time.

In like manner Endeavour is to be conceived as Motion; but so, as that neither the quantity of the Time in which, nor of the Line in which it is made may in demonstration be at all brought into comparison with the quantity of that Time, or of that Line of which it is a part. And yet, as a Point may be compared with a Point, so one Endeavour may be compared with another Endeavour, and one may be found to be greater or lesse then another. For if the Vertical points of two Angles be compared, they will be equal or unequal in the same proportion which the Angles themselves have to one another. Or if a straight Line cut many Circumferences of Concentrick Circles, the inequality of the points of intersection will be in the same proportion which the Perimeters have to one another. And in the same manner, if two Motions begin and end both together, their Endeavours will be Equal or Unequal according to the proportion of their Velocities; as we

see

see a bullet of Lead descend with greater Endeavour then a ball of Wooll,

Secondly, I define IMPETUS *or Quickness of Motion, to be the Swiftness or Velocity of the Body moved, but considered in the several points of that time in which it is moved; In which sense* Impetus *is nothing else but the quantity or velocity of Endeavour. But considered with the whole time, it is the whole velocity of the Body moved, taken together throughout all the time, and equal to the Product of a Line representing the time multiplyed into a Line representing the arithmetically mean* Impetus *or Quickness.* Which Arithmetical Mean what it is, is defined in the 29th *Article* of the 13th Chapter.

And because in equal times the wayes that are passed are as the Velocities, and the *Impetus* is the Velocity they go withal reckoned in all the several points of the times, it followeth that during any time whatsoever, howsoever the *Impetus* be encreased or decreased, the length of the way passed over shall be encreased or decreased in the same proportion; and the same Line shall represent both the way of the Body moved, and the several *Impetus* or degrees of Swiftness wherewith the way is passed over.

And if the Body moved be not a point, but a straight line moved so as that every point thereof make a several straight line, the Plain described by its motion, whether Uniform, Accelerated or Retarded, shall be greater or less (the time being the same) in the same proportion with that of the *Impetus* reckoned in one motion to the *Impetus* reckoned in the other. For the reason is the same in Parallelograms and their Sides,

For the same cause also if the Body moved be a Plain, the Solid described shall be still greater or less in the proportions of the several *Impetus* or Quicknesses reckoned through one Line, to the several *Impetus* reckoned through another.

This understood, let A B C D (in the first figure of the 17th Chapter) be a Parallelogram; in which suppose the side A B to be moved parallelly to the opposite side CD, decreasing al the way till it vanish in the point C, and so describing the figure ABE FC; the point B as A B decreaseth, will therefore describe the Line B E F C; and suppose the time of this motion designed by the line C D; and in the same time CD suppose the side A C to be moved parallelly and uniformly to BD. From the point O taken at adventure

ture in the Line C D, draw O R parallel to B D, cutting the Line B E F C in E, and the ſide A B in R. And again from the point Q taken alſo at adventure in the Line C D, draw Q S parallel to B D, cutting the Line B E F C in F, and the ſide A B in S; and draw E G and F H parallel to C D, cutting A C in G and H. Laſtly, ſuppoſe the ſame conſtruction done in all the points poſſible of the Line B E F C. I ſay, that as the proportions of the Swiftneſſes wherewith Q F, O E, D B, and all the reſt ſupopſed to be drawn parallel to D B, and terminated in the Line B E F C, are to the proportions of their ſeveral Times deſigned by the ſeveral parallels H F, G E, A B and all the reſt ſuppoſed to be drawn parallel to the Line of time C D, and terminated in the Line B E F C (the aggregate to the aggregate) ſo is the Area or Plain D B E F C to the Area or Plain A C F E B. For as A B decreaſing continually by the line B E F C vaniſheth in the time C D into the point C, ſo in the ſame time the line D C continually decreaſing vaniſheth by the ſame line C F E B into the point B; and the point D deſcribeth in that decreaſing motion the line D B equall to the line A C deſcribed by the point A in the decreaſing motion of A & B; & their ſwiftneſſes are therefore equal. Again, becauſe in the time G E the point O deſcribeth the line O E, and in the ſame time the point R deſcribeth the line RE, the line OE ſhall be to the line R E, as the ſwiftneſs wherewith O E is deſcribed to the ſwiftneſs wherwith RE is deſcribed. In like mãner, becauſe in the ſame time H F the point Q deſcribeth the Line Q F, and the point S the Line S F, it ſhall be as the ſwiftneſs by which Q F is deſcribed to the ſwiftneſs by which S F is deſcribed, ſo the Line it ſelf Q F to the Line it ſelf S F; and ſo in all the Lines that can poſſibly be drawn parallel to B D in the points where they cut the Line B E F C. But all the parallels to B D, as R E, S F, AC and the reſt that can poſſibly be drawn from the Line A B to the Line B E F C make the Area of the Plain A B E F C, and all the parallels to the ſame BD, as Q F, O E, D B & the reſt drawn to the points where they cut the ſame Line BEFC make the Area of the Plain BEFCD. As therefore the aggregate of the Swiftneſſes wherwith the Plain BEFCD is deſcribed is to the aggregate of the Swiftneſſes wherewith the Plain ACFEB is deſcribed, ſo is the Plain it ſelf B E F C D to the

Plain it ſelf ACFE B. But the aggregate of the Times repreſented by the parallels A B, G E, H F and the reſt, maketh alſo the Area A C F E B. And therefore as the aggregate of all the Lines Q F, O E, D B and all the reſt of the Lines parallel to B D and terminated in the Line B E F C is to the aggregate of all the Lines H F, G E, A B and all the reſt of the Lines parallel to CD & terminated in the ſame Line B E F C; that is, as the aggregate of the Lines of Swiftneſs to the aggregate of the lines of Time, or as the whole Swiftneſs in the parallels to DB to the whole Time in the parallels to C D, ſo is the Plain B E F C D to the Plain A C F E B. And the proportions of Q F to F H, and of O E to E G, and of D B to B A, and ſo of all the reſt taken together, are the proportion of the Plain D B E F C to the Plain A B E F C. But the Lines Q F, OE, D B and the reſt are the Lines that deſigne the Swifneſs; and the Lines H F, G E, AB & the reſt are the Lines that deſigne the Times of the motions; and therefore the proportion of the Plain D B E FC to the Plain A B E F C is the proportions of all the Velocities taken together, to all the Times taken together. Wherefore as the proportions of the Swiftneſſes, &c. which was to be demonſtrated.

The ſame holds alſo in the diminution of the Circles whereof the lines of Time are the Semidiameters, as may eaſily be conceived by imagining the whole Plain A B CD turned round upon the Axis BD; for the Line B E F C will be every where in the Superficies ſo made, and the Lines H F, G E, A B which here are Parallelograms will be there Cylinders, the Diameters of whoſe baſes are the lines H F, G E, A B, &c. and the Altitude a point, that is to ſay, a quantity leſs then any quantity that can poſſibly be named; and the Lines Q F, O E, D B, &c. ſmall ſolids whoſe lengths and breadths are leſs then any quantity that can be named.

But this is to be noted, that unleſs the proportion of the ſumme of the Swiftneſſes to the proportion of the ſumme of the Times be determined, the proportion of the Figure D B E F C to the Figure A B E F C cannot be determined.

Thirdly, I define RESISTANCE *to be the endeavour of one moved Body, either wholly or in part contrary to the endeavour of another moved Body, which toucheth the ſame.* I ſay wholly contrary, when the endeavour of two Bodies proceeds in the ſame ſtraight Line from the oppoſite extremes, and contrary in part, when two Bodies have

have their endeavour in two Lines, which proceeding from the extreme points of a straight Line, meet without the same.

Fourthly, that I may define what is is to PRESSE, I say that *Of two moved Bodies one Presses the other, when with its Endeavour it makes either all or part of the other Body to go out of its place.*

Fifthly, *A Body which is pressed and not wholly removed is said to* RESTORE *it self, when (the pressing Body being taken away) the parts which were moved, do by reason of the internal constitution of the pressed Body, return every one into its own place.* And this we may observe in Springs, in blown Bladders, and in many other Bodies, whose parts yeild more or less to the Endeavour which the pressing Body makes at the first arrival; but afterwards (when the pressing Body is removed) they do by some force within them *Restore* themselves, and give their whole Body the same figure it had before.

Sixthly, I define FORCE *to be the* Impetus *or Quickness of Motion multiplyed either into it self, or into the Magnitude of the Movent, by means wherof the said Movent works more or less upon the Body that resists it.*

3 Having premised thus much, I shal now demonstrate, First, That if a point moved come to touch another point which is at rest, how little soever the *Impetus* or quickness of its motion be, it shall move that other point. For if by that *Impetus* it do not at all move it out of its place, neither shall it move it, with double the same *Impetus*; for nothing doubled is still nothing; and for the same reason it shall never move it with that *Impetus* how many times soever it be multiplyed, because nothing how soever it be multiplyed will for ever be nothing. Wherefore when a point is at rest, if it do not yeild to the least *Impetus*, it will yeild to none, and consequently it will be impossible that that which is at rest should ever be moved.

Secondly, that when a point moved, how little soever the *Impetus* thereof be, falls upon a point of any Body at rest, how hard soever that Body be, it will at the first touch make it yeild a little. For if it do not yeild to the *Impetus* which is in that point, neither will it yeild to the *Impetus* of never so many points, which have all their *Impetus* severally equal to the *Impetus* of that point. For seeing all those points together work equally, if any one of them have no effect, the aggregate of them all together shall have no effect as many times told as there are points in the whole Body, that is, still no effect at all; and by con-

ſequent there would be ſome Bodies ſo hard that it would be impoſſible to break them; that is, a finite hardneſſe, or a finite force would not yeild to that which is infinite; which is abſurd.

Corollary. It is therefore manifeſt, that Reſt does nothing at all, nor is of any efficacy; and that nothing but Motion gives Motion to ſuch things as be at Reſt, and takes it from things moved,

Thirdly, that Ceſſation in the Movent does not cauſe Ceſſation in that which was moved by it. For (by the 11th Number of the 1 *Article* of this Chapter) whatſoever is moved, perſevers in the ſame way, & with the ſame Swiftneſs, as long as it is not hindered by ſome thing that is moved againſt it. Now it is manifeſt, that Ceſſation is not contrary Motion; and therefore it follows, that the ſtanding ſtill of the Movent, does not make it neceſſary that the thing moved ſhould alſo ſtand ſtill.

Corollary. They are therefore deceived, that reckon the taking away of the impediment or reſiſtance, for one of the cauſes of Motion.

4 Motion is brought into account for divers reſpects; Firſt, as in a Body *Undivided*, (that is, conſidered as a point); or, as in a *Divided* Body. In an Undivided Body, when we ſuppoſe the way by which the Motion is made, to be a Line; and in a Divided Body, when we compute the Motion of the ſeveral parts of that Body, as of Parts.

Secondly, From the diverſity of the regulation of Motion, it is in a Body conſidered as Undivided, ſometimes *Uniform*, and ſometimes *Multiform*. *Uniform* is that by which equal Lines are alwayes tranſmitted in equal times; & *Multiform*, when in one time more, in another time leſs ſpace is tranſmitted. Again, of Multiform Motions, there are ſome in which the degrees of Acceleration and Retardation proceed in the ſame proportions which the Spaces tranſmitted have, whether duplicate, or triplicate, or by whatſoever number multiplyed; and others in which it is otherwiſe.

Thirdly, from the number of the Movents; that is, one Motion is made by one Movent onely, and another by the concourſe of many Movents.

Fourthly, from the poſition of that Line in which a Body is moved, in reſpect of ſome other Line; and from hence one Motion is called *Perpendicular*, another *Oblique*, another *Parallel*.

Fifthly,

Fifthly, from the position of the Movent in respect of the Moved Body; from whence one Motion is *Pulsion* or Driving; another *Traction* or Drawing. PULSION, when the Movent makes the Moved Body goe before it; and TRACTION, when it makes it follow. Again, there are two sorts of *Pulsion*; one, when the motions of the Movent and Moved Body begin both together, which may be called TRUSION or *Thrusting* and VECTION; the other, when the Movent is first moved, and afterwards the Moved Body, which Motion is called PERCUSSION or *Stroke*.

Sixthly, Motion is considered sometimes from the Effect onely which the Movent works in the Moved Body, which is usually called *Moment*. Now MOMENT *is the Excess of Motion which the Movent has, above the Motion or Endeavour of the Resisting Body*.

Seventhly, it may be considered from the diversity of the *Medium*; as one Motion may be made in *Vacuity* or *empty Place*; another in a *fluid*; another in a *consistent Medium*, that is, a *Medium* whose parts are by some power so *consistent* and *cohering*, that no part of the same will yeild to the Movent, unless the whole yeild also.

Eighthly, when a Moved Body is considered as having parts, there arises another distinction of Motion into *Simple* and *Compounded*. *Simple*, when all the several parts describe several equal lines; *Compounded*, when the lines described are Unequal.

5 All Endeavour tends towards that part, that is to say, in that way which is determined by the Motion of the Movent, if the Movent be but one; or, if there be many Movents, in that way which their concourse determines. For example, if a Moved Body have direct Motion, its first Endeavour will be in a Straight line; if it have Circular Motion, its first Endeavour will be in the Circumference of a Circle; & whatsoever the line be in which a Body has its Motion from the concourse of two Movents, as soon as in any point thereof the force of one of the Movents ceases, there immediately the former Endeavour of that Body will be changed into an Endeavour in the line of the other Movent.

6 Wherefore, when any Body is carried on by the concourse of two Winds, one of those Winds ceasing, the Endeavour and Motion of that Body will be in that line, in which it would have been carried by that Wind alone which blows still. And in the de-

scribing

ſcribing of a Circle, where that which is moved has its Motion determined by a Movent in a Tangent, and by the Radius which keeps it in a certain diſtance from the Center, if the retention of the Radius ceaſe, that Endeavour which was in the Circumference of the Circle, will now be in the Tangent, that is, in a Straight line. For ſeeing Endeavour is computed in a leſſe part of the Circumference then can be given, that is, in a point, the way by which a Body is moved in the Circumference is compounded of innumerable Straight lines; of which every one is leſs then can be given, which are therefore called Points. Wherefore when any Body which is moved in the Circumference of a Circle, is freed from the retention of the Radius, it will proceed in one of thoſe Straight lines, that is, in a Tangent.

7 All Endeavour, whether ſtrong or weak, is propagated to infinite diſtance; for it is Motion. If therefore the firſt Endeavour of a Body be made in Space which is empty, it will alwayes proceed with the ſame Velocity; for it cannot be ſuppoſed that it can receive any reſiſtance at all from empty Space; and therefore (by the 7 *Article* of the 9 Chapter) it will alwayes proceed in the ſame way and with the ſame Swiftneſs. And if its Endeavour be in Space which is filled, yet ſeeing Endeavour is Motion, that which ſtands next in its way ſhall be removed, and endeavour further, and again remove that which ſtands next, & ſo infinitely. Wherefore the propagation of Endeavour from one part of full Space to another, proceeds infinitely. Beſides, it reaches in any inſtant to any diſtance, how great ſoever; For in the ſame inſtant in which the firſt part of the full *Medium* removes that which is next it, the ſecond alſo removes that part which is next to it; and therefore all Endeavour, whether it be in empty or in full Space, proceeds not onely to any diſtance how great ſoever, but alſo in any time how little ſoever, that is, in an inſtant. Nor makes it any matter, that Endeavour by proceeding growes weaker and weaker, till at laſt it can no longer be perceived by Senſe; for Motion may be inſenſible; and I do not here examine things by Senſe and Experience, but by Reaſon.

8 When two Movents are of equal Magnitude, the ſwifter of them works with greater force then the ſlower upon a Body that re-

resists their Motion. Also if two Movents have equal Velocity, the greater of them works with more force then the less. For where the Magnitude is equal, the Movent of greater Velocity makes the greater impression upon that Body upon which it falls; and where the Velocity is equal, the Movent of greater Magnitude falling upon the same point, or an equal part of another Body, loses less of its Velocity, because the resisting Body works onely upon that part of the Movent which it touches, and therefore abates the *Impetus* of that part onely, whereas in the mean time the parts which are not touched proceed, and retein their whole force till they also come to be touched, and their force has some effect. Wherfore (for example) in Batteries, a longer then a shorter piece of Timber of the same thickness and velocity, and a thicker then a slenderer piece of the same length and velocity, works a greater effect upon the Wall.

CHAP.

Chap. XVI.

Of Motion Accelerated *and* Uniform, *and of* Motion *by* Concourse.

1 *The Velocity of any Body, in what Time soever it be computed, is that which is made of the multiplication of the* Impetus, *or Quickness of its Motion into the Time.* 2, *&c. In all Motion, the Lengths which are passed through, are to one another, as the Products made by the* Impetus *multipyed into the Time.* 6 *If two Bodies be moved with Uniform Motion through two Lengths, the proportion of those Lengths to one another will be compounded of the proportions of Time to Time, and* Impetus *to* Impetus, *directly taken.* 7 *If two Bodies pass through two Lengths with Uniform Motion, the proportion of their Times to one another will be compounded of the proportions of Length to Length and* Impetus *to* Impetus, *reciprocally taken; also the proportion of their* Impetus *to one another will be compounded of the proportions of Length to Length and Time to Time, reciprocally taken.* 8 *If a Body be carried on with Uniform Motion by two Movents together, which meet in an Angle, the line by which it passes will be a straight line subtending the complement of that Angle to two right Angles.* 9, *&c. If a Body be carried by two Movents together, one of them being moved with Uniform, the other with Accelerated Motion, and the proportion of their Lengths to their Times being explicable in numbers, How to find out what line that Body describes.*

1 THe *Velocity* of any Body, in whatsoever Time it be moved, has its quantity determined by the sum of all the several Quicknesses or *Impetus* which it hath in the several points of the Time of the Bodies Motion. For seeing Velocity (by the Definition of it *Chap.*8.*Art.*15.) is that Power by which a Body can in a certain time pass through a certain length; and Quickness of *Motion*, or *Impetus* (by the 15 *Chap. Artic.*2.*Numb.*2.) is Velocity taken in one point of time onely, all the *Impetus* together taken in all the points of time, will be the same thing with the Mean *Impetus* multiplyed into the whole Time, or which is all one, will be the Velocity of the whole Motion.

Corol-

Corollary. If the *Impetus* be the same in every point, any straight line representing it may be taken for the measure of Time; and the Quicknesses or *Impetus* applyed ordinately to any straight line making an Angle with it, and representing the way of the Bodies motion, will designe a parallelogram which shall represent the Velocity of the whole Motion. But if the *Impetus* or Quickness of Motion begin from Rest, and increase Uniformly, that is, in the same proportion continually with the times which are passed, the whole Velocity of the Motion shall be represented by a Triangle, one side whereof is the whole time, and the other the greatest *Impetus* acquired in that time; or else by a parallelogram, one of whose sides is the whole time of Motion, and the other, half the greatest *Impetus*; or lastly by a parallelogram having for one side a mean proportional between the whole time & the half of that time, & for the other side the half of the greatest *Impetus*. For both these parallelograms are equal to one another, & severally eqnal to the triangle which is made of the whole line of time, and the greatest acquired *Impetus*; as is demonstrated in the Elements of Geometry.

2 In all Uniform Motions the Lengths which are transmitted are to one another, as the product of the mean *Impetus* multiplyed into its time, to the product of the mean *Impetus* multiplyed also into its time.

For let A B (in the first Figure) be the Time, and A C the *Impetus* by which any Body passes with Uniform Motion through the Length D E; & in any part of the time AB, as in the time A F, let another Body be moved with Uniform Motion, first, with the same *Impetus* A C. This Body therefore in the time A B with the *Impetus* A C will pass through the length A F. Seeing therefore, when Bodies are moved in the same Time, & with the same Velocity & *Impetus* in every part of their motion, the proportion of one Length transmitted to another Length trāsmitted, is the same wth that of Time to Time, it followeth, that the Length transmitted in the time A B with the *Impetus* AC will be to the Length transmitted in the time A F with the same *Impetus* A C, as A B it self is to AF, that is, as the parallelogram A I is to the parallelogram A H, that is, as the product of the time A B into the mean *Impetus* A C is to the product of the time A F into the same *Impetus* A C. Again, let it be supposed that a Body be moved in the time A F,

not with the ſame but with ſome other Uniform *Impetus*, as A L. Seeing therfore one of the Bodies has in all the parts of its motion the *Impetus* A C, and the other in like manner the *Impetus* A L, the Length traſmitted by the Body moved with the *Impetus* AC will be to the Length tranſmitted by the Body moved with the *Impetus* A L, as A C it ſelf is to A L, that is, as the parallelogram A H is to the parallelogram F L. Wherefore, by ordinate proportion it will be, as the parallelogram A I to the parallelogram F L, that is, as the product of the mean *Impetus* into the Time is to the product of the mean I*mpetus* into the Time, ſo the Length tranſmitted in the time A B with the *Impetus* A C, to the length tranſmitted in the time A F with the *Impetus*, A L; which was to be demonſtrated.

Corollary. Seeing therefore in Uniform Motion (as has been ſhewn) the Lengths tranſmitted are to one another as the parallelograms which are made by the multiplication of the mean *Impetus* into the Times, that is, (by reaſon of the equality of the *Impetus* all the way) as the Times themſelves, it will alſo be by permutation, as to Time to Length, ſo Time to Length; and in general, to this place are applicable all the properties and tranſmutations of Analogiſmes which I have ſet down and demonſtrated in the 13 Chapter.

3 In Motion begun from Reſt, and Uniformly Accelerated (that is, where the *Impetus* encreaſeth continually according to the proportion of the Times) it will alſo be, as one product made by the Mean *Impetus* multiplyed into the Time, to another product made likewiſe by the Mean *Impetus* multiplyed into the Time, ſo the Length tranſmitted in the one Time, to the Length tranſmitted in the other Time.

For let A B (in the ſame 1 figure) repreſent a Time; in the beginning of which Time A, let the *Impetus* be as the point A; but as the Time goes on, ſo let the *Impetus* encreaſe Uniformly till in the laſt point of that Time AB, namely in B, the *Impetus* acquired be BI. Again, let A F repreſent another Time, in whoſe beginning A, let the *Impetus* be as the point it ſelf A; but as the Time proceeds, ſo let the I*mpetus* encreaſe Uniformly till in the laſt point F of the Time A F the *Impetus* acquired be F K; and let D E be the Length paſſed through in the Time A B with *Impetus* Uniformly encreaſed. I ſay the Length D E, is to the Length tranſmitted in the Time AF, as the Time A B multiplyed into the Mean of the *Impetus* encrea-
ſing

ſing through the time A B, is to the Time A F multiplyed into the Mean of the *Impetus* encreaſing through the time A F.

For ſeeing the Triangle A B I is the whole Velocity of the Body moved in the Time A B till the *Impetus* acquired be B I, and the Triangle A F K the whole Velocity of the Body moved in the Time A F with *Impetus* encreaſing till there be acquired the *Impetus* F K; the Length D E to the Length acquired in the Time A F with *Impetus* encreaſing from Reſt in A till there be acquired the Impetus F K, will be as the Triangle A B I to the Triangle A F K, that is, if the Triangles A B I and A F K be like, in duplicate proportion of the Time A B to the Time A F; but if unlike, in the proportion compounded of the proportions of A B to B I, & of A K to A F. Wherefore, as A B I is to A F K, ſo let D E be to D P; for ſo, the Length tranſmitted in the Time A B with *Impetus* encreaſing to B I, will be to the Length tranſmitted in the Time A F with *Impetus* encreaſing to F K, as the triangle A B I is to the triangle A F K; But the triangle A B I is made by the multiplication of the Time A B into the Mean of the *Impetus* encreaſing to B I, and the triangle A F K is made by the multiplication of the Time A F into the Mean of the *Impetus* encreaſing to F K; and therefore the Length D E which is tranſmitted in the Time A B with *Impetus* encreaſing to B I, to the Length D P which is tranſmitted in the Time A F with *Impetus* encreaſing to F K, is as the product which is made of the Time A B multiplyed into its mean *Impetus*, to the product of the Time A F multiplyed alſo into its mean *Impetus*; which was to be proved.

Corol. 1 In Motion Uniformly accelerated, the proportion of the Lengths tranſmitted, to that of their Times, is compounded of the proportions of their Times to their Times and *Impetus* to *Impetus*.

Corol. 2 In Motion Uniformly accelerated, the Lengths tranſmitted in equal times taken in continual ſucceſſion from the beginning of Motion, are as the differences of ſquare numbers beginning from Unity, namely, as 3, 5, 7, &c. For if in the firſt time the Length tranſmitted be as 1, in the firſt and ſecond times the Length tranſmitted will be as 4, which is the Square of 2, and in the three firſt times, it will be as 9, which is the Square of 3, and in the four firſt times as 16, and ſo on. Now the differences of theſe Squares are 3, 5, 7, &c.

Corol. 3 In Motion Uniformly accelerated from Reſt, the Length

transmitted, is to another Length transmitted uniformly in the same Time, but with such *Impetus* as was acquired by the accelerated Motion in the last point of that Time, as a triangle to a parallelogram which have their altitude and base common. For seeing the Length D E (in the same 1 figure) is passed through with Velocity as the triangle A B I, it is necessary that for the passing through of a Length which is double to DE, the Velocity be as the parallelogram A I; for the parallelogram A I is double to the triangle A B I.

4 In Motion which beginning from Rest, is so accelerated, that the *Impetus* thereof encrease continually in proportion duplicate to the proportion of the times in which it is made, a Length transmitted in one time will be to a Length transmitted in another time, as the product made by the Mean *Impetus* multiplyed into the time of one of those Motions, to the product of the Mean *Impetus* multiplyed into the time of the other Motion.

For let A B (in the 2d. figure) represent a Time, in whose first instant A let the *Impetus* be as the point A; but as the time proceeds, so let the *Impetus* encrease continually in duplicate proportion to that of the times, till in the last point of time B the *Impetus* acquired be BI; then taking the point F any where in the time AB, let the *Impetus* F K acquired in the time A F be ordinately applyed to that point F. Seeing therefore the proportion of F K to B I is supposed to be duplicate to that of A F to A B, the proportion of A F to AB will be subduplicate to that of F K to B I; and that of A B to A F will be (by Chap. 13. *Article* 16) duplicate to that of B I to F K, and consequently the point K will be in a parabolical line whose diameter is A B and base B I; and for the same reason, to what point soever of the time A B the *Impetus* acquired in that time be ordinately applyed, the straight line designing that *Impetus* will be in the same parabolical line A K I. Wherefore the mean *Impetus* multiplyed into the whole time A B will be the Parabola A K I B, equal to the parallelogram A M, which parallelogram has for one side the line of time A B and for the other the line of the *Impetus* A L, which is two thirds of the *Impetus* B I; for every Parabola is equal to two thirds of that parallelogram with which it has its altitude and base common. Wherefore the whole Velocity in A B will be the parallelogram A M, as being made by the multiplication of the *Impetus* A L into the time A B. And in like manner, if F N

be

be taken, which is two thirds of the *Impetus* F K, and the parallelogram F O be completed, F O will be the whole Velocity in the time A F, as being made by the Uniform Impetus A O or F N multiplyed into the time A F. Let now the length transmitted in the time A B and with the Velocity A M be the straight line D E; and lastly, let the Length transmitted in the time AF with the Velocity A N, be DP; I say that as A M is to A N, or as the Parabola A K I B to the Parabola A F K, so is D E to D P. For as A M is to F L (that is, as A B is to A F) so let D E be to D G. Now the proportion of A M to A N is compounded of the proportions of A M to F L, and of F L to A N. But as A M to F L, so (by construction) is D E to D G; and as F L is to A N (seeing the time in both is the same, namely, A F), so is the Length D G to the Length D P; for Lengths transmitted in the same time are to one another as their Velocities are. Wherefore by ordinate proportion, as A M is to A N, that is, as the mean *Impetus* A L multiplyed into its time A B, is to the mean *Impetus* A O multiplyed into A F, so is D E to D P; which was to be proved.

Corol. 1 Lengths transmitted with Motion so accelerated that the *Impetus* encrease continually in duplicate proportion to that of their times, if the base represent the *Impetus*, are in triplicate proportion of their *Impetus* acquired in the last point of their times. For as the Length D E is to the Length D P, so is the parallelogram A M to the parallelogram A N, and so the Parabola A B I K to the Parabola A F K; But the proportion of the Parabola A B I K to the Parabola A F K is triplicate to the proportion which the base B I has to the base F K. Wherefore allso the proportion of D E to D P, is triplicate to that of B I to F K.

Corol. 2 Lengths transmitted in equal Times succeeding one another from the beginning, by Motion so accelerated, that the proportiō of the *Impetus* be duplicate to the proportiō of the times, are to one another as the differences of Cubique Numbers beginning at Unity, that is, as 7, 19, 37, &c. For if in the first time the Length transmitted be as 1, the Length at the end of the second time will be as 8, at the end of the third time as 27, and at the end of the fourth time as 64, &c. which are Cubique Numbers, whose differences are 7, 19, 37, &c.

Corol. 3 In Motion so accelerated, as that the Length transmitted be

be alwayes to the Length transmitted in duplicate proportion to their Times, the Length Uniformly transmitted in the whole time and with *Impetus* all the way equal to that which is last acquired, is as a Parabola to a parallelogram of the same altitude & base, that is, as 2 to 3. For the Parabola A B I K is the *Impetus* encreasing in the time A B; and the parallelogram A I is the greatest Uniform *Impetus* multiplyed into the same time A B. Wherefore the Lengths transmitted will be as a Parabola to a parallelogram &c. that is, as 2 to 3.

5 If I should proceed to the explication of such Motions as are made by *Impetus* encreasing in proportion triplicate, quadruplicate, quintuplicate, &c. to that of their times, it would be a labour infinite and unnecessary. For by the same method by which I have computed such Lengths as are transmitted with *Impetus* encreasing in single and duplicate proportion, any man may compute such as are transmitted with *Impetus* encreasing in triplicate, quadruplicate or what other proportion he pleases.

In making which computation he shall finde, that where the *Impetus* encrease in proportion triplicate to that of the times, there the whole Velocity will be designed by the first Parabolaster (of which see the next Chapter); and the Lengths transmitted will be in proportion quadruplicate to that of the times. And in like manner, where the *Impetus* encrease in quadruplicate proportion to that of the times, that there the whole Velocity will be designed by the second Parabolaster, and the Lengths transmitted will be in quintuplicate proportion to that of the times; and so on continually.

6 If two Bodies with Uniform Motion transmit two Lengths, each with its own *Impetus* and Time, the proportion of the Lengths transmitted will be compounded of the proportions of Time to Time, and *Impetus* to *Impetus*, directly taken.

Let two Bodies be moved Uniformly (as in the 3d figure) One in the time A B with the I*mpetus* A C, the other in the time A D with the I*mpetus* A E. I say the Lengths transmitted have their proportion to one another compounded of the proportions of A B to A D, and of A C to A E. For let any Length whatsoever, as Z, be transmitted by one of the Bodies in the time A B with the I*mpetus* A C; and any other Length, as X, be transmitted by the other Body in the time A D with the I*mpetus* A E; and let the parallelo-

grams

grams A F and A G be completed. Seeing now Z is to X (by the 2d *Article*) as the *Impetus* A C multiplyed into the time A B is to the *Impetus* A E multiplyed into the time A D, that is, as A F to A G; the proportion of Z to X will be compounded of the same proportions, of which the proportion of A F to A G is compounded; But the proportion of A F to A G is compounded of the proportions of the side A B to the side A D, and of the side A C to the side A E (as is evident by the Elements of *Euclide*), that is, of the proportions of the time A B to the time A D, and of the *Impetus* A C to the *Impetus* A E. Wherefore also the proportion of Z to X is compounded of the same proportions of the time A B to the time A D, and of the *Impetus* A C to the *Impetus* A E; which was to be demonstrated.

Corol. 1 When two Bodies are moved with Uniform Motion, if the Times and *Impetus* be in reciprocal proportion, the Lengths transmitted shall be equal. For if it were as A B to A D (in the same 3d figure) so reciprocally A E to A C, the proportion of A F to A G would be compounded of the proportions of A B to A D and of A C to A E, that is, of the proportions of A B to A D and of A D to A B. Wherefore, A F would be to A G as A B to A B, that is equal; and so the two products made by the multiplication of *Impetus* into Time would be equal; and by consequent, Z would be equal to X.

Corol. 2 If two Bodies be moved in the same Time, but with different *Impetus*, the Lengths transmitted will be as *Impetus* to *Impetus*. For if the Time of both of them be A D, and their different *Impetus* be A E and A C, the proportion of A G to D C will be compounded of the proportions of A E to A C and of A D to A D, that is, of the proportions of A E to A C and of A C to A C; and so the proportion of A G to D C, that is, the proportion of Length to Length will be as A E to A C, that is, as that of *Impetus* to *Impetus*. In like manner, if two Bodies be moved Uniformly, and both of them with the same *Impetus*, but in different times, the proportion of the Lengths transmitted by them will be as that of their times. For if they have both the same *Impetus* A C, and their different times be A B & A D, the proportion of A F to D C will be compounded of the proportions of A B to A D and of A C to

A C;

A C; that is, of the proportions of A B to A D and of A D, to A D; and therefore the proportion of A F to D C, that is, of Length to Length, will be the same with that of A B to A D, which is the proportion of Time to Time.

7 If two Bodies pass through two Lengths with Uniform Motion, the proportion of the Times in which they are moved will be compounded of the proportions of Length to Length and *Impetus* to *Impetus* reciprocally taken.

For let any two Lengths be given, as (in the same 3d figure) Z and X, and let one of them be transmitted with the *Impetus* A C, the other with the *Impetus* A E. I say the proportion of the Times in which they are transmitted, will be compounded of the proportions of Z to X, and of A E (which is the *Impetus* with which X is transmitted) to A C (the *Impetus* with which Z is transmitted.) For seeing AF is the product of the *Impetus* AC multiplyed into the Time A B, the time of Motion through Z will be a line wch is made by the applicatiō of the parallelogram AF to the straight line AC, which line is A B; and therefore A B is the time of motion through Z. In like manner, seeing AG is the product of the *Impetus* AE multiplied into the Time AD, the time of motion through X wil be a line which is made by the application of A G to the straight line A D; but AD is the time of motiō through X. Now the proportion of AB to A D is cōpounded of the proportions of the parallelogram A F to the parallelogram A G, and of the *Impetus* A E to the *Impetus* A C; which may be demonstrated thus. Put the parallelograms in order A F, A G, D C; and it will be manifest that the proportion of A F to D C is compounded of the proportions of A F to A G and of A G to D C; bnt A F is to D C as A B to A D; wherefore also the proportion of A B to A D is compounded of the proportrions of A F to AG & of AG to DC. And because the Length Z is to the Length X as AF is to AG, & the *Impetus* A E to the *Impetus* A C as AG to DC, therefore the proportion of AB to AD will be compounded of the proportions of the Length Z to the Length X, and of the *Impetus* A E to the *Impetus* A C; which was to be demonstrated.

In the same manner it may be proved, that in two Uniform Motions the proportion of the *Impetus* is compounded of the proportions,

ons of Length to Length, and of Time to Time reciprocally taken.

For if we ſuppoſe A C (in the ſame 3d figure) to be the Time, and A B the *Impetus* with which the Length Z is paſſed through; and A E to be the Time, and A D the *Impetus* with which the Length X is paſſed through, the demonſtration will proceed as in the laſt Article.

8 If a Body be carried by two Movents together which move with ſtraight and Uniform Motion, and concurre in any given angle, the line by which that Body paſſes will be a ſtraight line.

Let the Movent A B (in the 4th figure) have ſtraight and Uniform Motion, and be moved till it come into the place C D; and let another Movent A C, having likewiſe ſtraight and Uniform Motion, and making with the Movent A B any given angle C A B, be underſtood to be moved in the ſame time to D B; and let the Body be placed in the point of their concourſe A. I ſay the line which that Body deſcribes with its Motion is a ſtraight line. For let the parallelogram A B D C be completed, and its diagonal A D be drawn; and in the ſtraight line A B let any point E be taken; and from it let E F be drawn parallel to the ſtraight lines AC and BD, cutting AD in G; and through the point G let H I be drawn parallel to the ſtraight lines A B and C D; and laſtly, let the meaſure of the time be AC. Seeing therefore both the Motions are made in the ſame time, when A B is in C D, the Body alſo will be in C D; and in like manner, when A C is in B D, the Body will be in B D. But A B is in C D at the ſame time when A C is in B D; and therefore the Body will be in C D and B D at the ſame time; Wherefore it will be in the common point D. Again, ſeeing the Motion from A C to B D is Uniform, that is, the Spaces tranſmitted by it are in proportion to one another as the Times in which they are tranſmitted, when A C is in E F, the proportion of A B to AE will be ſame with that of E F to E G, that is, of the Time A C to the Time A H. Wherefore A B will be in H I in the ſame time in which A C is in E F, ſo that the Body will at the ſame time be in E F and in H I, and therefore in their common point G. And in the ſame manner it will, be whereſoever the point E be taken between A and B. Wherefore the Body will alwayes be in the Diagonal A D; which was to be demonſtrated.

Corollary. From hence it is manifest, that the Body will be carried through the same straight line AD, though the Motion be not Uniform, provided it have like acceleration; for the proportion of A B to A E will alwayes be the same with that of A C to A H.

9 If a Body be carried by two Movents together, which meet in any given angle, and are moved, the one Uniformly, the other with Motion Uniformly accelerated from Rest (that is, that the proportion of their *Impetus* be as that of their Times) that is, that the proportion of their Lēgths be duplicate to that of the lines of their Times, till the line of greatest *Impetus* acquired by acceleration be equal to that of the line of Time of the Uniform Motion; the line in which the Body is carried will be the crooked line of a Semiparabola, whose base is the *Impetus* last acquired, and Vertex the point of Rest.

Let the straight line A B (in the 5th Figure) be understood to be moved with Uniform Motion to C D; and let another Movent in the straight line A C be supposed to be moved in the same time to B D, but with motion Uniformly accelerted, that is, with such motion, that the proportion of the Spaces which are transmitted be alwayes duplicate to that of the Times, till the *Impetus* acquired be B D equal to the straight line A C; and let the Semiparabola A G D B be described. I say that by the concourse of those two Movents, the Body will be carried through the Semipabolical crooked line A G D. For let the parallelelogram A B D C be completed; & from the point E taken any where in the straight line A B let E F be drawn parallel to A C, and cutting the crooked line in G; and lastly, through the point G let A I be drawn parallel to the straight lines A B and C D. Seeing therefore the proportion of A B to A E is by supposition duplicate to the proportion of E F to E G, that is, of the time A C to the time A H, at the same time when A C is in E F, A B will be in H I; and therefore the moved Body will be in the common point G. And so it will alwayes be in what part soever of A B the point E be taken. Wherefore the moved Body will always be found in the parabolical line A G D; which was to be demonstreted.

10 If a Body be carried by two Movents together, which meet in any given angle, and are moved, the one Uniformly, the other with *Impetus* encreasing from Rest till it be equal to that of the Uniform

niform Motion, and with such acceleration, that the proportion of the Lengths transmitted be every where triplicate to that of the Times in which they are transmitted, The line in which that Body is moved, will be the crooked line of the first Semiparabolaster of two Means, whose base is the *Impetus* last acquired.

Let the straight line A B (in the 6th. Figure) be moved Uniformly to C D; and let another Movent A C be moved at the same time to B D with motion so accelerated, that the proportion of the Lengths transmitted by every where triplicate to the proportion of their Times; and let the *Impetus* acquired in the end of that motion be B D, equal to the straight line A C; & lastly, let A D be the crooked line of the first Semiparabolaster of two Means. I say that by the concourse of the two Movents together, the Body will be alwayes in that crooked line A D. For let the parallelogram A B D C be completed; and from the point E taken any where in the straight line A B let E F be drawn parallel to A C and cutting the crooked line in G; and through the point G let H I be drawn parallel to the straight lines A B and C D. Seeing therefore the proportion of A B to A E is (by supposition) triplicate to the proportion of E F to E G, that is, of the time A C to the time A H, at the same time when A C is in E F, A B will be in H I; and therefore the moved Body will be in the common point G. And so it will alwayes be in what part soever of A B the point E be taken; and by conseqnent the Body will always be in the crooked line A G D; which was to be demonstrated.

11 By the same method it may be shewn what line it is that it made by the motion of a Body carried by the concourse of any two Movents, which are moved, one of them Uniformly, the other with acceleration, but in such proportions of Spaces and Times as are explicable by Numbers, as *duplicate*, *triplicate* &c. or such as may be designed by any broken number whatsover. For which this is the Rule.

Let the two numbers of the Length & Time be added together; & let their Sum be the Denominator of a Fraction, whose Numerator must be the number of the Length. Seek this Fraction in the Table of the third Article of the 17th Chapter, and the line sought will be that which denominates the three-sided Figure noted on the left hand, and the kind of it will be that which is numbred

red above over the Fraction. For example, Let there be a concourse of two Movents, whereof one is moved Uniformly, the other with motion so accelerated that the Spaces are to the Times as 5 to 3. Let a Fraction be made whose Denominator is the Sum of 5 and 3, and the Numerator 5, namely the Fraction $\frac{5}{8}$. Seek in the Table, and you will find $\frac{5}{8}$ to be the third in that row which belongs to the three-sided Figure of four Means. Wherfore the line of Motion made by the concourse of two such Movents as are last of all described, will be the crooked line of the third Parabolaster of four Means.

12 If Motion be made by the concourse of two Movents whereof one is moved Uniformly, the other beginning from Rest in the Angle of concourse with any acceleration whatsoever; the Movent which is Moved Uniformly shall put forward the moved Body in the several parallel Spaces, lesse, then if both the Movents had Uniform motion; and still lesse and lesse, as the Motion of the other Movent is more and more accelerated.

Let the Body be placed in A (in the 7th figure) and be moved by two Movents, by one with Uniform Motion from the straight line A B to the straight line C D parallel to it; and by the other with any acceleration from the straight line A C to the straight line B D parallel to it; and in the parallelogram A B D C let a Space be taken between any two parallels E F and G H. I say, that whilest the Movent A C passes through the latitude which is between E F and G H, the Body is lesse moved forwards from A B towards C D, then it would have been, if the Motion from A C to BD had been Uniform.

For suppose that whilest the Body is made to descend to the parallel E F by the power of the Movent from A C towards B D, the same Body in the same time is moved forwards to any point F in the line E F by the power of the Movent from A B towards C D; and let the straight line A F be drawn and produced indeterminately, cutting G H in H. Seeing therefore it is as A E to A G, so E F to G H; if A C should descend towards B D with uniform Motion, the Body in the time G H (for I make A C and its parallels the measure of time) would be found in the point H. But because A C is supposed to be moued towards B D which motion continually accelerated, that is, in greater proportion of Space to

Space

Space then of Time to Time, in the time G H the Body will be in some parallel beyond it, as between G H and B D. Suppose now that in the end of the time G H it be in the parallel I K, & in I K let I L be taken equal to G H. When therefore the Body is in the parallel I K, it will be in the point L. Wherefore when it was in the parallel G H, it was in some point between G and H, as in the point M; but if both the Motions had been Uniform it had been in the point H; and therefore whilest the Movent AC passes over the latitude which is between E F and G A, the Body is less moved forwards from A B towards C D, then it would have been if both the Motions had been Uniform; which was to be demonstrated.

13 Any Length being given whch is passed through in a given time with uniform motion, To find out what Length shall be passed through in the same time with Motion uniformly accelerated, that is, with such Motion, that the proportion of the Lengths passed through be continually duplicate to that of their Times, and that the line of the *Impetus* last acquired, be equal to the line of the whole time of the Motion.

Let A B (in the 8th. figure) be a Length transmitted with Uniform Motion in the time A C; and let it be required to find another Length which shall be transmitted in the same time with Motion Uniformly accelerated, so, that the line of the *Impetus* last acquired be equal to the straight line A C.

Let the parallelogram A B D C be completed; and let B D be divided in the middle at E; and between B E and B D let B F be a mean proportional; and let A F be drawn and produced till it meet with C D produced in G; and lastly, let the parallelogram A C G H be completed. I say A H is the Length required.

For as duplicate proportion is to single proportion, so let A H be to A I, that is, let A I be the half of A H; and let I K be drawn parallel to the straight line A C, and cutting the Diagonal A D in K, and the straight line A G in L. Seeing therefore A I is the half of A H, I L will also be the half of B D, that is, equal to B E, and I K equal to B F; for B D, (that is, G H), B F, and B E (that is, I L) being continual proportionals, A H, A B, and A I will also be continual proportionals. But as A B is to A I, that is, as A H is to A B, so is B D to I K, and so also is G H, that is, B D to B F; and therefore B F and I K are equal. Now the proportion of A H to AI is duplicate to the proportion of A B to A I, that is, to that

of

of B D to I K, or of G H to I K. Wherefore the point K will be in a Parabola, whose diameter is A H & base G H, which G H is equal to A C. The Body therefore proceeding from Rest in A with motion Uniformly accelerated in the time A C, when it has passed through the Length A H, will acquire the *Impetus* G H equal to the time A C; that is, such *Impetus*, as that with it the Body will pass through the Length A C in the time A C. Wherefore any Length being given, &c. which was propounded to be done.

14 Any Length being given which in a given Time is transmitted with Uniform Motion, To find out what Length shall be transmitted in the same Time with Motion so accelerated, that the Lengths transmitted be continually in triplicate proportion to that of their Times, and the line of the *Impetus* last of all acquired be equal to the Line of Time given.

Let the given Length A B (in the 9th figure) be transmitted with Uniform motion in the Time A C; and let it be required to find what Length shall be transmitted in the same time with motion so accelerated, that the Lengths transmitted be continually in triplicate proportion to that of their Times, and the *Impetus* last acquired be equal to the Time given.

Let the parallelogram A B D C be completed; and let B D be so divided in E, that B E be a third part of the whole B D; and let B F be a mean proportional between B D and B E; and let A F be drawn and produced till it meet the straight line C D in G; and lastly, let the parallelogram A C G H be completed. I say A H is the Length required.

For as triplicate proportion is to single proportion, so let A H be to another line A I, that is, make A I a third part of the whole AH; and let I K be drawn parallel to the straight line A C, cutting the Diagonal A D in K, and the straight line A G in L; then, as A B is to A I, so let A I be to another A N; and from the point N let N Q be drawn parallel to A C, cutting A G, A D, and F K produced, in P, M and O; and last of all let F O and L M be drawn, which will be equal and parallel to the straight lines B N and I N. By this construction, the Lengths transmitted A H, A B, A I and A N will be continual proportionals; and in like manner, the Times G H, B F, I L and N P, that is, N Q, N O, N M and N P will be continual proportionals, and in the same proportion with

AH,

A H, A B, A I, and A N. Wherefore the proportion of A H to A N is the same with that of B D, that is, of N Q to N P; and the proportion of N Q to N P triplicate to that of N Q to N O, that is, triplicate to that of B D to I K; Wherefore also the Length A H is to the Length A N in triplicate proportion to that of the Time B D to the Time I K; and therefore the crooked line of the first three sided figure of two means, whose Diameter is A H, and base G H equal to AC, shall pass through the point O; and consequently A H shall be transmitted in the time A C, and shall have its last acquired *Impetus* G H equal to A C, and the proportions of the Lengths acquired in any of the times triplicate to the proportions of the times themselves. Wherefore A H is the Length required to be found out,

By the same method, if a Length be given which is transmitted with Uniform Motion in any given Time, another Length may be found out, which shall be transmitted in the same Time with motion so accelerated, that the Lengths transmitted shall be to the Times in which they are transmitted, in proportion quadruplicate, quintuplicate, and so on infinitely. For if B D be divided in E, so, that B D be to B E as 4 to 1; and there be taken between B D and B E a mean proportional F B; and as A H is to A B, so A B be made to a third, and again so that third to a fourth, and that fourth to a fifth A N, so that the proportion of A H to A N be quadruplicate to that of A H to A B, and the parallelogram N B F O be completed; the crooked line of the first three-sided Figure of three Means will pass through the point O; and consequently the Body moved will acquire the *Impetus* G H equal to A C in the time A C. And so of the rest.

15 Also, if the proportion of the Lengths transmitted, be to that of their Times, as any number to any number, the same method serves for the finding out of the Length transmitted with such *Impetus*, and in such Time.

For let A C (in the 10 figure) be the time, in which a Body is transmitted with Uniform Motion from A to B; and the parallelogram A B D C being completed, let it be required to find out a Length in which that Body may be moved in the same time A C, frō A wth motion so accelerated, that the proportion of the Lengths transmitted, to that of the Times be continually as 3 to 2.

Let

Let B D be ſo divided in E, that B D be to B E as 3 to 2; and between B D and B E let B F be a mean proportionall; and let A F be drawn and produced till it meet with C D produced in G; and making A M a mean proportional between A H and A B, let it be as A M to AB, ſo A B to A I; and ſo the proportion of A H to A I will be to that of A H to A B, as 3 to 2. (for of the proportions of which that of A H to A M is one, that of A H to A B is two, and that of AH to A I is three;) & conſequently as 3 to 2 to that of G H to B F, & (F K being drawn parallel to B I, and cutting A D in K) ſo likewiſe to that of G H or B D to I K; Wherefore the proportion of the Length A H to A I is to the proportion of the Time B D to I K, as 3 to 2; and therefore, if in the time A C, the Body be moved with accelerated motion, as was propounded, till it acquire the *Impetus* H G equal to A C, the Length tranſmitted in the ſame Time will be A H.

16 But if the proportion of the Lengths to that of the Times had been as 4 to 3, there ſhould then have been taken two mean proportionals between A H and A B, and their proportion ſhould have been continued one term further, ſo that A H to A B might have three of the ſame proportions, of which A H to AI has four; and all things elſe ſhould have been done as is already ſhewn. Now the way how to interpoſe any number of Means between two Lines given, is not yet found out. Nevertheleſs, this may ſtand for a general Rule; *If there be a Time given, and a Length be tranſmitted in that Time with Uniform Motion; as for example, if the Time be* A C, *and the Length* A B; *the ſtraight line* A G, *which determines the Length* C G *or* A H *tranſmitted in the ſame Time* A C *with any accelerated motion, ſhall ſo cut* B D *in* F, *that* B F *ſhall be a mean proportional between* B D *and* B E, B E *being ſo taken in* B D, *that the proportion of Length to Length be every where to the proportion of Time to Time, as the whole* B D *is to its part* B E.

17 If in a given Time, two Lengths be tranſmitted, One with uniform motion, the other with motion accelerated in any proportion of the Lengths to the Times, and again in part of the ſame Time, parts of the ſame Lengths be tranſmitted with the ſame motions, the whole Length will exceed the other Length in the ſame proportion in which one part exceeds the other part.

For example, let A B (in the 8th. figure) be a Length tranſmitted

ted in the time A C with uniform Motion; and let A H be another Length transmitted in the same time with Motion uniformly accelerated; so that the *Impetus* last acquired be G H equal to A C; and in A H let any part A I be taken, and transmitted in part of the time A C with uniform Motion; and let another part A B be taken, and transmitted in the same part of the time A C with Motion uniformly accelerated. I say, that as A H is to AB, so will A B be to A I.

Let B D be drawn parallel and equal to H G, and divided in the midst at E, and between B D and B E, let a mean proportional be taken as B F; & the straight line A G (by the demonstration of the 13th Art.) shal pass through F. And dividing AH in the midst at I, A B shall be a mean proportional between A H and A I. Again (because A I and A B are described by the same Motions) if I K be drawn parallel and equal to B F or A M, and divided in the midst at N, and between I K and I N be taken the mean proportional I L, the straight line A F will (by the demonstration of the same 13th Art.) pass through L. And dividing A B in the midst at O, the line A I will be a mean proportional between A B and A O. Whereas A B is divided in I and O in like manner as A H is divided in B and I; and as A H to A B so is A B to A I. Which was to be proved.

Coroll. Also as A H to A B, so is H B to B I; and so also B I to I O.

And as this (where one of the Motions is uniformly accelerated) is proved out of the demonstration of the 13th Article; so (when the accelerations are in double proportion to the times) the same may be proved by the demonstration of the 14th Art. and by the same method in all other accelerations, whose proportions to the times are explicable in numbers.

18 If two sides which contain an Angle in any Parallelogram, be moved in the same time to the sides opposite to them, one of them with Uniform Motion, the other with Motion Uniformly accelerated; that side which is moved Uniformly will effect as much with its concourse through the whole Length transmitted, as it would do if the other Motion were also Uniform, and the Length transmitted by it in the same time were a mean proportional between the whole and the half.

Let the side A B of the Parallelogram A B D C (in the 11th Figure) be understood to be moved with Uniform Motion till it be coincident with C D; and let the time of that Motion be AC

or B D. Alſo in the ſame time let the ſide A C be underſtood to be moved with Motion uniformly accelerated, till it be coincident with B D; then dividing A B in the middle in E, let A F be made a mean proportional between A B and A E; and drawing F G parallel to A C, let the ſide A C be underſtood to be moved in the ſame time AC with uniform Motion till it be coincident with F G. I ſay the whole AB confers as much to the velocity of the Body placed in A when the Motion of AC is uniformly accelerated till it come to B D, as the part A F confers to the ſame when the ſide AC is moved Uniformly and in the ſame time to F G.

For ſeeing A F is a mean proportional between the whole A B & it is half AE, BD wil (by the 13th *Art.*) be the laſt *Impetus* acquired by AC with motion uniformly accelerated till it come to the ſame B D; and conſequently the ſtraight line F B will be the exceſs by w^ch^ the Length tranſmitted by AC with motion uniformly accelerated, will exceed the Length tranſmitted by the ſame AC in the ſame time with Uniform Motion, and with *Impetus* every where equal to B D. Wherefore if the whole A B be moved Uniformly to C D in the ſame time in which A C is moved Uniformly to F G, the part F B (ſeeing it concurs not at all with the Motion of the ſide A C which is ſuppoſed to be moved onely to F G) will cōfer nothing to its motion. Again, ſuppoſing the ſide A C to be moved to BD with Motion Uniformly accelerated, the ſide AB with its uniform Motion to CD will leſs put forwards the Body when it is accelerated in all the parallels, then when it is not at all accelerated; & by how much the greater the acceleration is, by ſo much the leſs it will put it forwards (as is ſhewn in the 12th *Artic.*) When therefore AC is in FG with accelerated Motion, the Body will not be in the ſide C D at the point G, but at the point D; ſo that G D wil be the exceſs by which the Length tranſmitted with accelerated Motion to BD, exceeds the Length tranſmitted with Uniform Motion to FG; ſo that the Body by its acceleration avoids the action of the part A F, & comes to the ſide C D in the time A C, and makes the Length C D, which is equal to the Length AB. Wherefore Uniform Motion from A B to C D in the time A C works no more in the whole Length A B upon the Body uniformly accelerated from A C to B D , then if A C were moved in the ſame time with uniform Motion to FG; the difference conſiſting onely in this, that when AB works upon the Body uniformly moved from AC to

FG,

F G, that by which the accelerated Motion exceeds the Uniform Motion, is altogether in F B, or G D; but when the same A B works upon the Body accelerated, that by which the accelerated Motion exceeds the Uniform Motion, is dispersed through the whole Length A B or C D, yet so that if it were collected and put together, it would be equal to the same F B or G D. Wherefore, If two sides which contain an angle &c; which was to be demonstrated.

19 If two transmitted Lengths have to their Times any other proportion explicable by number, & the side A B be so divided in E, that A B be to A E in the same proportion which the Lengths transmitted have to the Times in which they are transmitted, and between A B and A E there be taken a mean proportional A F, it may be shewn by the same method, that the side which is moved with Uniform Motion, works as much with its concourse through the whole Length A B, as it would do if the other Motion were also Uniform, and the Length transmitted in the same Time A C were that mean proportional A F.

And thus much concerning Motion by concourse.

F G, that by which the accelerated Motion exceeds the Uniform Motion, is altogether in F B, or G D; but when the same A B works upon the Body accelerated, that by which the accelerated Motion exceeds the Uniform Motion, is dispersed through the whole Length A B or C D, yet so that if it were collected and put together, it would be equal to the same F B or G D. Wherefore, the two sides which contain an angle &c; which was to be demonstrated.

19. If two transmitted Lengths have to their Times any other proportion explicable by numbers, & the side A B be so divided in E, that A B be to A E in the same proportion which the Lengths transmitted have to the Times in which they are transmitted; and between A B and A E there be taken a mean proportional A F, it may be shewn by the same method, that the side which is moved with Uniform Motion, works as much with its concourse through the whole Length A B, as it would do if the other Motion were also Uniform, and the Length transmitted in the same Time A C were that mean proportional A F.

And thus much concerning Motion by concourse.

Chap. XVII.

Of Figures Deficient.

1 *Definitions of a Deficient Figure; of a Complete Figure; of the Complement of a Deficient Figure; and of Proportions which are Proportional and Commensurable to one another.* 2 *The proportion of a Deficient Figure to its Complement.* 3 *The proportions of Deficient Figures to the Parallelograms in which they are described, set forth in a Table.* 4 *The Description and Production of the same Figures.* 5 *The drawing of Tangents to them.* 6 *In what proportion the same Figures exceed a straight lined Triangle of the same Altitude and Base.* 7 *A Table of Solid Deficient Figures described in a Cylinder.* 8 *In what proportion the same Figures exceed a Cone of the same Altitude and Base.* 9 *How a plain Deficient Figure may be described in a Parallelogram, so, that it be to a Triangle of the same Base and Altitude, as another Deficient Figure (plain or solid) twice taken, is to the same Deficient Figure together with the Complete Figure in which it is described.* 10 *The transferring of certain properties of Deficient Figures described in a Parallelogram to the proportions of the Spaces transmitted with several degrees of Velocity.* 11 *Of Deficient Figures described in a Circle.* 12 *The proposition demonstrated in the 2d. Article, confirmed from the Elements of Philosophy.* 13 *An unusual way of reasoning concerning the Equality between the superficies of a portion of a Sphere, and a Circle.* 14 *How from the description of Deficient Figures in a Parallelogram, any number of mean Proportionals may be found out between two given straight lines.*

1 I Call those *Deficient Figures*, which may be understood to be generated by the Uniform Motion of some quantity, which decreases continually, till at last it have no magnitude at all.

And I call that a *Complete Figure*, answering to a Deficient Figure, which is generated with the same motion, and in the same time, by a Quantity which retaines alwayes its whole magnitude.

The *Complement* of a Deficient Figure is that, which being added to the Deficient Figure, makes it Complete.

Four Proportions are said to be *Proportionall*, when the first of them

them is to the second, as the third is to the fourth. For example, if the first proportion be duplicate to the second; and again the third be duplicate to the fourth, those Proportions are said to be *Proportionall.*

And *Commensurable* Proportions are those, which are to one another as number to number. As when to a proportion given, one proportion is duplicate, another triplicate, the duplicate proportion will be to the triplicate proportion as 2 to 3; but to the given proportion it will be as 2 to 1; and therefore I call those three proportions *Commensurable.*

2 A Deficient Figure, which is made by a Quantity continually decreasing to nothing by proportions every where proportionall and commensurable, is to its Complement, as the proportion of the whole altitude, to an altitude diminished in any time, is to the proportion of the whole Quantity which describes the Figure, to the same Quantity diminished in the same time.

Let the quantity A B (in the 1 figure) by its motion through the altitude A C, describe the Complete Figure A D; and againe, let the same quantity, by decreasing continually to nothing in C, describe the Deficient Figure A B E F C, whose Complement will be the Figure B D C F E. Now let A B be supposed to be moved till it lie in G K, so that the altitude diminished be G C, and A B diminished be G E; and let the proportion of the whole altitude A C to the diminished altitude G C, be (for example) triplicate to the proportion of the whole quantity A B or G K, to the diminished quantity G E. And in like manner, let H I be taken equal to G E, & let it be diminished to H F; and let the proportion of G C to H C be triplicate to that of H I to H F; & let the same be done in as many parts of the straight line A C as is possible; and a line be drawn through the points B, E, F and C. I say the Deficient Figure A B E F C, is to its Complement B D C E F as 3 to 1, or as the proportion of A C to G C is to the proportion of A B, that is, of G K to G E.

For (by the second Article of the 15. Chap.) the proportion of the complement B E F C D to the deficient figure A B E F C, is all the proportions of D B to O E, and of D B to Q F, and of all the lines parallel to D B terminated in the line B E F C, to all the parallels to A B terminated in the same points of the line B E F C.

And seeing the proportions of D B to O E, and of D B to Q F &c. are every where triplicate of the proporions of A B to G E, and of A B to H F &c. the proportions of H F to A B, and of G E to A B &c. (by the 16 Article of the 13 Chap.) are triplicate of the proportions of Q F to D B, and of O E to D B &c. and therefore the deficient figure A B E F C which is the aggregate of all the lines H F, G E, A B, &c. is triple to the complement B E F C D made of all the lines Q F, O E, D B, &c. which was to be proved.

It follows from hence, That the same complement B E F C D is $\frac{1}{4}$ of the whole Parallelogram. And by the same method may be calculated in all other Deficient Figures generated as above declared, the proportion of the Parallelogram to either of its parts; as that when the parallels encrease fom a point in the same proportion, the Parallelogram will be divided into two equal Triangles; when one encrease is double to the other, it will be divided into a Semiparabola and its Complement, or into 2 and 1.

The same construction standing, the same conclusion may otherwise be demonstrated, thus.

Let the straight line C B be drawn cutting G K in L, & through L let M N be drawn parallel to the straight line A C; wherefore the Parallelograms G M and L D will be equal. Then let L K be divided into three equal parts, so that it may be to one of those parts in the same proportion which the proportion of A C to G C or of G K to G L hath to the proportion of G K to G E. Therefore L K will be to one of those three parts as the Arithmetical proportion between G K and G L is to the Arithmetical proportion between G K and the same G K want the third part of L K; and K E will be somwhat greater then a third of L K. Seeing now the altitude A G or M L is by reason of the continual decrease, to be supposed less then any quantity that can be given; L K (which is intercepted between the Diagonal B C and the side B D) will be also less then any quantity that can be given; and consequently, if G be put so neer to A in *g*, as that the difference between C *g* and C A be less then any quantity that can be assigned, the difference also between C *l* (removing L to *l*) and C B, will be less then any quantity that can be assigned; and the line *g l* being drawn & produced to the line B D in *k* cutting the crooked line

in

in *e*, the proportion of G *k* to G *l* will ſtill be triplicate to the proportion of G *k* to G *e*, and the difference between *k* and *e* the third part of *k l* will be leſs then any quantity that can be given; and therefore the Parallelogram *e* D will differ from a third part of the Parallelogram A *e* by a leſs difference then any quantity that can be aſſigned. Again, let H I be drawn parallel and equal to *g e*, cutting C B in P, the crooked line in F, and B D in I, and the proportion of C *g*, to C H will be triplicate to the proportion of H F to H P, and I F will be greater then the third part of P I. But again, ſetting H in *h* ſo neer to *g*, as that the difference between C *h* and C *g* may be but as a point, the point P will alſo in *p* be ſo neer to *l*, as that the difference between C *p* and C *l* will be but as a point; and drawing *h p* till it meet with *g k* in *i*, cutting the crooked line in *f*, and having drawn *e o* parallel to B D, cutting D C in *o*, the Parallelogram *f o* will differ leſs from the third part of the Parallelogram *g f*, then by any quantity that can be given. And ſo it will be in all other Spaces generated in the ſame manner. Wherefore the differences of the Arithmetical and Geometrical Means, which are but as ſo many points B, *e*, *f*, &c. (ſeeing the whole Figure is made up of ſo many indiviſible Spaces) will conſtitute a certain line, ſuch as is the line B E F C, which will divide the complete Figure A D into two parts; whereof one, namely A B E F C, which I call a Deficient Figure, is triple to the other, namely B D C E F, which I call the Complement thereof. And whereas the proportion of the altitudes to one another, is in this caſe everywhere triplicate to that of the decreaſing quantities to one another; in the ſame manner if the proportion of the altitudes had been every where quadruplicate to that of the decreaſing quantities it might have been demonſtrated, that the Deficient Figure had been quadruple to its Complement; and ſo in any other proportion, Wherefore, a Deficient Figure, which is made, &c. Which was to be demonſtrated.

The ſame rule holdeth alſo in the diminution of the Baſes of Cylinders, as is demonſtrated Chap. 15. Art. 2.

3 By this Propoſition, the magnitudes of all Deficient Figures (when the proportions by which their baſes decreaſe continually, are proportionall to thoſe by which their altitudes decreaſe) may be compared with the magnitudes of their Complements;

and

and consequently, with the magnitudes of their Complete Figures. And they will be found to be as I have set them down in the following Tables; in which I compare a Parallelogram with three-sided Figures; and first with a straight lined triangle, made by the base of the Parallelogram continually decreasing in such manner, that the altitudes be alwayes in proportion to one another as the bases are, and so the triangle will be equal to its Complement; or the proportions of the altitudes and bases wil be as 1 to 1, and then the triangle will be half the Parallelogram. Secondly, with that three-sided Figure which is made by the continual decreasing of the bases in subduplicate proportion to that of the altitudes; and so the Deficient Figure will be double to its Complement, and to the Parallelogram as 2 to 3. Then, with that, where the proportion of the altitudes is triplicate to that of the bases; and then the Deficient Figure will be triple to its Complement, and to the Parallelogram as 3 to 4. Also the proportion of the altitudes to that of the bases may be as 3 to 2; and then the Deficient Figure will be to its Complement as 3 to 2, & to the Parallelogram as 3 to 5; and so forwards according as more mean proportionals are taken, or as the proportions are more multiplyed, as may be seen in the following Table. For example, if the bases decrease so, that the proportion of the altitudes to that of the bases be alwayes as 5 to 2, and it be demanded what proportion the Figure made has to the Parallelogram, which is supposed to be Unity; then, seeing that where the proportion is taken five times, there must be four Means; look in the Table amongst the three-sided figures of four Means, and seeing the proportion was as 5 to 2, look in the uppermost row for the number 2, and descending in the 2d Columne till you meet with that three-sided Figure, you will finde $\frac{5}{7}$; which shews that the Deficient Figure is to the Parallelogram as $\frac{5}{7}$ to 1, or as 5 to 7.

	1	2	3	4	5	6	7
Parallelogram :	1						
Straight-ſided Triangle	$\frac{1}{2}$						
Three-ſided figure of 1 Mean	$\frac{2}{3}$						
Three-ſided figure of 2 Means	$\frac{3}{4}$	$\frac{3}{5}$					
Three-ſided figure of 3 Means	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$				
Three-ſided figure of 4 Means	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$			
Three-ſided figure of 5 Means	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$	$\frac{6}{11}$		
Three-ſided figure of 6 Means	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$	$\frac{7}{11}$	$\frac{7}{12}$	$\frac{7}{13}$	
Three-ſided figure of 7 Means	$\frac{8}{9}$	$\frac{8}{10}$	$\frac{8}{11}$	$\frac{8}{12}$	$\frac{8}{13}$	$\frac{8}{14}$	$\frac{8}{15}$

4 Now for the better underſtanding of the nature of theſe three-ſided figures, I will ſhew how they may be deſcribed by points; and firſt, thoſe which are in the firſt column of the Table. Any Parallelogram being deſcribed, as A B C D (in the 2d. figure,) let the Diagonal B D be drawn; and the ſtraight-lined triangle B C D will be half the Parallelogram; Then let any number of lines, as E F, be drawn parallel to the Side B C, and cutting the Diagonal B D in G; & let it be every where, as E F to E G, ſo E G to another E H; and through all the points H let the line B H H D be drawn; and the Figure B H H D C will be that which I call a Three-ſided Figure of one Mean, becauſe in three proportionals, as E F, E G and E H, there is but one Mean, namely, E G; and this three-ſided figure will be $\frac{2}{3}$ of the Parallelogram, and is called a *Parabola*. Again, let it be as E G to E H, ſo E H to another E I, and let the line B I I D be drawn, making the three-ſided figure B I I D C; & this will be $\frac{3}{4}$ of the Parallelogram, and is by many called a *Cubique Parabola*. In like manner, if the proportions be further continued in E F, there will be made the reſt of the three-ſided

ſided figures of the firſt Column; which I thus demonſtrate. Let there be drawn ſtraight lines, as H K and G L parallel to the baſe D C. Seeing therefore the proportion of E F to E H is duplicate of that of E F to E G, or of B C to B L, that is, of C D to L G, or of K M (producing K H to A D in M) to K H, the proportion of B C to B K will be duplicate to that of K M to K H; but as B C is to B K, ſo is D C, or K M to K N; and therefore the proportion of K M to K N is duplicate to that of K M to K H; and ſo it will be whereſoever the parallel K M be placed. Wherefore the Figure B H H D C is double to its Complement B H H D A, and conſequently $\frac{2}{3}$ of the whole Parallelogram. In the ſame manner if through I, be drawn O P I Q parallel and equal to C D, it may be demonſtrated that the proportion of O Q to O P, that is, of B C to B O, is triplicate to that of O Q to O I, and therefore that the Figure B I I D C is triple to its Complement B I I D A, and conſequently $\frac{3}{4}$ of the whole Parallelogram, &c.

Secondly, ſuch three-ſided figures as are in any of the tranſverſe rowes, may be thus deſcribed. Let A B C D (in the 3d. *Figure*) be a Parallelogram, whoſe Diagonal is B D. I would deſcribe in it ſuch figures, as in the preceding Table I call Three-ſided Figures of three Means. Parallel to D C, I draw E F as often as is neceſſary, cutting B D in G; and between E F and E G I take three proportionals E H, E I and E K. If now there be drawn lines through all the points H, I & K; that through all the points H will make the figure B H D C, which is the firſt of thoſe three-ſided figures; and that through all the points I, will make the figure B I D C, which is the ſecond; and that which is drawn through all the points K, will make the figure B K D C the third of thoſe three-ſided figures. The firſt of theſe (ſeeing the proportion of E F to E G is quadruplicate of that of E F to E H) will be to its Complement as 4 to 1, and to the Parallelogram as 4 to 5. The ſecond (ſeeing the proportion of E F to E G is to that of E F to E I as 4 to 2) will be double to its Complement, and $\frac{4}{6}$ or $\frac{2}{3}$ of the Parallelogram. The third (ſeeing the proportion of E F to E G is to that of E F to E K as 4 to 3) will be to its Complement as 4 to 3, and to the Parallelogram as 4 to 7.

Any of theſe figures being deſcribed, may be produced at pleaſure, thus; Let A B C D (in the 4th figure) be a Parallelogram, and in it let the figure B K D C be deſcribed, namely, the third three-ſided figure of three Means. Let B D be produced indefinitely to E, and let E F be made parallel to the baſe D C, cutting

A D produced in G, and B C produced in F; and in G E let the point H be so taken, that the proportion of F E to F G may be quadruplicate to that of F E to F H (which may be done by making F H the greatest of three proportionals between F E and F G); the crooked line B K D produced, will pass through the point H. For if the straight line B H be drawn, cutting C D in I, and H L be drawn parallel to G D, and meeting C D produced in L; it will be as F E to F G, so C L to C I; that is, in quadruplicate proportion to that of F E to F H, or of C D to C I. Wherefore if the line B K D be produced according to its generation, it will fall upon the point H.

5 A straight line may be drawn so, as to touch the crooked line of the said figure in any point, in this manner. Let it be required to draw a Tangent to the line B K D H (in the 4th figure) in the point D. Let the points B and D be connected, and drawing D A equal and parallel to B C, let B and A be connected; and because this figure is by construction the third of three Means, let there be taken in A B three points, so, that by them the same A B be divided into four equal parts; of which take three, namely, A M, so that A B may be to A M, as the figure B K D C is to its Complement. I say the straight line M D, will touch the figure in the point given D. For let there be drawn any where between A B and D C a parallel, as R Q, cutting the straight line B D, the crooked line B D, the straight line M D, and the straight line A D in the points P, K, O and Q. R K will therefore (by construction) be the least of three Means in Geometrical proportion between R Q and R P. Wherefore (by the *Coroll.* of the 28th Article of the 13th Chapter) R K will be less then R O; and therefore M D will fall without the figure. Now if M D be produced to N, F N will be the least of three Means in Arithmetical proportion between F E and F G; and F H will be the greatest of three Means in Geometrical proportion between the same F E and F G. Wherefore (by the same *Coroll.* of the 28 *Artic.* of the 13th Chap.) F H will be less then F N; and therefore D N will fall without the figure, and the straight line M N will touch the same figure onely in the point D.

6 The proportion of a Deficient Figure to its Complement being known, it may also be known what proportion a straight-lined

lined triangle has to the excess of the Deficient Figure above the same triangle; and these proportions I have set down in the following Table; where if you seek (for example) how much the fourth three-sided figure of five Means exceeds a triangle of the same altitude and base, you will find in the concourse of the fourth column with the three-sided figures of five Means, $\frac{2}{10}$; by which is signified, that that three-sided figure exceeds the triangle by two tenths, or by one fifth part of the same triangle.

		1	2	3	4	5	6	7
	The Triangle	1						
The Excess of	a Three-sided figure of one Mean	$\frac{1}{3}$						
	a Three-sided figure of 2 Means	$\frac{2}{4}$	$\frac{1}{5}$					
	a Three-sided figure of 3 Means	$\frac{3}{5}$	$\frac{2}{6}$	$\frac{1}{7}$				
	a Three-sided figure of 4 Means	$\frac{4}{6}$	$\frac{3}{7}$	$\frac{2}{8}$	$\frac{1}{9}$			
	a Three-sided figure of 5 Means	$\frac{5}{7}$	$\frac{4}{8}$	$\frac{3}{9}$	$\frac{2}{10}$	$\frac{1}{11}$		
	a Three-sided figure of 6 Means	$\frac{6}{8}$	$\frac{5}{9}$	$\frac{4}{10}$	$\frac{3}{11}$	$\frac{2}{12}$	$\frac{1}{13}$	
	a Three-sided figure of 7 Means	$\frac{7}{9}$	$\frac{6}{10}$	$\frac{5}{11}$	$\frac{4}{12}$	$\frac{3}{13}$	$\frac{2}{14}$	$\frac{1}{15}$

7 In the next Table are set down the proportion of a Cone, and the Solids of the said three-sided figures, namely, the proportions between them and a Cylinder. As for example, in the concourse of the second Column with the three-sided figures of four Means, you have $\frac{5}{9}$; which gives you to understand, that the Solid of the second three-sided figure of four Means is to the Cylinder as $\frac{5}{9}$ to 1, or as 5 to 9,

The

		1	2	3	4	5	6	7
	The Cylinder	1						
	A Cone	1/3						
The Solids of	a Three-sided figure of one Mean	2/4						
	a Three-sided figure of 2 Means	3/5	3/7					
	a Three-sided figure of 3 Means	4/6	4/8	4/10				
	a Three-sided figure of 4 Means	5/7	5/9	5/11	5/13			
	a Three-sided figure of 5 Means	6/8	6/10	6/12	6/14	6/16		
	a Three-sided figure of 6 Means	7/9	7/11	7/13	7/15	7/17	7/19	
	a Three-sided figure of 7 Means	8/10	8/12	8/14	8/16	8/18	8/20	8/22

8 Lastly, the Excess of the Solids of the said three-sided figures, above a Cone of the same altitude and base, are set down in the Table which follows

		1	2	3	4	5	6	7
	The Cone	1						
The Excesses of the Solids of these three-sided figures above a C ne	Of the Solid of a three-sided figure of one Mean	6/12						
	Of the Solid of a three-sided figure of 2 Means	12/15	6/21					
	Of the Solid of a three-sided figure of 3 Means	18/18	12/24	6/30				
	Of the Solid of a three-sided figure of 4 Means	24/21	18/27	12/33	6/39			
	Of the Solid of a three-sided figure of 5 Means	30/24	24/30	18/36	12/42	6/48		
	Of the Solid of a three-sided figure of 6 Means	36/27	30/33	24/39	18/45	12/51	6/57	
	Of the Solid of a three-sided figure of 7 Means	42/30	36/36	20/42	24/48	18/54	12/60	6/66

9 If any of these Deficient Figures, of which I have now spoken, as A B C D (in the 5th figure) be inscribed within the Complete figure B E, having A D C E for its Complement; and there be made upon C B produced, the triangle A B I; and the Parallelogram A B I K be completed; and there be drawn parallel to the straight line C I, any number of lines as M F, cutting every one of them the crooked line of the Deficient Figure in D, and the straight lines A C, A B and A I in H, G and L; and as G F is to G D, so G L be made to another G N; and through all the points N there be drawn the line A N I, there will be a Deficient Figure A N I B, whose Complement will be A N I K. I say the figure A N I B is to the triangle A B I, as the Deficient Figure A B C D twice taken, is to the same Deficient Figure together with the Complete figure B E.

For as the proportion of A B to A G, that is, of G M to G L, is to the proportion of G M to G N; so is the magnitude of the figure A N I B, to that of its Complement A N I K (by the 2d. *Art.* of this Chapter.)

But (by the same Article), As the proportion of A B to A G, that is, of G M to G L, is to the proportion of GF to G D, that is, (by construction) of G L to G N; so is the figure A B C D to its Complement A D C E.

And by Composition, As the proportion of G M to G L, together with that of G L to G N, is to the proportion of G M to G L; so is the complete figure B E, to the Deficient Figure A B C D.

And by Conversion, As the proportion of G M to G L, is to both the proportions of G M to G L and of G L to G N, that is, to the proportion of G M to G N (which is the proportion compounded of both); so is the Deficient Figure A B C D, to the complete Figure B E.

But it was, As the proportion of G M to G L, to that of G M to G N; so the figure A N I B to its Complement A N I K. And therefore, A B C D. B E : : A N I B. A N I K are proportionals. And by Composition, A B C D + B E. A B C D : : B K. A N I B are proportionals.

And

And by doubling the Consequents ABCD+BE. 2 ABCD : : BK. 2 ANIB are proportionals.

And by taking the halfes of the third & the fourth ABCD+BE. 2 ABCD : : ABI. ANIB are also proportionals; which was to be proved.

10 From what has been said of Deficient Figures described in a Parallelogram, may be found out what proportions Spaces transmitted with accelerated Motion in determined times, have to the times themselves, according as the moved Body is accelerated in the several times with one or more degrees of Velocity.

For, let the Parallelogram A B C D (in the 6th figure) and in it the three-sided figure D E B C be described; and let F G be drawn any where parallel to the base, cutting the Diagonal B D in H, and the crooked line B E D in E; & let the proportion of B C to B F be (for example) triplicate to that of F G to F E; whereupon the figure D E B C will be triple to its Complement B E D A; and in like manner, I F being drawn parallel to B C, the three-sided figure E K B F will be triple to its Complement B K E I. Wherefore, the parts of the Deficient Figure cut off from the Vertex by straight lines parallel to the base, namely D E B C and E K B F, will be to one another as the Parallelograms A C and I F; that is, in proportion compounded of the proportions of the altitudes and bases. Seeing therefore the proportion of the altitude B C to the altitude B F is triplicate to the proportion of the base D C to the base F E, the figure D E B C to the figure E K B F will be quadruplicate to the proportion of the same D C to F E. And by the same method, may be found out, what proportion any of the said three-sided figures, has to any part of the same cut off from the Vertex by a straight line parallel to the base.

Now as the said figures are understood to be described by the continual decreasing of the base, as of C D (for example) till it end in a point, as in B; so also they may be understood to be described by the continual encreasing of a point, as of B, till it acquire any magnitude, as that of C D.

Suppose now the figure B E D C to be described by the encreasing of the point B to the magnitude C D. Seeing therefore the propor ion of B C to B F is triplica te to that of C D to F E, the pro-

proportion of F E to C D will by Conversion (as I shall presently demonstrate) be triplicate to that B F to B C. Wherefore, if the straight line B C be taken for the measure of the time in which the point B is moved, the Figure E K B F will represent the Sum of all the encreasing Velocities in the time B F; and the figure D E B C will in like manner represent the Summe of all the encreasing Velocities in the time B C. Seeing therefore the proportion of the figure E K B F to the figure D E B C, is compounded of the proportions of altitude to altitude, and base to base; and seeing the proportion of F E to C D is triplicate to that of B F to B C; the proportion of the figure E K B F to the figure D E B C, will be quadruplicate to that of BF to BC; that is, the proportion of the Sum of the Velocities in the time BF, to the Sum of the Velocities in the time BC wil be quadruplicate to the proportion of BF to BC. Whertore if a Body be moved from B with Velocity so encreasing, that the Velocity acquired in the time B F, be to the Velocity acquired in the time B C in triplicate proportion to that of the times themselves B F to B C, and the Body be carried to F in the time B F; the same Body in the time B C will be carried through a line equal to the fifth proportional from B F in the continual proportion of B F to B C. And by the same manner of working, we may determine, what Spaces are transmitted by Velocities encreasing according to any other proportions.

It remains, that I demonstrate the proportion of F E to C D, to be triplicate to that of B F to B C. Seeing therefore the proportion of C D, that is of F G to F E is subtriplicate to that of B C to B F; the proportion of F G to F E will also be subtriplicate to that of F G to F H. Wherefore the proportion of F G to F H is triplicate to that of F G, that is, of C D to F E. But in four continual proportionals, of which the least is the first, the proportion of the first to the fourth (by the 16 *Art.* of the 13 Chap.) is subtriplicate to the proportion of the third to the same fourth. Wherefore the proportion of F H to G F is subtriplicate to that of F E to C D; and therefore the proportion of F E to C D is triplicate to that of F H to F G, that is, of B F to B C, which was to be proved.

It may from hence be collected, that when the Velocity of a

 Body,

Body, is encreased in the same proportion with that of the times, the degrees of Velocity above one another proceed as numbers do in immediate succession from Unity, namely, as 1,2,3,4, &c. And when the Velocity is encreased in proportion duplicate to that of the times, the degrees proceed as numbers from Unity skipping One, as 1, 3, 5, 7,&c. Lastly, when the proportions of the Velocities are triplicate to those of the times, the progression of the degrees is as that of numbers from Unity skipping Two in every place, as 1, 4, 7, 10, &c. and so of other proportions. For Geometrical proportionals, when they are taken in every point, are the same with Arithmetical proportionals.

11 Moreover, it is to be noted, that as in quantities which are made by any magnitudes decreasing, the proportions of the figures to one another, are as the proportions of the altitudes to those of the bases; so also it is in those which are made with motion decreasing, which motion is nothing else but that power by which the described figures are greater or less. And therefore in the description of *Archimedes* his *Spiral*, which is done by the continual diminution of the Semidiameter of a Circle in the same proportion in which the Circumference is diminished, the Space which is contained within the Semidiameter and the Spiral Line, is a third part of the whole Circle. For the Semidiameters of Circles, in as much as Circles are understood to be made up of the aggregate of them, are so many Sectors; and therefore in the description of a Spiral, the Sector which describes it, is diminished in duplicate proportions to the diminutions of the Circumference of the Circle in which it is inscribed; so that the Complement of the Spiral (that is, that space in the Circle which is without the Spiral Line,) is double to the space within the Spiral Line. In the same manner, if there be taken a mean proportional every where between the Semidiameter of the Circle which contains the Spiral, and that part of the Semidiameter which is within the same, there will be made another figure, which will be half the Circle. And to conclude, this Rule serves for all such Spaces as may be described by a Line or Superficies decreasing either in magnitude or power; so that if the proportions in which they decrease, be commensurable to the proportions of the times in which they de-

crease,

G

crease, the magnitudes of the figures they describe will be known.

12 The truth of that proposition which I demonstrated in the second *Article* (which is the foundation of all that has been said concerning Deficient Figures) may be derived from the Elements of Philosophy, as having its original in this, *That all equality and inequality between two effects, (that is, all Proportion) proceeds from, and is determined by the equal and unequal causes of those effects, or from the proportion which the causes concurring to one effect have to the causes which concurre to the producing of the other effect*; and that therefore the proportions of Quantities are the same with the proportions of their causes. Seeing therefore two Deficient Figures (of which one is the Complement of the other) are made, one by motion decreasing in a certain time and proportion, the other by the loss of Motion in the same time, the causes which make and determine the quantities of both the figures, so, that they can be no other then they are, differ onely in this, that the proportions by which the quantity which generates the figure proceeds in describing of the same, (that is, the proportions of the remainders of all the times and altitudes) may be other proportions then those by which the same generating quantity decreases in making the Complement of that Figure, (that is, the proportions of the quantity which generates the Figure continually diminished.) Wherefore, as the proportions of the quantity in which Motion is lost, is to that of the decreasing quantities by which the Deficient Figure is generated, so will the Defect or Complement be to the Figure it self which is generated.

13 There are also other quantities which are determinable from the knowledge of their causes, namely, from the comparison of the Motions by which they are made, and that more easily then from the common Elements of Geometry. For example, That the Superficies of any portion of a Sphere, is equal to that Circle, whose Radius is a straight Line drawn from the Pole of the portion to the Circumference of its base, I may demonstrate in this manner. Let B A C (in the 7 Figure) be a portion of a Sphere, whose Axis is A E, & whose base is B C; & let A B be the straight line drawn from the Pole A to the base in B; and let A D, equal to A B, touch the great Circle B A C in the Pole A. It is to be proved

 that

that the Circle made by the Radius A D, is equal to the Superficies of the portion B A C. Let the plain A E B D be understood to make a revolution about the Axis A E; & it is manifest that by the straight line A D a Circle will be described; and by the arch A B the Superficies of a portion of a Sphere, and lastly, by the Subtense A B the Superficies of a right Cone. Now seeing both the straight line A B, and the arch A B make one and the same revolution, and both of them have the same extreme points A and B, the cause why the the Spherical Superficies which is made by the arch, is greater then the Conical Superficies which is made by the Subtense, is, that A B the arch, is greater then A B the Subtense; and the cause why it is greater consists in this, that although they be both drawn from A to B, yet the Subtense is drawn straight, but the arch angularly, namely according to that angle which the arch makes with the Subtense, which angle is equal to the angle D A B (for an angle of contingence adds nothing to an angle of a Segment, as has been shewn in the 14 Chapter at the 16th Article.) Wherefore the magnitude of the angle D A B is the cause why the Superficies of the portion described by the arch A B, is greater then the Superficies of the right Cone described by the Subtense A B.

Again, the cause why the Circle described by the Tangent A D is greater then the Superficies of the right Cone described by the Subtense A B (notwithstanding that the Tangent and the Subtense are equal, and both moved round in the same time) is this, that AD stands at right angles to the Axis, but A B obliquely; which obliquity consists in the same angle D A B. Seeing therefore the quantity of the angle D A B is that which makes the excess both of the Superficies of the Portion, and of the Circle made by the Radius AD, above the superficies of the Right Cone described by the subtense A B; it follows, that both the Superficies of the Portion, and that of the Circle, do equally exceed the Superficies of the Cone. Wherefore, the Circle made by A D, or A B, and the Spherical Superficies made by the arch A B, are equal to one another; which was to be proved.

14 If these Deficient Figures which I have described in a Parallelogram, were capable of exact description, then any number of

of mean proportionals might be found out between two ſtraight lines given. For example, in the Parallelogram A B C D, (in the 8th. Figure) let the three-ſided figure of two Means be deſcribed, (which many call a *Cubical Parabola*); and let R and S be two given ſtraight lines; between which, if it be required to find two mean proportionals, it may be done thus. Let it be as R to S, ſo B C to B F; and let F E be drawn parallel to B A, and cut the crooked line in E; then through E let G H be drawn parallel and equal to the ſtraight line A D, and cut the Diagonal B D in I; for thus we have G I the greateſt of two Means between G H and G E, as appears by the deſcription of the figure in the 4th Article. Wherefore if it be as G H to G I, ſo R to another line T, that T will be the greateſt of two Means between R and S. And therefore if it be again as R to T, ſo T to another line X, that will be done which was required.

In the ſame manner, four mean proportionals may be found out, by the deſcription of a three-ſided figure of four Means; and ſo, any other number of Means, &c.

CHAP. XVIII.

Of the Equation of Straight Lines with the Crooked Lines of Parabolas and other Figures made in imitation of Parabolas.

1 To find a straight Line equal to the crooked Line of a Semiparabola. 2 To find a straight Line equal to the Crooked Line of the first Semiparabolaster, or to the Crooked Line of any other of the Deficient Figures of the Table of the 3d. Article of the precedent Chapter.

1 A *Parabola* being given, to find a Straight Line equal to the Crooked Line of the Semiparabola.

Let the Parabolical Line given be A B C (in the first Figure), and the Diameter found be A D, and the base drawn D C, and the Parallelogram A D C E being completed, draw the straight Line A C. Then dividing A D into two equal parts in F, draw F H equal and parallel to D C, cutting A C in K, and the parabolical line in O; and between F H and F O take a mean proportional F P, and draw A O, A P and P C. I say that the two Lines A P and P C taken together as one Line, is equal to the parabolical line A B O C.

For the line A B O C being a parabolical line, is generated by the concourse of two Motions, one Uniform from A to E, the other in the same time uniformly accelerated from rest in A to D. And because the motion from A to E is uniform, A E may represent the times of both those motions from the beginning to the end. Let therefore A E be the time; and consequently the lines or

ordinately applyed in the Semiparabola, will defigne the parts of time wherein the Body that describeth the line A B O C is in every point of the same; so that as at the end of the time A E or D C it is in C, so at the end of the time F O it will be in O. And because the Velocity in A D is encreased uniformly, that is, in the same proportion with the time, the same lines ordinately applyed in the Semiparabola will defigne also the continual augmentations of the *Impetus*, till it be at the greatest, designed by the base D C. Therefore supposing Uniform motion in the line A F, in the time F K the Body in A by the concourse of the two uniform motions in A F and F K will be moved uniformly in the line A K; and K O will be the encrease of the *Impetus* or Swiftness gained in the time F K; and the line A O will be uniformly described by the concourse of the two uniform motions in A F and F O in the time F O. From O draw OL parallel to E C, cutting AC in L; & draw LN parallel to D C, cutting E C in N, and the parabolical line in M; and produce it on the other side to A D in I; and I N, I M and I L will be (by the construction of a Parabola) in continual proportion, & equal to the three lines F H, F P and F O; and a straight line parallel to E C passing through M will fall on P; and therefore O P will be the encrease of *Impetus* gained in the time F O or I L. Lastly, produce P M to C D in Q; and Q C, or M N, or P H will be the encrease of *Impetus* proportional to the time F P, or I M, or D Q. Suppose now uniform motion from H to C in the time P H. Seeing therefore in the time F P with uniform motion and the *Impetus* encreased in proportion to the times, is described the straight line A P; and in the rest of the time and *Impetus*, namely P H, is described the line C P uniformly; it followeth that the whole line A P C is described with the whole *Impetus*, and in the same time wherewith is described the parabolicall line A B C; and therefore the line A P C, made of the two straight lines A P and P C, is equal to the parabolical line A B C; which was to be proved.

2 To find a Straight line equal to the Crooked line of the first Semiparabolaster.

Let A B C be the Crooked line of the first Semiparabolaster; A D the Diameter; D C the Base; and let the Parallelogram com-

completed be A D C E, whose Diagonal is A C. Divide the Diameter into two equal parts in F, and draw F H equal and parallel to D C, cutting A C in K; the Crooked line in O, and E C in H. Then draw O L parallel to E C, cutting A C in L; and draw L N parallel to the base D C, cutting the Crooked line in M, and the straight line E C in N; and produce it on the other side to A D in I. Lastly, through the point M draw P M Q parallel and equal to H C, cutting F H in P; and joyn C P, A P and A O. I say the two Straight lines A P and P C are equal to the Crooked line A B O C.

For the line A B O C being the Crooked line of the first Semiparabolaster, is generated by the concourse of two Motions, one uniform from A to E, the other in the same time accelerated from rest in A to D, so as that the *Impetus* encreaseth in proportion perpetually triplicate to that of the encrease of the time, or (which is all one) the lengths transmitted are in proportion triplicate to that of the times of their transmission; for as the *Impetus* or Quicknesses encrease, so the Lengths transmitted encrease also. And because the motion from A to E is uniform, the line A E may serve to represent the time, and consequently the lines ordinately drawn in the Semiparabolaster, will designe the parts of time wherein the Body beginning from rest in A, describeth by its motion the Crooked line A B O C. And because D C which represents the greatest acquired *Impetus* is equal to A E, the same ordinate lines will represent the several augmentations of the *Impetus* encreasing from rest in A. Therefore supposing uniform Motion from A to F in the time F K, there will be described by the concourse of the two uniform Motions A F and F K the line A K uniformly, and K O will be the encrease of *Impetus* in the time F K; And by the concourse of the two uniform Motions in A F and F O, will be described the line A O uniformly. Through the point L draw the straight line L M N parallel to D C, cutting the straight line A D in I, the crooked line A B C in M, and the straight line E C in N; and through the point M the straight line P M Q parallel and equal to H C, cutting D C in Q, and F H in P. By the concourse therefore of the two uniform Motions in A F and F P in the time F P will be uniformly described the straight line A P; and L M or O P will be the encrease of *Impetus* to be added for the time F O. And because the proportion of I N

to I L is triplicate to the proportion of I N to I M, the proportion of F H to F O will also be triplicate to the proportion of F H to F P; and the proportional *Impetus* gained in the time F P is P H. So that F H being equal to P C which designed the whole *Impetus* acquired by the acceleration, there is no more encrease of *Impetus* to be computed. Now in the time P H suppose an uniform motion from H to C; and by the two uniform motions in C H and H P will be described uniformly the Straight line P C. Seeing therefore the two Straight lines A P and P C are described in the time A E with the same encrease of *Impetus* wherewith the Crooked line A B C is described in the same time A E, that is, seeing the Line A P C and the Line ABC are transmitted by the same Body in the same Time, & with equal Velocities, the Lines themselves are equal; which was to be demonstrated.

By the same method, if any of the Semiparabolasters in the Table of the 3d Article of the precedent Chapter be exhibited, may be found a Straight line equal to the Crooked line thereof, namely, by dividing the Diameter into two equal parts, and proceeding as before. Yet no man hitherto hath compared any Crooked with any Straight Line, though many Geometricians of every Age have endeavoured it. But the cause why they have not done it may be this, that there being in *Euclide* no Definition of Equality, nor any mark by which to judge of it besides Congruity (which is the 8th. *Axiome* of the first Book of his Elements) a thing of no use at all in the comparing of Straight and Crooked; and others after *Euclide* (except *Archimedes* and *Apollonius*, and in our time *Bonaventura*) thinking the industry of the Ancients had reached to all that was to be done in Geometry, thought also, that all that could be propounded, was either to be deduced from what they had written, or else that it was not at all to be done. It was therefore disputed by some of those Ancients themselves, whether there might be any Equality at all between Crooked and Straight Lines; Which question *Archimedes* (who assumed that some Straight line was equal to the Circumference of a Circle) seems to have despised, as he had reason. And there is a late Writer that granteth that between a Straight and a Crooked Line there is Equality; but now, now sayes he, since the fall of *Adam*, without the special assistance of Divine Grace, it is not to be found.

CHAP. XIX.

Of Angles of Incidence and Reflection, equal by supposition.

1 *If two straight lines falling upon another straight line be parallel, the lines reflected from them shall also be parallel.* 2 *If two straight lines drawn from one point, fall upon another straight line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle made by the lines of Incidence.* 3 *If two straight parallel lines drawn (not oppositely but) from the same parts, fall upon the Circumference of a Circle, the lines reflected from them, if produced they meet within the Circle, will make an angle double, to that which is made by two straight lines drawn from the Center to the points of Incidence.* 4 *If two straight lines drawn from the same point without a Circle fall upon the Circumference, and the lines reflected from them, being produced meet within the Circle, they will make an angle equal to twice that angle which is made by two straight lines drawn from the Center to the points of Incidence, together with the angle which the incident lines themselves make.* 5 *If two straight lines drawn from one point fall upon the concave Circumference of a Circle, and the angle they make be less then twice the angle at the Center, the lines reflected from them, and meeting within the Circle, will make an angle which being added to the angle of the incident lines, will be equal to twice the angle at the Center.* 6 *If through any one point two unequal Chords be drawn cutting one another, and the Center of the Circle be not placed between them, and the lines reflected from them concurre wheresoever, there cannot through the point through which the two former lines were drawn, be drawn any other straight line, whose reflected line shall pass through the common point of the two former lines reflected.* 7 *In equal Chords the same is not true.* 8 *Two points being given in the Circumference of a Circle, to draw two straight lines to them, so as that their reflected lines may contain any angle given.* 9 *If a straight line falling upon the Circumference of a Circle be produced till it reach the Semidiameter, and that part of it which is intercepted between the Circumference and the Semidiameter, be equal to that part of the Semidiameter which is between the point of concourse & the center, the reflected line will be parallel to the Semidiameter.* 10 *If from a point within a*

Circle, two straight lines be drawn to the Circumference, and their reflected lines meet in the Circumference of the same Circle, the angle made by the reflected lines, will be a third part of the angle made by the incident lines.

Whether a Body, falling upon the superficies of another Body and being reflected from it, do make equal angles at that superficies, it belongs not to this place to dispute, being a knowledge which depends upon the natural causes of Reflection; of which hitherto nothing has been said, but shall be spoken of hereafter.

In this place therefore let it be supposed, that the angle of Incidence is equal to the angle of Reflection, that our present search may be applyed not to the finding out of the causes, but some consequences of the same.

I call an *Angle of Incidence*, that which is made between a straight line and another line (straight or crooked) upon which it falls, and which I call the *Line Reflecting*; and an *Angle of Reflection* equal to it, that which is made at the same point between the straight line which is reflected, and the line reflecting.

1 If two straight lines which fall upon another straight line be be parallel, their reflected lines shall be also parallel.

Let the two straight lines A B and CD (in the 1 figure) which fall upon the straight line E F, at the points B and D, be parallel; and let the lines reflected from them be B G and D H. I say B G and D H are also parallel.

For the angles A B E and C D E are equal by reason of the parallellelisme of A B and C D; and the angles G B F and H D F are equal to them by supposition; for the lines B G and D H are reflected from the lines A B and C D. Wherefore B G and D H are parallel.

2 If two straight lines drawn from the same point, fall upon another straight line, the lines reflected from them, if they be drawn out the other way, will meet in an angle equal to the angle of the Incident lines.

From the point A C (in the 2d. figure) let the two straight lines A B and A D be drawn; and let them fall upon the straight line E K at the points B and D; and let the lines B I and D G be reflected from them. I say, I B and G D do converge, and that if they be

be produced on the other side of the line E K they shall meet, as in F; and that the angle B F D shal be equal to the angle B A D.

For the angle of Reflection I B K is equal to the angle of Incidence A B E; and to the angle I B K, its vertical angle E B F is equal; and therefore the angle A B E is equal to the angle E B F. Again, the angle A D E is equal to the angle of Reflection G D K, that is, to its vertical angle E D F; and therefore the two angles A B D and A D B of the triangle A B D, are one by one equal to the two angles F B D and F D B of the triangle F B D; Wherfore also the third angle B A D is equal to the third angle B F D; which was to be proved.

Corollary 1. If the straight line A F be drawn, it will be perpendicular to the straight line E K. For both the angles at E will be equal, by reason of the equality of the two angles A B E and F B E, and of the two sides A B and F B.

Corollary 2. If upon any point between B and D there fall a straight line, as A C, whose reflected line is C H, this also produced beyond C, will fall upon F; which is evident by the demonstration above.

3 If from two points taken without a Circle, two straight parallel lines drawn (not oppositely but) from the same parts, fall upon the Circumference; the lines reflected from them, if produced they meet within the Circle, will make an angle double to that which is made by two straight lines drawn from the Center to the points of Incidence.

Let the two straight parallels A B and D C (in the 3d figure) fall upon the Circumference B C at the points B and C; and let the Center of the Circle be E; and let A B reflected be B F, and D C reflected be C G; and let the lines F B and G C produced meet within the Circle in H; and let E B and E C be connected. I say the angle F H G is double to the angle B E C.

For seeing A B and D C are parallels, and E B cuts A B in B, the same E B produced will cut D C somewhere; let it cut it in D, & let D C be produced howsoever to I, and let the intersection of D C & B F be at K. The angle therefore I C H (being external to the triangle C K H,) will be equal to the two opposite angles C K H and C H K. Again, I C E (being external to the triangle C D E) is

is equal to the two angles at D and E. Wherefore the angle I C H, being double to the angle I C E, is equal to the angles at D and E twice taken; and therefore the two angles C K H and C H K are equal to the two angles at D and E twice taken. But the angle C K H is equal to the angles D and A B D, that is, D twice taken, (for A B and D C being parallels, the altern angles D, and A B D are equal). Wherefore C H K, that is the angle, F H G is also equal to the angle at E twice taken; which was to be proved.

Corollary. If from two points taken within a circle, two straight parallels fall upon the circumference, the lines reflected from them shall meet in an angle, double to that which is made by two straight lines drawn from the center to the points of Incidence. For the parallels L B and I C falling upon the points B and C, are reflected in the lines B H and C H, and make the angle at H double to the angle at E, as was but now demonstrated.

4 If two straight lines drawn from the same point without a circle, fall upon the circumference, and the lines reflected from them being produced meet within the circle, they will make an angle equal to twice that angle which is made by two straight lines drawn from the center to the points of Incidence together with the angle which the incident lines themselves make.

Let the two straight lines A B and A C (in the 4th figure) be drawn from the point A to the circumference of the circle, whose center is D; and let the lines reflected from them be BE and CG, and being produced make within the circle the angle H; also let the two straight lines D B and D C be drawn from the center D to the points of Incidence B and C. I say the angle H is equal to twice the angle at D together with the angle at A.

For let A C be produced howsoever to I. Therefore the angle C H (which is external to the triangle C K H) will be equal to the two angles G K H and C H K. Again, the angle I C D (which is external to the triangle C L D) will be equal to the two angles C L D and C D L. But the angle I C H is double to the angle I C D, and is therefore equal to the angles C L D and C D L twice taken. Wherefore the angles C K H and C H K are equal to the angles C L D and C D L twice taken. But the angle C L D (being exter-

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nal to the triangle AL B) is equal to the two angles LAB & LBA; & consequently CLD twice taken is equal to LAB & L A B twice taken. Wherefore C K H & C H K are equal to the angle C D L together with L A B and L B A twice taken. Also the angle CKH is equal to the angle L A B once, and A B K, that is, L B A twice taken. Wherefore the Angle C H K is equal to the remaining angle C D L (that is, to the angle at D) twice taken, and the angle L A B (that is, the angle at A) once taken; which was to be proved.

Corollary. If two straight converging lines, as I C and M B fall upon the concave circumference of a circle, their reflected lines, as C H and B H, will meet in the angle H, equal to twice the angle D, together with the angle at A made by the Incident lines produced. Or, if the Incident lines be H B and I C, whose reflected lines C H and B M meet in the point N, the angle C N B will be equal to twice the angle D, together with the angle C K H made by the lines of Incidence. For the angle C N B is equal to the angle H (that is, to twice the angle D) together with the two angles A and N B H (that is K B A). But the angles K B A and A are equal to the angle C K H. Wherefore the angle C N B is equal to twice the angle D, together with the angle C K H made by the lines of Incidence I C and H B produced to K.

5 If two straight lines drawn from one point, fall upon the concave circumference of a circle, and the angle they make be lesse then twice the angle at the center; the lines reflected from them, and meeting within the circle, will make an angle, which being added to the angle of the incident lines, will be equal to twice the angle at the center.

Let the two Lines A B and A C (in the 5th figure) drawn from the point A, fall upon the concave circumference of the circle whose center is D; & let their reflected Lines BE and C E meet in the point E; also let the angle A be less then twice the angle D. I say the angles A and E together taken are equal to twice the angle D.

For let the straight Lines A B and E C cut the straight Lines D C and D B in the points G and H; and the angle B H C will be equal to the two angles E B H and E; also the same angle B H C will be equal to the two angles D and D C H; and in like manner the angle B G C will be equal to the two angles AC D & A, & the

same

ſame angle B G C will be alſo equal to the two angles D B G and D. Wherefore the four angles E B H, E, A C D and A are equal to the four angles D, D C H, D B G and D. If therefore equals be taken away on both ſides, namely, on one ſide A C D and E B H, and on the other ſide D C H and D B G (for the angle E B H is equal to the angle D B G, and the angle A C D equal to the angle D C H) the remainders on both ſides will be equal, namely, on one ſide the angles A and E, and on the other the angle D twice taken. Wherefore the angles A and E are equal to twice the angle D.

Corollary. If the angle A be greater then twice the angle D, their reflected Lines will diverge. For, by the *Corollary* of the third Propoſition, if the angle A be equal to twice the angle D, the reflected Lines B E and C E will be parallel; and if it be leſſe, they will concurre, as has now been demonſtrated; and therefore if it be greater, the reflected Lines B E and C E will diverge, and conſequently, if they be produced the other way, they will concurre, and make an angle equal to the exceſſe of the angle A above twice the angle D; as is evident by the fourth Article.

6 If through any one point, two unequal chords be drawn, cutting one another, either within the circle, or (if they be produced) without it, and the center of the circle be not placed between them, and the Lines reflected from them concurre whereſoever; there cannot through the point through which the former Lines were drawn, be drawn another ſtraight Line, whoſe reflected Line ſhall paſſe through the point where the two former reflected Lines concurre.

Let any two unequal chords, as B K and C H (in the 6th Figure) be drawn through the point A in the circle B C; and let their reflected Lines B D and C E meet in F; and let the center not be between A B and A C; and from the point A let any other ſtraight Line as A G be drawn to the circumference between B and C. I ſay G N, which paſſes through the point F, where the reflected Lines BD and C E meet, will not be the reflected Line of A G.

For let the arch B L be taken equal to the arch B G, and the ſtraight Line B M equal to the ſtraight Line B A; and L M being drawn, let it be produced to the circumference in O. Seeing therefore

fore B A and B M are equal, and the arch B L equal to the arch B G, and the angle M B L equal to the angle A B G, A G and M L will also be equal, and (producing G A to the circumference in I) the whole lines L O and G I will in like manner be equal. But L O is greater then G F N (as shall presently be demonstrated) and therefore also G I is greater then G N. Wherefore the angles N G C and I G B are not equal. Wherefore the Line G F N is not reflected from the Line of Incidence A G; and consequently no other straight Line (besides A B and A C) which is drawn through the point A, and fails upon the circumference B C, can be reflected to the point F, which was to be demonstrated.

It remains that I prove L O to be greater then G N; which I shall do in this manner. L O and G N cut one another in P; and P L is greater then P G. Seeing now L P. P G : : P N. P O are proportionals, therefore the two Extremes L P and P O together taken, (that is L O), are greater then P G and P N together taken, (that is, G N;) which remained to be proved.

7 But if two equal chords be drawn through one point within a circle, and the Lines reflected from them meet in another point, then another straight Line may be drawn between them through the former point, whose reflected Line shall pass through the later point.

Let the two equal chords B C and E D (in the 7th figure) cut one another in the point A within the circle B C D; and let their reflected Lines C H and D I meet in the point F. Then dividing the arch C D equally in G, let the two chords G K and G L be drawn through the points A and F. I say G L will be the Line reflected from the chord K G. For the four chords B C, C H, E D and D I, are by supposition all equal to one another; and therefore the arch B C H is equal to the arch E D I; as also the angle B C H to the angle E D I; & the angle A M C to its vertical angle F M D; and the straight Line D M to the straight Line C M; and in like manner, the straight Line A C to the straight Line F D; and the chords C G and G D being drawn, will also be equal; as also the angles F D G and A C G, in the equal Segments G D I and G C B. Wherefore the straight Lines F G and A G are equal; and therefore the angle F G D is equal to the angle A G C, that is, the angle

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gle of Incidence equal to the angle of Reflection. Wherefore the line G L is reflected from the incident Line K G; which was to be proved.

Corollary. By the very sight of the figure, it is manifest, that if G be not the middle point between C and D, the reflected Line G L will not pass through the point F.

8 Two points in the circumference of a circle being given, to draw two straight Lines to them, so as that their reflected Lines may be parallel, or contain any angle given.

In the circumference of the circle whose center is A (in the 8th. figure) let the two points B and C be given; and let it be required to draw to them from two points taken without the circle, two incident Lines, so, that their reflected Lines may (first) be parallel.

Let A B and A C be drawn; as also any incident Line D C, with its reflected Line C F; and let the angle E C D be made double to the angle A; and let H B be drawn parallel to E C, and produced till it meet with D C produced in I. Lastly, producing A B indefinitely to K, let G B be drawn, so, that the angle G B K may be equal to the angle H B K, and then G B will be the reflected Line of the incident Line H B. I say D C and H B are two incident Lines, whose reflected Lines C F and B G are parallel.

For seeing the angle E C D is double to the angle B A C, the angle H I C is also (by reason of the parallels E C and H I) double to the same B A C; Therefore also F C and G B (namely the lines reflected from the incident lines D C and H B are parallel. Wherefore the first thing required, is done.

Secondly, let it be required to draw to the points B & C two straight lines of Incidence, so, that the lines reflected from them may contain the given angle Z.

To the angle E C D made at the point C, let there be added on one side the angle D C L equal to half Z, and on the other side the angle E C M equal to the angle D C L; and let the straight Line B N be drawn parallel to the straight line C M; and let the angle K B O be made equal to the angle N B K; which being done, B O will be the Line of Reflection from the Line of Incidence N B. Lastly, from the incident Line L C, let the reflected Line C O be drawn, cutting B O at O, and making the angle C O B. I say the angle C O B is equal to the angle Z.

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Let N B be produced till it meet with the straight line L C produced in P. Seeing therefore the angle L C M is by construction equal to twice the angle BAC together with the angle Z; the angle N P L (which is equal to L C M by reason of the parallels N P and M C) will also be equal to twice the same angle B A C together with the angle Z. And seeing the two straight lines O C and O B fall from the point O upon the points C and B; and their reflected lines L C and N B meet in the point P; the angle N P L will be equal to twice the angle B A C together with the angle COP. But I have already proved the angle NPL to be equal to twice the angle B A C together with the angle Z. Therefore the angle C O P is equal to the angle Z; Wherefore, Two points in the circumference of a Circle being given, I have drawn, &c. which was to be done.

But if it be required to draw the incident Lines from a point within the circle, so, that the Lines reflected from them may contain an angle equal to the angle Z, the same method is to be used, saving that in this case the angle Z is not to be added to twice the angle B A C, but to be taken from it.

9 If a straight line falling upon the circumference of a circle, be produced till it reach the Semidiameter, and that part of it which is intercepted between the circumference and the Semidiameter, be equal to that part of the Semidiameter which is between the point of concourse and the center, the reflected Line will be parallel to the Semidiameter.

Let any Line A B (in the *9*th figure) be the Semidiameter of the circle whose center is A; and upon the circumference B D let the straight Line CD fall, and be produced till it cut A B in E, so, that E D and E A may be equal; & from the incident Line C D let the Line D F be reflected. I say A B and D F will be parallel.

Let A G be drawn through the point D. Seeing therefore E D and E A are equal, the angles E D A and E A D will also be equal. But the angles F D G and E D A are equal (for each of them is half the angle E D H or F D C.) Wherefore the angles F D G and E A D are equal; and consequently D F and A B are parallel; which was to be proved.

Corollary. If E A be greater then E D, then D F and A B being produced will concurre; but if E A be less then E D, then B A and D H being produced will concurre.

10 If from a point within a circle, two straight Lines be drawn to the Circumference, and their reflected Lines meet in the Circumference of the same circle, the angle made by the Lines of Reflection, will be a third part of the angle made by the Lines of Incidence.

From the point B (in the 10th figure) taken within the circle whose center is A, let the two straight lines B C and B D be drawn to the circumference; and let their reflected Lines C E and D E meet in the circumference of the same circle at the point E. I say the angle C E D will be a third part of the angle C B D.

Let A C and A D be drawn. Seeing therefore the angles C E D and C B D together taken, are equal to twice the angle C A D (as has been demonstrated in the 5th article); and the angle CAD twice taken is quadruple to the angle C E D; the angles C E D and C B D together taken, will also be equal to the angle C E D four times taken; and therefore if the angle C E D be taken away on both sides, there will remain the angle C B D on one side, equal to the angle C E D thrice taken on the other side; which was to be demonstrated.

Coroll. Therefore a point being given within a Circle, there may be drawn two Lines from it to the Circumference, so as their reflected Lines may meet in the Circumference. For it is but trisecting the Angle C B D; which how it may be done, shall be shewn in the following Chapter.

CHAP. XX.

Of the Dimension of a Circle, and the Division of Angles or Arches.

1 The Dimension of a Circle neer determined in Numbers by Archimedes *and others. 2 The first attempt for the finding out of the Dimension of a Circle by Lines. 3 The second attempt for the finding out of the Dimension of a Circle from the consideration of the nature of Crookedness. 4 The third attempt; and some things propounded to be further searched into. 5 The Equation of the Spiral of* Archimedes *with a straight Line. 6 Of the Analysis of Geometricians by the Powers of Lines.*

1 IN the comparing of an Arch of a Circle with a Straight Line, many and great Geometricians, even from the most ancient times, have exercised their wits; and more had done the same, if they had not seen their pains, though undertaken for the common good, if not brought to perfection, vilified by those that envy the prayses of other men. Amongst those Ancient Writers whose Works are come to our hands, *Archimedes* was the first that brought the Length of the Perimeter of a Circle within the limits of Numbers very litle differing from the truth; demonstrating the same to be less then three Diameters and a seventh part, but greater then three Diameters and ten seventy one parts of the Diameter. So that supposing the Radius to consist of 10000000 equal parts, the Arch of a Quadrant will be between 15714285 and 15704225 of the same parts. In our times *Ludovicus Van Cullen* & *Willebrordus Snellius* with joint endeavour have come yet neerer to the truth; and pronounced from true Principles, that the Arch of a Quadrant (putting, as before 10000000 for Radius) differs not one whole Unity from the number 15707963; which, if they had ex-

exhibited their arithmetical operations(and no man had discovered any errour in that long work of theirs) had been demonstrated by them. This is the furthest progress that has been made by the way of Numbers; and they that have proceeded thus far deserve the praise of Industry. Neverthelels, if we consider the benefit (which is the scope at which all Speculation should aime) the improvement they have made has been little, or none. For any ordinary man may much sooner, & more accurately find a Straight Line equal to the Perimeter of a Circle, and consequently square the Circle, by winding a small thred about a given Cylinder, then any Geometrician shall do the same by dividing the Radius into 10000000 equal parts. But though the length of the Circumference were exactly set out, either by Numbers, or mechanically, or onely by chance, yet this would contribute no help at all towards the Section of Angles, unless happily these two Problemes, *To divide a given Angle according to any proportion assigned*, and *To finde a Straight Line equal to the Arch of a Circle*, were reciprocal, and followed one another. Seeing therefore the benefit proceeding from the knowledge of the Length of the Arch of a Quadrant, consists in this, that we may thereby divide an Angle according to any proportion, either accurately, or at least accurately enough for common use; and seeing this cannot be done by Arithmetick, I thought fit to attempt the same by Geometry; and in this Chapter to make trial whether it might not be performed by the drawing of Straight and Circular Lines.

2 Let the Square A B C D (in the first figure) be described, and with the *Radii* A B, B C and D C the three Arches B D, C A and A C; of which let the two B D and C A cut one another in E, and the two B D and A C in F. The Diagonals therefore B D and A C being drawn will cut one another in the center of the Square G, and the two Arches B D and C A into two equal parts in H and Y; and the Arch B H D will be trisected in F and E. Through the Center G let the two Straight Lines K G L and M G N be drawn parallel and equal to the sides of the Square A B and A D, cutting the four sides of the same Square in the points K, L, M and N; which being done, K L will pass through F, and M N through E. Then let O P be drawn parallel and equal to the side B C, cutting the Arch B F D in F, and the sides A B and D C in

in O and P. Therefore O F will be the Sine of the arch B F, which is an arch of 30 degrees; and the same O F will be equal to half the Radius. Lastly, dividing the arch B F in the middle in Q, let R Q the Sine of the arch B Q be drawn and produced to S, so that Q S be equal to R Q, and consequently R S be equal to the chord of the arch B F; and let F S be drawn and produced to T in the side B C. I say, the Straight Line B T is equal to the Arch B F; and consequently that B V the triple of B T is equal to the Arch of the Quadrant B F E D.

Let T F be produced till it meet the side B A produced in X; and dividing O F in the middle in Z, let Q Z be drawn and produced till it meet with the side B A produced. Seeing therefore the Straight Lines R S and O F are parallel, and divided in the midst in Q and Z, Q Z produced will fall upon X, and X Z Q produced to the side B C will cut B T in the midst in *a*.

Upon the Straight line F Z the fourth part of the Radius A B let the equilateral triangle *a* Z F be constituted; & upon the center *a*, with the Radius *a* Z let the arch Z F be drawn; which arch Z F will therefore be equal to the arch Q F the half of the arch B F. Again, let the straight line Z O be cut in the midst in *b*, and the straight line *b* O in the midst in *c*; and let the bisection be continued in this manner till the last part O *c* be the least that can possibly be taken; and upon it, and all the rest of the parts equal to it into which the straight line O F may be cut, let so many equilateral triangles be understood to be constituted; of which let the last be *d* O *c*. If therefore upon the center *d*, with the Radius *d* O be drawn the arch O *c*, and upon the rest of the equal parts of the straight line OF be drawn in like manner so many equal arches, all those arches together taken will be equal to the whole arch B F; & the half of them, namely, those that are comprehended between O & Z, or between Z & F will be equal to the arch B Q or Q F and in summe, what part soever the straight line O *c* be of the straight line O F, the same part will the arch O *c* be of the arch O F, though both the arch and the chord be infinitely bisected. Now seeing the arch O *c* is more crooked then that part of the arch B F which is equal to it; and seeing also that the more the straight line X *c* is produced the more it diverges from the straight line X O, if

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the points O and *r* be understood to be moved forwards with straight motion in X O and X *r*, the arch O *r* will thereby be extended by little and little, till at the last it come some-where to have the same crookedness with that part of the arch B F which is equal to it. In like manner, if the straight line X *b* be drawn, and the point *b* be understood to be moved forwards at the same time, the arch *c b* will also by little and little be extended, till its crookedness come to be equal to the crookedness of that part of the arch B F which is equal to it. And the same will happen in all those small equal arches which are described upon so many equal parts of the straight line O F. It is also manifest, that by straight motion in X O and X Z all those small arches will lie in the arch B F in the points B, Q and F. And though the same small equall arches should not be coincident with the equall parts of the arch B F in all the other points thereof, yet certainly they will constitute two crooked lines, not onely equall to the two arches B Q and Q F and equally crooked, but also having their cavity towards the same parts; which how it should be, unlesse all those small arches should be coincident with the arch B F in all its points, is not imaginable. They are therefore coincident, and all the straight lines drawne from X & passing through the points of division of the straight line O F, will also divide the arch B F into the same proportions into which O F is divided.

Now seeing X *b* cuts off from the point B the fourth part of the arch B F, let that fourth part be B *e*; and let the Sine thereof *f e* be produced to F T in *g*, for so *f e* will be the fourth part of the straight line *f g*, because as O *b* to O F, so is *f e* to *f g*. But B T is greater then *f g*; and therefore the same B T is greater then four Sines of the fourth part of the arch B F. And in like manner, if the arch B F be subdivided into any number of equal parts whatsoever, it may be proved that the straight line B T is greater then the Sine of one of those small arches so many times taken as there be parts made of the whole arch B F. Wherefore the straight line B T is not lesse then the Arch B F. But neither can it be greater, because if any straight line whatsoever, lesse then B T, be drawn below B T parallel to it and terminated in the straight lines X B and X T, it would cut the arch B F; and

so

ſo the Sine of ſome one of the parts of the arch B F taken ſo often as that ſmall arch is found in the whole arch B F, would be greater then ſo many of the ſame arches; which is abſurd. Wherefore the Straight line B T is equal to the Arch B F; & the Straight line B V equal to the Arch of the Quadrant B F D; and B V four times taken, equal to the Perimeter of the Circle deſcribed with the Radius A B. Alſo the Arch B F and the Straight line BT are every where divided into the ſame proportions; and conſequently any given Angle, whether greater or leſs then B A F may be divided into any proportion given.

But the ſtraight line B V (though its magnitude fall within the terms aſſigned by *Archimedes*) is found, if computed by the Canon of Sines, to be ſomwhat greater then that wch is exhibited by the *Ludolphine* numbers. Nevertheleſs, if in the place of BT, another ſtraight line, though never ſo little leſs, be ſubſtituted, the diviſion of Angles is immediatly loſt, as may by any man be demonſtrated by this very Scheme.

Howſoever, if any man think this my Straight line BV to be too great, yet, ſeeing the Arch and all the Parallels are every where ſo exactly divided, and B V comes ſo neer to the truth, I deſire he would ſeach out the reaſon, Why (granting BV to be preciſely true) the Arches cut off ſhould not be equal.

But ſome man may yet ask the reaſon why the ſtraight lines drawn from X through the equal parts of the arch B F ſhould cut off in the Tangent B V ſo many ſtraight lines equal to them, ſeeing the connected ſtraight line XV paſſes not through the point D, but cuts the ſtraight line A D produced in *l*; and conſequently require ſome determination of this Probleme. Concerning which, I will ſay what I think to be the reaſon, namely, that whileſt the magnitude of the Arch doth not exceed the magnitude of the Radius, that is, the magnitude of the Tangent B C, both the Arch and the Tangent are cut alike by the ſtraight lines drawn from X; otherwiſe not. For AV being connected, cutting the arch B H D in I, if X C being drawn ſhould cut the ſame arch in the ſame point I, it would be as true that the Arch B I is equal to the Radius B C, as it is true that the Arch B F is equal to the ſtraight line B T, and drawing X K it would cut the arch B I in the midſt in *i*; Alſo

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drawing A *i* and producing it to the Tangent B C in *k*, the straight line B *k* will be the Tangent of the arch B *i*, (which arch is equal to half the Radius) and the same straight line B *k* will be equal to the straight line *k* I. I say all this is true, if the preceding demonstration be true; and consequently the proportional section of the Arch and its Tangent proceeds hitherto. But it is manifest by the Golden Rule, that taking B *h* double to B T, the line X *h* shall not cut off the arch B E which is double to the arch B F, but a much greater. For the magnitude of the straight lines X M, X B and M E being known (in numbers) the magnitude of the straight line cut off in the Tangent by the straight line X E produced to the Tangent may also be known; and it will be found to be less then B *h*; Wherfore the straight line X*h* being drawn will cut off a part of the arch of the Quadrant greater then the arch B E. But I shall speak more fully in the next Article concerning the magnitude of the arch B I.

And let this be the first attempt for the finding out of the dimension of a Circle by the Section of the arch B F.

3 I shall now attempt the same by arguments drawn from the nature of the Crookedness of the Circle it self; but I shall first set down some Premisses necessary for this speculation; and

First, If a Straight line be bowed into an Arch of a Circle equal to it, as when a stretched thred which toucheth a Right Cylinder, is so bowed in every point, that it be every where coincident with the Perimeter of the base of the Cylinder, the Flexion of that line will be equal, in all its points; and consequently the Crookedness of the Arch of a Circle is every where Uniform; which needs no other demonstration then this, That the Perimeter of a Circle is an Uniform line.

Secondly, and consequently, If two unequal Arches of the same Circle be made by the bowing of two straight lines equal to them, the Flexion of the longer line (whilest it is bowed into the greater Arch) is greater then the Flexion of the shorter line (whilest it is bowed into the lesser Arch) according to the proportion of the Arches themselves; and consequently, the Crookedness of the greater Arch is to the Crookedness of the lesser Arch; as the greater Arch is to the lesser Arch.

Thirdly, If two unequal Circles and a straight line touch one ano-

another in the ſame point, the Crookedneſs of any Arch taken in the leſſer Circle, will be greater then the Crookedneſs of an Arch equal to it taken in the greater Circle, in reciprocal proportion to that of the *Radii* with which the Circles are deſcribed; or, which is all one, any ſtraight line being drawn from the point of Contact till it cut both the circumferences, as the part of that ſtraight line cut off by the circumference of the greater Circle to that part which is cut off by the circumference of the leſſer Circle.

For let A B and A C (in the ſecond figure) be two Circles, touching one another and the ſtraight line A D in the point A; and let their Centers be E and F; and let it be ſuppoſed, that as A E is to A F, ſo is the Arch A B to the Arch A H. I ſay the Crookedneſs of the Arch A C is to the Crookedneſs of the Arch A H, as A E is to A F. For let the ſtraight line A D be ſuppoſed to be equal to the Arch A B, and the ſtraight line A G to the Arch A C; and let A D (for example) be double to A G. Therefore by reaſon of the likeneſs of the Arches A B and A C, the ſtraight line A B will be double to the ſtraight line A C, and the Radius A E double to the Radius A F, and the Arch A B double to the Arch A H. And becauſe the ſtraight line A D is ſo bowed to be coincident with the Arch A B equal to it, as the ſtraight line A G is bowed to be coincident with the Arch A C equal alſo to it, the Flexion of the ſtraight line A G into the Crooked line A C will be equal to the Flexion of the ſtraight Line A D into the Crooked line A B. But the Flexion of the ſtraight line A D into the Crooked line A B is double to the the Flexion of the ſtraight line A G into the Crooked line A H; and therefore the Flexion of the ſtraight line A G into the Crooked line A C is double to the Flexion of the ſame ſtraight line A G into the Crooked line A H. Wherefore, as the Arch A B is to the Arch A C or A H; or as the Radius A E is to the Radius A F; or as the Chord A B is to the Chord A C; ſo reciprocally is the Flexion or Uniform Crookedneſs of the Arch A C, to the Flexion or Uniform Crookedneſs of the Arch A H, namely, here double. And this may by the ſame method be demonſtrated in Circles whoſe Perimeters are to one another triple, quadruple, or in whatſoever given proportion. The Crookedneſs therefore of two equal Arches

taken in ſeveral Circles are in proportion reciprocall to that of their *Radii*, or like Arches, or like Chords; which was to be demonſtrated.

Let the Square A B C D be again deſcribed (in the third Figure,) and, in it the Quadrants A B D, B C A and D A C; and dividing each ſide of the Square A B C D in the midſt in E, F, G and H, let E G and F H be connected, which will cut one another in the center of the Square at I, and divide the arch of the Quadrant A B D into three equal parts in K and L. Alſo the Diagonals A C and B D being drawn will cut one another in I, and divide the arches B K D and C L A into two equal parts in M and N. Then with the Radius B F let the arch F E be drawn, cutting the Diagonal B D in O; and dividing the arch B M in the midſt in P, let the ſtraight line E *a* equal to the chord B P be ſet off from the point E in the arch E F, and let the arch *a b* be taken equal to the arch O *a*, and let B *a* and B *b* be drawn and produced to the arch A N in *c* and *d*, and laſtly, let the ſtraight line A *d* be drawn. I ſay the Straight line A *d* is equal to the Arch A N or B M.

I have proved in the preceding article, that the arch E O is twice as crooked as the arch B P, that is to ſay, that the arch E O is ſo much more crooked then the arch B P, as the arch B P is more crooked then the ſtraight line E *a*. The crookedneſs therefore of the chord E *a*, of the arch B P, and of the arch E O are as 0, 1, 2. Alſo the difference between the arches E O and E O, the difference between the arches E O and E *a*, and the difference between the arches E O and E *b* are as 0, 1, 2. So alſo the difference between the arches A N and A N, the difference between the arches A N and A *c*, and the difference between the arches A N and A *d* are as 0, 1, 2; and the ſtraight line A *c* is double to the chord B P or E *a*; and the ſtraight line A *d* double to the chord E *b*.

Again, let the ſtraight line B F be divided in the midſt in Q, and the arch B P in the midſt in R; and deſcribing the Quadrant B Q S (whoſe arch Q S is a fourth part of the arch of the Quadrant B M D, as the arch B R is a fourth part of the arch B M which is the arch of the Semiquadrant A B M) let the chord S *e* equal to the chord B R be ſet off from the point S in the arch S Q; and let B *e* be drawn and produced to the arch A N in *f*; which being done, the ſtraight line A *f* will be quadruple to the chord

chord B R or S *e*. And seeing the crookedness of the arch S *e* or of the arch A *c* is double to the crookedness of the arch B R, the excess of the crookedness of the arch A *f* above the crookedness of the arch A *c* will be subduple to the excess of the crookedness of the arch A *c* above the crookedness of the arch A N; and therefore the arch N *c* will be double to the arch *cf*. Wherefore the arch *cd* is divided in the midst in *f*, and the arch N *f* is $\frac{3}{4}$ of the arch N *d*. And in like manner if the arch B R be bisected in V, and the straight Line BQ in X, and the quadrant B X Y be described, and the straight Line Y *g* equal to the chord B V be set off from the point Y in the arch Y X, it may be demonstrated that the straight Line B *g* being drawn and produced to the arch A N will cut the arch *fd* into two equal parts, and that a straight Line drawn from A to the point of that Section, will be equal to eight chords of the arch B V, and so on perpetually; and consequently, that the straight Line A *d* is equal to so many equal chords of equal parts of the arch B M, as may be made by infinite bisections. Wherefore the Straight Line A *d* is equal to the Arch B M or A N, that is, to half the Arch of the Quadrant A B D or B C A.

Corollary. An Arch being given not greater then the arch of a Quadrant (for being made greater it comes again towards the Radius B A produced, from which it receded before) if a straight Line double to the chord of half the given arch be adapted from the beginning of the arch, and by how much the arch that is subtended by it is greater then the given arch, by so much a greater arch be subtended by another straight Line, this Straight Line shall be equal to the first given Arch.

Supposing the Straight Line B V (in the first Figure) be equal to the arch of the Quadrant B H D , and A V be connected cutting the arch B H D in I, it may be asked what proportion the arch B I has to the arch I D. Let therefore the arch A Y be divided in the midst in *o*, and in the straight line A D let A *p* be taken equal, and A *q* double to the drawn chord A *o*. Then upon the center A, with the Radius A *q* let an arch of a circle be drawne cutting the arch A Y in *r*, and let the arch Y *r* be doubled at *t*; which being done, the drawne straight line A *t* (by what has been last demonstrated) will be equall to the arch A Y.

A-

Again, upon the Center A with the Radius A *t* let the arch *t u* be drawne cutting A D in *u*; and the ſtraight line A *u* will be equall to the arch A Y. From the point *u* let the ſtraight line *u s* be drawn equal and parallell to the ſtraight line A B, cutting M N in *x*, and biſected by M N in the ſame point *x*. Therefore the ſtraight line A *x* being drawn and produced till it meet with B C produced in V, it will cut off B V double to B *s*, that is, equal to the arch B H D. Now let the point where the ſtraight line A V cuts the arch B H D, be I; and let the arch D I be divided in the midſt in *y*; and in the ſtraight line D C, let D *z* be taken equal, and D ʎ double to the drawn chord D *y*; and upon the center D with the Radius D ʎ let an arch of a circle be drawn cutting the arch B H D in the point *n*; and let the arch *n m* be taken equal to the arch I *n*; which being done, the ſtraight line D *m* will (by the laſt foregoing *Corollary*) be equal to the arch D I. If now the ſtraight lines D *m* and C V be equal, the arch B I will be equal to the Radius A B or B C; and conſequently X C being drawn will paſs through the point I. Moreover, if the ſemicircle B H D *c* being completed, the ſtraight lines *c* I and B I be drawn making a right angle (in the Semicircle) at I, and the arch B I be divided in the midſt at *i*, it will follow that A *i* being connected will be parallel to the ſtraight line *c* I, and being produced to B C in *k*, will cut off the ſtraight line B *k* equal to the ſtraight line *k* I, and equal alſo to the ſtraight line A *y* cut off in A D by the ſtraight line *c* I. All which is manifeſt, ſuppoſing the arch B I and the Radius B C to be equal.

But that the arch B I and the Radius B C are preciſely equal, cannot (how true ſoever it be) be demonſtrated, unleſs that be firſt proved wch is contained in the firſt article, namely, that the ſtraight lines drawn from X through the equal parts of O F, (produced to a certain length) cut off ſo many parts alſo in the Tangent B C ſeverally equal to the ſeveral arches cut off; which they do moſt exactly as far as BC in the Tangent, and BI in the arch BE; in ſo much that no inequality between the arch B I and the Radius B C can be diſcovered either by the hand or by ratiocination. It is therefore to be further enquired, whether the ſtraight line A V cut the arch of the Quadrant in I in the ſame proportion as the point C divides the

the ſtraight line B V which is equal to the arch of the Quadrant. But however this be, it has been demonſtrated that the ſtraight line B V is equal to the arch B H D.

4 I ſhall now attempt the ſame dimenſion of a Circle another way, aſſuming the two following *Lemma's*.

Lemma 1. If to the Arch of a Quadrant, and the Radius, there be taken in continual proportion a third Line Z; then the Arch of the Semiquadrant, Half the chord of the Quadrant, and Z will alſo be in continual proportion.

For ſeeing the Radius is a mean proportional between the Chord of a Quadrant and its Semichord, and the ſame Radius a mean proportional between the Arch of the Quadrant and Z, the Square of the Radius will be equal as well to the Rectangle made of the Chord and Semichord of the Quadrant, as to the Rectangle made of the Arch of the Quadrant and Z; and theſe two Rectangles will be equal to one another. Wherfore, as the Arch of a Quadrant is to its Chord, ſo reciprocally is half the Chord of the Quadrant to Z. But as the Arch of the Quadrant is to its Chord, ſo is half the Arch of the Quadrant to half the Chord of the Quadrant. Wherefore, as half the arch of the Quadrant is to half the Chord of the Quadrant (or to the Sine of 45 degrees) ſo is half the Chord of the Quadrant to Z; which was to be proved.

Lemma 2. The Radius, the Arch of the Semiquadrant, the Sine of 45 degrees, and the Semiradius are proportional.

For ſeeing the Sine of 45 degrees is a mean proportional between the Radius and the Semiradius; and the ſame Sine of 45 degrees is alſo a mean proportional (by the precedent *Lemma*) between the Arch of 45 degrees and Z; the Square of the Sine of 45 degrees will be equal as well to the Rectangle made of the Radius and Semiradius, as to the Rectangle made of the Arch of 45 degrees and Z. Wherefore, as the Radius is to the Arch of 45 degrees, ſo reciprocally is Z to the Semiradius; which was to be demonſtrated.

Let now A B C D (in the fourth Figure) be a Square; and with the *Radii* A B, B C and D A let the three Quadrants A B D, B C A and D A C be deſcribed; and let the ſtraight lines E F and G H drawn parallel to the Sides B C & A B, divide the Square A B C D into foure equal Squares. They will therefore cut the arch of the

Qua-

Quadrant A B D into three equal parts in I and K, and the arch of the Quadrant B C A into three equal parts in K and L. Also let the Diagonals A C and B D be drawn, cutting the arches B I D and A L C in M and N. Then upon the center H with the Radius H F equal to half the Chord of the arch B M D, or to the Sine of 45 degrees, let the arch F O be drawn cutting the arch C K in O; and let A O be drawn and produced till it meet with B C produced in P; also let it cut the arch B M D in Q, and the straight line D C in R. If now the straight line H Q be equal to the straight line D R, and being produced to D C in S cut off D S equal to half the straight line B P; I say then the Straight Line B P will be equal to the Arch B M D.

For seeing P B A and A D R are like triangles, it will be as P B to the Radius B A or A D, so A D to D R; and therefore as well P B, A D and DR, as P B, A D (or A Q) and Q H are in continuall proportion; and producing H O to D C in T, DT will be equal to the Sine of 45 degrees, as shall by and by be demonstrated. Now D S, DT and D R are in continual porportion by the first *Lemma*; and by the second *Lemma* D C. D S: : D R. D F are proportionals. And thus it will be, whether B P be equal or not equal to the arch of the Quadrant B M D. But if they be equal, it will then be, as that part of the arch B M D which is equal to the Radius, is to the remainder of the same arch B M D; so A Q to H Q, or so B C to C P. And then will B P and the arch B M D be equal. But it is not demonstrated that the Straight Lines H Q and D R are equal; though if from the point B there be drawn (by the construction of the first figure) a Straight Line equal to the arch B M D, then D R to H Q, and also the half of the Straight Line B P to D S, will always be so equal, that no inequality can be discovered between them. I will therefore leave this to be further searched into. For though it be almost out of doubt, that the Straight Line B P and the arch B M D are equal, yet that may not be received without demonstration; and means of Demonstration the Circular Line admitteth none that is not grounded upon the nature of Flexion, or of Angles. But by that way I have already exhibited a Straight Line equal to the Arch of a Quadrant in the First and Second aggression.

It

It remains that I prove DT to be equal to the Sine of 45 degrees.

In B A produced let AV be taken equal to the Sine of 45 degrees; and drawing and producing VH, it will cut the arch of the Quadrant C N A in the midst in N, and the same arch again in O, and the Straight line D C in T, so, that D T will be equal to the Sine of 45 degrees, or to the straight line AV; also the Straight line V H will be equal to the straight line H I or the Sine of 60 degrees.

For the square of AV is equal to two squares of the Semiradius; and consequently the square of VH is equal to three Squares of the Semiradius. But H I is a mean proportional between the Semiradius and three *Semiradii*; and therefore the square of H I is equal to three Squares of the Semiradius. Wherefore H I is eqval to H V. But because A D is cut in the midst in H, therefore V H and H T are equal; and therefore also D T is equal to the Sine of 45 degrees. In the Radius B A let B X be taken equal to the Sine of 45 degrees; for so V X will be equal to the Radius; and it will be as VA to A H the Semiradius, so V X the Radius to X N the Sine of 45 degrees. Wherefore V H produced passes through N. Lastly, upon the center V with the Radius VA let the arch of a circle be drawn cutting VH in Y; which being done, VY will be equal to H O (for H O is by construction equal to the Sine of 45 degrees) and YH will be equal to OT; & therefore VT passes through O. All which was to be demonstrated.

I will here add certain Problemes, of which if any Analyst can make the construction, he will thereby be able to judge clearly of what I have now said concerning the dimension of a Circle. Now these Problemes are nothing else (at least to sense) but certain symptomes accompanying the construction of the first and third figure of this Chapter.

Describing therefore again the Square A B C D (in the fifth figure) and the three Quadrants A B D, B C A and D A C, let the Diagonals A C & B D be drawn, cutting the arches B H D & CIA in the middle in H and I, & the straight lines E F and G L, dividing the square A B C D into four equal squares, and trisecting the arches B H D and C I A, namely, B H D in K and M, and CIA in

in M and O. Then dividing the arch B K in the midst in P, let Q P the Sine of the arch B P be drawn and produced to R, so that Q R be double to Q P; and connecting K R, let it be produced one way to B C in S, and the other way to B A produced in T. Also let B V be made triple to B S, and consequently (by the second article of this Chapter) equall to the arch B D. This construction is the same with that of the first figure, which I thought fit to renew discharged of all lines but such as are necessary for my present purpose.

In the first place therefore, if A V be drawn, cutting the arch B H D in X, and the side D C in Z, I desire some Analyst would (if he can) give a reason, Why the straight lines T E and T C should cut the arch B D the one in Y, the other in X, so as to make the arch B Y equal to the arch Y X; or if they be not equal, that he would determine their difference.

Secondly, if in the side D A, the straight line D *a* be taken equal to D Z, and V *a* be drawn; Why V *a* and V B should be equal; or if they be not equal, What is the difference.

Thirdly, drawing Z *b* parallel and equal to the side C B, cutting the arch B H D in *c*, and drawing the straight line A *c*, and producing it to B V in *d*; Why A *d* should be equal and parallel to the straight line *a* V, and consequently equal also to the arch B D.

Fourthly, drawing *e* K the Sine of the arch B K; & taking (in *e* A produced) *e f* equal to the Diagonal A C, and connecting *f* C; Why *f* C should pass through *a* (which point being given, the length of the arch B H D is also given) and *c*; and why *f e* and *f c* should be equal; or if not, why unequal.

Fifthly, drawing *f* Z, I desire he would shew, Why it is equal to B V, or to the arch B D; or if they be not equal, What is their difference.

Sixtly, granting *f* Z to be equal to the arch B D, I desire he would determine whether it fall all without the arch B C A, or cut the same, or touch it, and in what point.

Seventhly, the Semicircle B D *g* being completed; Why *g* I being drawn and produced, should pass through X (by which point X the length of the arch B D is determined). And the same *g* I being yet further produced to D C in *h*; Why A *d* (which is equal

to

to the arch B D) should pass through that point h.

Eighthly, upon the Center of the square A B C D, which let be *k*, the arch of the quadrant E *i* L being drawn, cutting *e* K produced in *i*; Why the drawn straight line *i* X should be parallel to the side C D.

Ninthly, in the sides B A and B C taking B *l* and B *m* severally equal to half B V, or to the arch B H, and drawing *m n* parallel and equal to the side B A, cutting the arch B D in *o*; Why the straight line wich connects V *l* should pass through the point *o*,

Tenthly, I would know of him, Why the straight line which connects *a* H should be equal to B *m*; or if not, how much it differs from it.

The Analyst that can solve these Problemes without knowing first the length of the arch B D, or using any other known Method then that which proceeds by perpetual bisection of an angle, or is drawn from the consideration of the nature of Flexion, shall do more then ordinary Geometry is able to perform. But if the Dimension of a Circle cannot be found by any other Method; then I have either found it, or it is not at all to be found.

From the known Length of the Arch of a Quadrant, and from the proportional Division of the Arch and of the Tangent BC, may be deduced the Section of an Angle into any given proportion; as also the Squaring of the Circle, the Squaring of a given Sector, and many the like propositions, which it is not necessary here to demonstrate. I will therefore onely exhibit a Straight line equal to the Spiral of *Archimedes*, and so dismiss this speculation.

5 The length of the Perimeter of a Circle being found, that Straight line is also found, which touches a Spiral at the end of its first conversion. For upon the center A (in the sixth figure) let the circle B C D E be described; and in it let *Archimedes* his Spiral A F G H B be drawn, beginning at A and ending at B. Through the center A let the straight line C E be drawn, cutting the Diameter B D at right angles; and let it be produced to I, so, that A I be equal to the Perimeter B C D E B. Therefore I B being drawn will touch the Spiral A F G H B in B; which is demonstrated by *Archimedes* in his book *de Spiralibus*.

And for a Straight Line equal to the given Spiral A F G H B, it may be found thus.

 Let

Let the ſtraight line AI (which is equal to the Perimeter BCDE) be biſected in K; and taking KL equal to the Radius AB, let the rectangle IL be completed. Let ML be underſtood to be the axis, and KL the baſe of a Parabola, and let MK be the crooked line thereof. Now if the point M be conceived to be ſo moved by the concourſe of two movents, the one frō IM to KL with velocity encreaſing continually in the ſame proportion with the Times, the other from ML to IK uniformly, that both thoſe motions begin together in M and end in K; *Galilæus* has demonſtrated that by ſuch motion of the point M, the crooked line of a Parabola will be deſcribed. Again, if the point A be conceived to be moved uniformly in the ſtraight line AB, and in the ſame time to be carried round upon the center A by the circular motion of all the points between A and B; *Archimedes* has demonſtrated that by ſuch motion will be deſcribed a Spiral line. And ſeeing the circles of all theſe motions are concentrick in A; and the interiour circle is alwayes leſſe then the exteriour in the proportion of the times in which AB is paſſed over with uniform motion; the velocity alſo of the circular motion of the point A, will continually encreaſe proportionally to the times. And thus far the generations of the Parabolical line MK, and of the Spiral line AFGHB, are like. But the Uniform motion in AB concurring with circular motion in the Perimeters of all the concentrick circles, deſcribes that circle, whoſe center is A, and Perimeter BCDE; and therefore that circle is (by the *Coroll.* of the firſt article of the 16 Chapter) the aggregate of all the Velocities together taken of the point A whilſt it deſcribes the Spiral AFGHB. Alſo the rectangle IKLM is the aggregate of all the Velocities together taken of the point M, whileſt it deſcribes the crooked line MK. And therefore the whole velocity, by which the Parabolicall line MK is deſcribed, is to the whole velocity with which the Spiral line AFGHB is deſcribed in the ſame time, as the rectangle IKLM, is to the Circle BCDE, that is to the triangle AIB. But becauſe AI is biſected in K & the ſtraight lines IM & AB are equal, therefore the rectangle IKLM and the triangle AIB are alſo equal. Wherefore the Spiral line AFGHB, and the Parabolical line MK, being deſcribed with equal velocity and in equal times, are equal to one another. Now in the firſt article of the 18 Chapter a ſtraight line is found

out

out equal to any Parabolical line. Wherefore alſo a Straight line is found out, equal to a given Spiral line of the firſt revolution deſcribed by *Archimedes*; which was to be done.

6 In the ſixth Chapter, which is of *Method*, that which I ſhould there have ſpoken of the *Analyticks* of Geometricians, I thought fit to deferre, becauſe I could not there have been underſtood, as not having then ſo much as named *Lines*, *Superficies*, *Solids*, *Equal and Unequal &c.* Wherefore I will in this place ſet down my thoughts concerning it.

Analyſis, is continual Reaſoning from the Definitions of the terms of a propoſition we ſuppoſe true, and again from the Definitions of the terms of thoſe Definitions, and ſo on, till we come to ſome things known, the Compoſition whereof is the demonſtration of the truth or falſity of the firſt ſuppoſition; and this Compoſition or Demonſtration is that we call *Syntheſis*. *Analytica* therefore is that art, by which our reaſon proceeds from ſomething ſuppoſed, to Principles, that is, to prime Propoſitions, or to ſuch as are known by theſe, till we have ſo many known Propoſitions as are ſufficient for the demonſtration of the truth or falſity of the thing ſuppoſed. *Synthetica* is the art it ſelf of Demonſtration. *Syntheſis* therefore and *Analyſis* differ in nothing, but in proceeding forwards or backwards; and *Logiſtica* comprehends both. So that in the *Analyſis* or *Syntheſis* of any queſtion, that is to ſay, of any Probleme, the Terms of all the Propoſitions ought to be convertible; or if they be enunciated Hypothetically, the truth of the Conſequent ought not onely to follow out of the truth of its Antecedent, but contrarily alſo the truth of the Antecedent muſt neceſſarily be inferred from the truth of the Conſequent. For otherwiſe, when by Reſolution we are arrived at Principles, we cannot by Compoſition return directly back to the thing ſought for. For thoſe Terms which are the firſt in *Analyſis*, will be the laſt in *Syntheſis*; as for example, when in *Reſolving*, we ſay, theſe two Rectangles are equal and therefore their ſides are reciprocally proportional, we muſt neceſſarily in *Compounding* ſay, the ſides of theſe Rectangles are reciprocally proportional and therefore the Rectangles themſelves are equal; Which we could not ſay, unleſs *Rectangles have their ſides reciprocally proportional*, and *Rectangles are equal*, were Terms convertible.

Now

Now in every *Analysis*, that which is sought, is the Proportion of two quantities; by which proportion (a figure being described) the quantity sought for may be exposed to Sense. And this *Exposition* is the end and Solution of the question, or the construction of the Probleme.

And seeing *Analysis* is reasoning from something supposed, till we come to Principles, that is, to Definitions, or to Theoremes formerly known; and seeing the same reasoning tends in the last place to some Equation; we can therefore make no end of Resolving, till we come at last to the causes themselves of Equality and Inequality, or to Theoremes formerly demonstrated from those causes; and so have a sufficient number of those Theoremes for the demonstration of the thing sought for.

And seeing also, that the end of the *Analyticks*, is either the construction of such a Probleme as is possible, or the detection of the impossibility thereof; whensoever the Probleme may be solved, the *Analyst* must not stay, till he come to those things which contain the efficient cause of that whereof he is to make construction. But he must of necessity stay when he comes to prime Propositions; and these are Definitions. These Definitions therefore must contain the efficient cause of his Construction; I say of his Construction, not of the Conclusion which he demonstrates; for the cause of the Conclusion is contained in the premised propositions; that is to say, the truth of the proposition he proves, is drawn from the propositions which prove the same. But the cause of his construction is in the things themselves, and consists in motion, or in the concourse of motions. Wherefore those propositions in which *Analysis* ends, are Definitions, but such, as signifie in what manner the construction, or generation of the thing proceeds. For otherwise, when he goes back by *Synthesis* to the proofe of his Probleme, he will come to no Demonstration at all; there being no true Demonstration but such as is scientificall; and no Demonstration is scientifical but that which proceeds from the knowledge of the causes from which the construction of the Probleme is drawne. To collect therefore what has been said into few words; ANALYSIS *is Ratiocination from the supposed construction or generation of a thing to the efficient cause, or coefficient causes of that which is constructed or generated. And* SYNTHESIS *is Ratiocination from the first causes*

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ses of the Construction, continued through all the middle causes till we come to the thing it selfe which is constructed or generated.

But because there are many means by which the same thing may be generated, or the same Probleme be constructed, therefore neither do all Geometricians, nor doth the same Geometrician alwayes use one and the same Method. For if to a certain quantity given, it be required to construct another quantity equal, there may be some that will enquire whether this may not be done by means of some motion. For there are quantities, whose equality and inequality may be argued from Motion and Time, as well as from Congruence; and there is motion, by which two quantities, whether Lines or Superficies, though one of them be crooked, the other straight, may be made congruous or coincident. And this method *Archimedes* made use of in his Book *de Spiralibus*. Also the equality or inequality of two quantities may be found out and demonstrated from the consideration of Waight, as the same *Archimedes* did in his Quadrature of the Parabola. Besides, equality and equality are found out often by the division of the two quantityes into parts which are considered as undivisible; as *Cavallerius Bonaventura* has done in our time, and *Archimedes* often. Lastly, the same is performed by the consideration of the *Powers* of lines, or the roots of those *Powers*, and by the multiplication, division, addition and substraction, as also by the extraction of the roots of those *Powers*, or by finding where straight lines of the same proportion terminate. For example, when any number of straight lines, how many soever, are drawne from a straight line, and passe all through the same point, looke what proportion they have, and if their parts continued from the point retaine every where the same proportion, they shall all terminate in a straight line. And the same happens if the point be taken between two Circles. So that the places of all their points of termination make either straight lines, or circumferences of Circles, and are called *Plain Places*. So also when straight parallel lines are applyed to one straight line, if the parts of the straight line to which they are applyed be to one another in proportion duplicate to that of the contiguous applyed lines, they will all terminate in a Conical Section; which Section being the place of their

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termination, is called a *Solid Place*, becauſe it ſerves for the finding out of the quantity of any Equation which conſiſts of three dimenſions. There are therfore three ways of finding out the cauſe of Equality or Inequality between two given quantities; namely, Firſt by the Computation of *Motions* (for by equal Motion, & equal Time equal Spaces are deſcribed,) and Ponderation is motion. Secondly By *Indiviſibles*; becauſe all the parts together taken are equal to the whole. And thirdly by the *Powers*; for when they are equall, their roots alſo are equall; and contrarily, the *Powers* are equall, when their roots are equal. But if the queſtion be much complicated, there cannot by any of theſe wayes be conſtituted a certaine Rule, from the ſuppoſition of which of the unknown quantities the *Analyſis* may beſt begin; nor out of the variety of Equations that at firſt appeare, which we were beſt to chooſe; but the ſucceſſe will depend upon dexterity, upon formerly acquired Science, and many times upon fortune.

For no man can ever be a good *Analyſt* without being firſt a good Geometrician; nor do the rules of *Analyſis* make a Geometrician, as *Syntheſis* doth; which begins at the very Elements, and proceeds by a Logical Uſe of the ſame. For the true teaching of Geometry is by *Syntheſis*, according to *Euclides* method; and he that hath *Euclide* for his Maſter, may be a Geometrician without *Vieta* (though *Vieta* was a moſt admirable Geometrician); but he that has *Vieta* for his maſter, not ſo, without *Euclide*.

And as for that part of *Analyſis* which works by the Powers, though it be eſteemed by ſome Geometricians (not the chiefeſt) to be the beſt way of ſolving all Problemes, yet it is a thing of no great extent; it being all contained in the doctrine of rectangles, and rectangled Solids. So that although they come to an Equation which determines the quantity ſought, yet they cannot ſometimes by art exhibit that quantity in a Plain, but in ſome Conique Section; that is, as Geometricians ſay, not Geometrically, but mechanically. Now ſuch Problemes as theſe, they call *Solid*; and when they cannot exhibit the quantity ſought for with the helpe of a conique Section, they call it a *Lineary* Probleme. And therefore in the quantities of angles, and of the arches of Circles, there is no uſe at all of the *Analyticks* which proceed by the *Powers*; ſo that the

the Antients pronounced it impoſſible, to exhibit in a plaine the Diviſion of Angles, except biſection, and the biſection of the biſected parts, otherwiſe then mechanically. For *Pappus*, (before the 31 propoſition of his fourth Book) diſtinguiſhing and defining the ſeveral kinds of Problemes, ſays that *ſome are* Plain, *others* Solid, *and others* Lineary. *Thoſe therefore which may be ſolved by ſtraight lines and the circumferences of Circles* (that is, which may be deſcribed with the Rule and Compaſs, without any other Inſtrument) *are fitly called* Plain; *for the lines by which ſuch Problemes are found out, have their generation in a Plain. But thoſe which are ſolved by the uſing of ſome one or more Conique Sections in their conſtruction, are called* Solid, *becauſe their conſtruction cannot be made without uſing the ſuperficies of ſolid figures, namely of Cones. There remains the third kinde, which is called* Lineary, *becauſe other lines beſides thoſe already mentioned are made uſe of in their conſtruction, &c.* And a little after he ſayes, *Of this kinde are the* Spiral *lines, the* Quadratrices, *the* Conchoeides, *and the* Ciſſoeides. *And Geometricians think it no ſmall fault, when for the finding out of a* Plain *Probleme any man makes uſe of* Coniques, *or new* Lines. Now he ranks the *Triſection* of an angle among *Solid* Problemes, and the *Quinqueſection* among *Lineary*. But what! are the ancient Geometricians to be blamed, who made uſe of the *Quadratrix* for the finding out of a ſtraight line equal to the arch of a Circle? and *Pappus* himſelf, was he faulty when he found out the triſection of an Angle by the help of an *Hyperbole*? Or am I in the wrong, who think I have found out the conſtruction of both theſe Problemes by the Rule and Compaſs onely? Neither they, nor I. For the Ancients made uſe of this *Analyſis* which proceeds by the Powers; and with them it was a fault to do that by a more remote Power, which might be done by a neerer; as being an argument that they did not ſufficiently underſtand the nature of the thing. The virtue of this kind of *Analyſis* conſiſts in the changing and turning and toſſing of Rectangles and Analogiſmes; and the skill of *Analyſts* is meer Logick, by which they are able methodically to find out whatſoever lies hid either in the Subject or Predicate of the Concluſiō ſought for. But this doth not properly belong to *Algebra*, or the *Analyticks Specious, Symbolical* or *Coſſick*; which are, as I may ſay, the *Brachygraphy* of the *Analyticks*, and an art, neither of teaching nor

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learning Geometry, but of registring with brevity and celerity, the inventions of Geometricians. For though it be easie to discourse by *Symbols* of very remote propositions; yet whether such discourse deserve to be thought very profitable, when it is made without any *Ideas* of the things themselves, I know not.

Chap. XXI.

Of Circular Motion.

1 *In Simple Motion, every Straight Line taken in the Body moved, is so carried, that it is always parallel to the places in which it formerly was.* 2 *If Circular Motion be made about a resting Center, and in that Circle there be an Epicyle, whose revolution is made the contrary way, in such manner, that in equal times it make equal angles, every Straight Line taken in that Epicycle will be so carried, that it will alwayes be parallel to the places in which it formerly was.* 3 *The properties of Simple Motion.* 4 *If a fluid Body be moved with simple Circular Motion, all the points taken in it will describe their Circles in times proportional to the distances from the Center.* 5 *Simple Motion dissipates Heterogeneous and congregates Homogeneous Bodies.* 6 *If a Circle made by a Movent moved with Simple Motion, be commensurable to another Circle made by a point which is carried about by the same Movent, all the points of both the Circles will at some time return to the same situation.* 7 *If a Sphere have Simple Motion, its Motion will more dissipate Heterogeneous Bodies by how much it is more remote from the Poles.* 8 *If the Simple Circular Motion of a fluid Body be hindered by a Body which is not fluid, the fluid Body will spread it self upon the Superficies of that Body.* 9 *Circular Motion about a fixed Center, casteth off by the Tangent such things as lie upon the Circumference and stick not to it.* 10 *Such things as are moved with Simple Circular Motion, beget Simple Circular Motion.* 11 *If that which is so moved have one side hard, and the other side fluid, its Motion will not be perfectly Circular.*

1 I Have already defined *Simple Motion* to be that, in which the several points taken in a moved Body, do in several equal times describe several equal arches. And therefore in Simple Circular Motion it is necessary that every Straight Line taken in the Moved Body be alwayes carried parallel to it self; which I thus demonstrate.

First, let A B (in the first figure) be any Straight Line taken in

any Solid Body; and let A D be any arch drawn upon any Center C and Radius C A. Let the point B be understood to describe towards the same parts the arch B E, like and equall to the arch A D. Now in the same time in which the point A transmits the arch A D, the point B (which by reason of its simple motion is supposed to be carried with velocity equall to that of A) will transmit the arch B E; and at the end of the same time the whole A B will be in D E; and therefore A B and D E are equall. And seeing the arches A D and B E are like and equall, their subtending straight lines A D and B E will also be equall; and therefore the four sided figure A B D E will be a parallelogram. Wherefore A B is carried parallel to it selfe. And the same may be proved by the same method, if any other straight line be taken in the same moved Body in which the straight line A B was taken. So that all straight lines taken in a Body moved with Simple Circular Motion will be carried parallel to themselves.

Coroll. 1 It is manifest that the same will also happen in any Body which hath Simple Motion, though not Circular. For all the points of any straight line whatsoever, will describe lines though not Circular, yet equall; so that though the crooked lines A D and B E were not arches of Circles, but of *Parabolas*, *Ellipses*, or of any other figures; yet both they, and their Subtenses, and the straight lines which joyne them, would be equal and parallel.

Coroll. 2 It is also manifest, that the *Radii* of the equall circles A D and B E, or the Axis of a Sphere, will be so carried, as to be allwayes parallel to the places in which they formerly were. For the straight line B F drawn to the center of the arch B E being equall to the Radius A C, will also be equall to the straight line F E or C D; and the angle B F E will be equall to the angle A C D. Now the intersection of the straight lines C A and B E, being at G, the angle C G E (seeing B E and A D are parallel) will be equal to the angle D A C. But the angle E B F is equal to the same angle D A C; and therefore the angles C G E and E B F are also equal. Wherefore A C and B F are parallel; which was to be demonstrated.

2. Let there be a Circle given (in the second figure), whose center

center is A, and Radius A B; and upon the center B and any Radius B C let the Epicycle C D E be described. Let the center B be understood to be carried about the center A, and the whole Epicycle with it till it be coincident with the Circle F G H, whose center is I; and let B A I be any angle given. But in the time that the center B is moved to I, let the Epicycle C D E have a contrary revolution upon its own center, namely from E by D to C according to the same proportions; that is, in such manner, that in both the Circles, equal angles be made in equal times. I say E C the Axis of the Epicycle will be alwayes carried parallel to it self. Let the angle F I G be made equal to the angle B A I; I F and A B will therefore be parallel; and how much the Axis A G has departed from its former place A C (the measure of which progression is the angle C A G, or C B D which I suppose equal to it), so much in the same time has the Axis I G (the same with B C) departed from its own former situation. Wherefore, in what time B C comes to I G by the motion from B to I upon the center A, in the same time G will come to F by the contrary motion of the Epicycle; that is, it will be turned backwards to F, & I G will lie in I F. But the angles F I G and G A C are equal; and therefore A C, that is, B C), and I G, (that is the Axis, though in different places) will be parallel. Wherefore, the Axis of the Epicycle E D C will be carried alwayes parallel to it self; which was to be proved.

Coroll. From hence it is manifest, that those two annual Motions which *Copernicus* ascribes to the Earth, are reducible to this one Circular Simple Motion, by which all the points of the moved Body are carried always with equal velocity, that is, in equal times they make equal revolutions uniformly.

This, as it is the most simple, so it is the most frequent of all Circular Motions; being the same which is used by all men when they turn any thing round with their arms, as they do in grinding or sifting. For all the points of the thing moved, describe lines which are like and equal to one another. So that if a man had a Ruler, in which many Pens points of equal length were fastned, he might with this one Motion write many lines at once.

3 Having shewed what Simple Motion is, I will here also set down some properties of the same.

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First, when a Body is moved with Simple Motion in a fluid Medium which hath no vacuity, it changes the situation of all the parts of the fluid ambient which resist its motion; I say there are no parts so small of the fluid ambient, how farre soever it be continued, but do change their situation, in such manner, as that they leave their places continually to other small parts that come into the same.

For (in the same second figure) let any Body, as KLMN, be understood to be moved with Simple Circular Motion; and let the Circle which every point thereof describes have any determined quantity, suppose that of the same K L M N. Wherefore the Center A, and every other point, and consequently the moved Body it self, will be carried sometimes towards the side where is K, and sometimes towards the other side where is M. When therefore it is carried to K, the parts of the fluid Medium on that side will go back; and (supposing all space to be full) others on the other side will succeed. And so it will be when the Body is carried to the side M, and to N, and every way. Now when the neerest parts of the fluid Medium go back, it is necessary that the parts next to those neerest parts go back also; and (supposing still all space to be full) other parts will come into their places with succession perpetual and infinite. Wherefore all, even the least parts of the fluid Medium change their places, &c. which was to be proved.

It is evident from hence, that Simple Motion, whether Circular, or not Circular, of Bodies which make perpetual returns to their former places, hath greater or less force to dissipate the parts of resisting Bodies, as it is more or less swift, and as the lines described have greater or less magnitude. Now the greatest Velocity that can be, may be understood to be in the least circuit, and the least in the greatest; and may be so supposed when there is need.

4 Secondly, supposing the same Simple Motion in the Aire, Water, or other fluid Medium; the parts of the Medium which adhere to the Moved Body will be carried about with the same Motion and Velocity, so that in what time soever any point of the Movent finishes its Circle, in the same time every part of the Medium which adheres to the Movent, shall also describe

ſcribe ſuch a part of its Circle, as is equal to the whole Circle of the Movent; I ſay it ſhall deſcribe a part, and not the whole Circle, becauſe all its parts receive their motion from an interiour concentrique Movent, and of Concentrique Circles the exteriour are alwayes greater then the interiour; nor can the motion imprinted by any Movent be of greater Velocity then that of the Movent it ſelf. From whence it follows, that the more remote parts of the fluid ambient,ſhall finiſh their Circles in times which have to one another the ſame proportion with their diſtances from the Movent. For every point of the fluid ambient, as long as it toucheth the Body which carries it about, is carried about with it, and would make the ſame Circle, but that it is left behind ſo much as the exteriour Circle exceeds the interiour. So that if we ſuppoſe ſome thing which is not fluid to float in that part of the fluid ambient which is neereſt to the Movent, it will together with the Movent be carried about. Now that part of the fluid ambient which is not the neereſt but almoſt the neereſt, receiving its degree of velocity from the neereſt, (which degree cannot be greater then it was in the giver) doth therefore in the ſame time make a Circular Line, not a whole Circle, yet equal to the whole Circle of the neereſt. Therefore in the ſame time that the Movent deſcribes its Circle,that which doth not touch it ſhall not deſcribe its Circle; yet it ſhall deſcribe ſuch a part of it, as is equal to the whole Circle of the Movent. And after the ſame manner,the more remote parts of the ambient will deſcribe in the ſame time ſuch parts of their Circles as ſhall be ſeverally equal to the whole Circle of the Movent; and by conſequent they ſhall finiſh their whole Circles in times proportional to their diſtances from the Movent; which was to be proved.

5 Thirdly, The ſame Simple Motion of a Body placed in a fluid Medium, congregates, or gathers into one place ſuch things as naturally float in that Medium, if they be Homogeneous;and if they be Heterogeneous, it ſeparates and diſſipates them. But if ſuch things as be Heterogeneous do not float, but ſettle, then the ſame Motion ſtirs and mingles them diſorderly together. For ſeeing Bodies which are unlike to one another, that is, Heterogeneous Bodies,are not unlike in that they are Bodies(for Bodies,as Bodies, have

have no difference) but onely from some special Cause, that is, from some internal Motion, or Motions of their smallest parts (for I have shewn in the 9th Chapter and 9th Article, that all Mutation is such Motion), it remains that Heterogeneous Bodies have their unlikeness or difference from one another from their internal or specifical Motions. Now Bodies w^ch^ have such difference, receive unlike & different Motions from the same external common Movent; and therefore they will not be moved together, that is to say, they will be dissipated. And being dissipated they will necessarily at some time or other meet with Bodies like themselves, and be moved alike and together with them; and afterwards meeting with more Bodies like themselves, they will unite and become greater Bodies. Wherefore Homogeneous Bodies are congregated, and Heterogenous dissipated by Simple Motion in a Medium where they naturally float. Again, such as being in a fluid Medium, do not float, but sink, if the Motion of the fluid Medium be strong enough, will be stirred up and carried away by that Motion, and consequently they will be hindred from returning to that place to which they sink naturally, and in which onely they would unite, and out of which they are promiscuously carried; that is, they are disorderly mingled.

Now this Motion by which Homogeneous Bodies are congregated, and Heterogeneous are scattered, is that which is commonly called *Fermentation*, from the Latine *Fervere*; as the Greeks have their Ζύμη (which signifies the same) from Ζέω *Ferveo*. For Seething makes all the parts of the Water change their places; and the parts of any thing that is thrown into it, will go several wayes according to their several natures. And yet all *Fervor* or Seething is not caused by Fire; for New Wine and many other things have also their Fermentation and *Fervor*, to which Fire contributes little, and some times nothing. But when in Fermentation we find Heat, it is made by the Fermentation.

6 Fourthly, in what time soever the Movent whose Center is A (in the 2d figure) moved in K L N shall by any number of revolutions (that is, when the Perimeters B I and K L N be commensurable) have described a Line equal to the Circle which passes through the points B and I; in the same time all the points of the float-

floating Body whose Center is B, shall return to have the same situation in respect of the Movent, from which they departed. For seeing it is as the distance B A, that is, as the Radius of the Circle which passes through B I, is to the Perimeter it self B I, so the Radius of the Circle K L N is to the Perimeter K L N; and seeing the velocities of the points B aud K are equal, the time also of the revolution in I B to the time of one revolution in K L N, will be as the Perimeter B I to the Perimeter K L N; and therefore so many revolutions in K L N as together taken are equal to the Perimeter B I, will be finished in the same time in which the whole Perimeter B I is finished; & therefore also the points L, N, F & H, or any of the rest, will in the same time return to the same situation from which they departed; and this may be demonstrated whatsoever be the points considered. Wherefore all the points shall in that time return to the same situation; which was to be proved.

From hence it follows, that if the Perimeters B I and L K N be not commensurable, then all the points wil never return to have the same situation or configuration in respect of one another.

7 In Simple Motion, if the Body moved be of a Spherical figure, it hath less force towards its Poles then towards its middle, to dissipate Heterogeneous, or to congregate Homogeneous Bodies.

Let there be a Sphere (as in the third figure) whose Center is A and Diameter B C; & let it be conceived to be moved with Simple Circular Motion; of which Motion let the Axis be the Straight Line D E, cutting the Diameter B C at right angles in A. Let now the Circle which is described by any point B of the Sphere, have B F for its Diameter; and taking F G equal to B C, and dividing it in the middle in H, the Center of the Sphere A, will when half a revolution is finished, lie in H. And seeing H F aud A B are equal, a Circle described upon the Center H with the Radius H F or H G, will be equal to the Circle whose Center is A and Radius A B. And if the same Motion be continued, the point B will at the end of another half revolution return to the place from whence it began to be moved; and therefore at the end of half a revolution, the point B will be carried to F, and the whole Hemisphere D B E into that Hemisphere in which are the points L, K and F. Wherfore that part of the fluid Medium which is cōtiguous to the

point F, will in the ſame time go back the length of the Straight Line B F; and in the return of the point F to B, that is, of G to C, the fluid Medium wil go back as much in a Straight Line from the point C. And this is the effect of Simple Motion in the middle of the Sphere, where the diſtance from the Poles is greateſt. Let now the point I be taken in the ſame Sphere neerer to the Pole E, and through it let the Straight Line I K be drawn parallel to the Straight Line BF, cutting the arch F L in K, & the Axis HL in M; then connecting HK, upon HF let the perpendicular KN be drawn. In the ſame time therefore that B comes to F, the point I will come to K, B F and I K being equal, and deſcribed with the ſame velocity. Now the Motion in I K to the fluid Medium upon which it works, namely to that part of the Medium which is contiguous to the point K, is oblique, whereas if it proceeded in the Straight Line H K, it would be perpendicular; and therefore the Motion which proceeds in I K has leſs power, then that which proceeds in H K with the ſame velocity. But the Motions in H K and H F do equally thruſt back the Medium; and therefore the part of the Sphere at K, moves the Medium leſs, then the part at F; namely ſo much leſs, as K N is leſs then H F. Wherefore alſo the ſame Motion hath leſs power to diſperſe Heterogeneous, and to congregate Homogeneous Bodies, when it is neerer, then when it is more remote from the Poles; which was to be proved.

Corollary. It is alſo neceſſary, that in Plains which are perpendicular to the Axis, and more remote then the Pole it ſelf from the middle of the Sphere, this Simple Motion have no effect. For the Axis D E with Simple Motion deſcribes the Superficies of a Cylinder; and towards the Baſes of the Cylinder there is in this Motion no endeavour at all.

8 If in a fluid Medium, moved about (as hath been ſaid) with Simple Motion, there be conceived to float ſome other Spherical Body which is not fluid, the parts of the Medium which are ſtopped by that Body, will endeavour to ſpread themſelves every way upon the Superficies of it. And this is manifeſt enough by experience, namely by the ſpreading of water poured out upon a pavement. But the reaſon of it may be this. Seeing the Sphere A (in the 3d figure) is moved towards B, the Medium alſo in which it is moved, will have the ſame Motion. But becauſe in this Motion it

it falls upon a Body not liquid, as G, so that it cannot go on; and seeing the small parts of the Medium can not go forwards, nor can they go directly backwards, against the force of the Movent; it remayns therefore that they diffuse themselves upon the Superficies of that Body, as towards O and P, Which was to be proved.

9 Compounded Circular Motion (in which all the parts of the moved Body do at once describe Circumferences, some greater, others less, according to the proportion of their several distances from the common Center) carries about with it such Bodies, as being not fluid, adhere to the Body so moved; and such as do not adhere, it casteth forwards in a Straight Line which is a Tangent to the point from which they are cast off.

For let there be a Circle whose Radius is A B (in the fourth figure); and let a Body be placed in the Circumference in B, which if it be fixed there, will necessarily be carried about with it, as is manifest of it self. But whilest the motion proceeds, let us suppose that Body to be unfixed in B. I say the Body wil cõtinue its motion in the Tangent BC. For let both the Radius AB, and the Sphere B, be conceived to consist of hard matter; and let us suppose the Radius A B to be stricken in the point B by some other Body which falls upon it in the Tangent D B. Now therefore there will be a motion made by the concourse of two things, the one, Endeavour towards C in the Straight Line D B produced, (in which the Body B would proceed, if it were not retained by the Radius A B); the other, the Retention it self. But the Retention alone causeth no endeavour towards the Center; and therefore the Retention being taken away, (which is done by the unfixing of B) there will remain but one Endeavour in B, namely, that in the Tangent B C. Wherefore the Motion of the Body B unfixed, will proceed in the Tangent B C; which was to be proved.

By this demonstration it is manifest, that Circular Motion about an unmoved Axis, shakes off, and puts further from the Center of its motion such things as touch, but do not stick fast to its Superficies; and the more, by how much the distance is greater from the Poles of the Circular Motion; and so much the more also, by how much the things that are shaken off, are less driven towards the Center by the fluid ambient, for other Causes.

10 If in a fluid Medium a Spherical Body be moved with simple Circular Motion;and in the same Medium there float another Sphere whose matter is not fluid; this Sphere also shall be moved with simple Circular Motion.

Let B C D (in the 5th figure) be a Circle, whose Center is A, and in whose Circumference there is a Sphere so moved that it describes with Simple Motion the Perimeter B C D. Let also E F G be another Sphere of Consistent matter, whose Semidiameter is E H, and Center H; and with the Radius A H let the Circle H I be described.I say the Sphere E FG will(by the Motion of the Body in B C D) be moved in the Circumference H I with Simple Motion.

For seeing the Motion in B C D (by the 4th Article of this Chapter) makes all the points of the fluid Medium describe in the same time Circular Lines equal to one another, the points E, H and G of the Straight Line E H G will in the same time describe with equal *Radii* equal Circles. Let E B be drawn equal and parallel to the Straight Line A H; and let A B be connected, which will therefore be equal and parallel to E H; and therefore also, if upon the Center B and Radius B E the arch E K be drawn equal to the arch H I, and the straight Lines A I, B K and I K be drawn, B K and A I will be equal, and they will also be parallel, because the two arches E K and H I,that is, the two angles K B E and I A H are equal;and consequently the Straight Lines A B and KI which connect them will also be equal and parallel. Wherefore K I and E H are parallel.Seeing therefore E and H are carried in the same time to K and I, the whole Straight Line I K will be parallel to E H, from whence it departed. And therefore, (seeing the Sphere E F G is supposed to be of consistent matter, so as all its points keep alwayes the same situation) it is necessary that every other Straight Line taken in the same Sphere, be carried alwayes parallel to the places in which it formerly was. Wherefore the Sphere E F G is moved with simple Circular Motion; which was to be demonstrated.

11 If in a fluid Medium,whose parts are stirred by a Body moved with Simple Motion, there float annother Body, which hath its Superficies either wholly hard, or wholly fluid; the parts of this

this Body shall approach the Center equally on all sides, that is to say, the motion of the Body shall be Circular, and Concentrique with the motion of the Movent. But if it have one side hard, and the other side fluid, then both those Motions shall not have the same center, nor shall the floating Body be moved in the Circumference of a perfect Circle.

Let a Body be moved in the Circumference of the Circle K L M N (in the 2d figure) whose center is A. And let there be another Body at I, whose Superficies is either all hard, or all fluid. Also let the Medium in which both the Bodies are placed, be fluid. I say the Body at I will be moved in the Circle I B about the Center A. For this has been demonstrated in the last Article.
Wherefore let the Superficies of the Body at I, be fluid on one side, and hard on the other. And first, let the fluid side be towards the Center. Seeing therefore the Motion of the Medium is such, as that its parts do continually change their places, (as hath been shewn in the 5th Article); if this change of place be considered in those parts of the Medium which are contiguous to the fluid Superficies, it must needs be, that the small parts of that Superficies enter into the places of the small parts of the Medium which are contiguous to them; And the like change of place will be made with the next contiguous parts towards A. And if the fluid parts of the Body at I, have any degree at all of tenacity (for there are degrees of tenacity, as in the Aire and Water) the whole fluid side will be lifted up a little; but so much the less, as its parts have less tenacity; whereas the hard part of the Superficies which is contiguous to the fluid part, has no cause at all of elevation, that is to say, no endeavour towards A.

Secondly, let the hard Superficies of the Body at I, be towards A. By reason therefore of the said change of place of the parts which are contiguous to it, the hard Superficies must of necessity (seeing by Supposition there is no empty Space) either come neerer to A, or else its smallest parts must supply the contiguous places of the Medium, which otherwise would be empty. But this cannot be by reason of the supposed hardness; and therefore the other must needs be, namely, that the Body come neerer to A. Wherefore the Body at I, has greater endeavour towards the center

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ter A, when its hard side is next it, then when it is averted from it. But the Body in I, while it is moving in the circumference of the Circle I B, has sometimes one side, sometimes another turned towards the center; and therefore it is sometimes neerer, sometimes further off from the center A. Wherefore the Body at I, is not carried in the circumference of a perfect Circle; which was to be demonstrated.

Chap. XXII.

Of other Variety of Motion.

1 Endeavour and Pressure how they differ. 2 Two kinds of Mediums in which Bodies are moved. 3 Propagation of Motion what it is 4 What motion Bodies have when they preß one another. 5 Fluid Bodies, when they are pressed together, penetrate one another. 6 When one Body presseth another, and doth not penetrate it, the action of the pressing Body is perpendicular to the Superficies of the Body pressed. 7 When a hard Body, pressing another Body, penetrates the same, it doth not penetrate it perpendicularly, unleß it fall perpendicularly upon it. 8 Motion sometimes opposite to that of the Movent. 9 In a full Medium, Motion is propagated to any distance. 10 Dilatation and Contraction what they are. 11 Dilatation and Contraction suppose Mutation of the smallest parts in respect of their situation. 12 All Traction is Pulsion. 13 Such things as being pressed, or bent, restore themselves, have motion in their internal parts. 14 Though that which carrieth another be stopped, the Body carried will proceed. 15,16 The effects of Percussion not to be compared with those of Waight. 17,18 Motion cannot begin first in the internal parts of a Body. 19 Action and Reaction proceed in the same Line. 20 Habit what it is.

1 I Have already (in the 15th Chap. at the 2d Article) defined Endeavour to be Motion through some Length, though not considered as Length, but as a Point. Whether therefore there be resistance or no resistance, the Endeavour will still be the same. For simply to Endeavour, is to Go. But when two Bodies having opposite Endeavours preß one another, then the Endeavour of either of them is that which we call Pressure, and is mutual when their pressures are opposite.

2. Bodies moved, and also the Mediums in which they are moved, are of two kinds. For either they have their parts coherent in such manner, as no part of the Moved Body will easily yeild to the Mouent, except the whole Body yeild also, and such are the things we

we call *Hard*; Or else their parts, while the whole remains unmoved, will easily yeild to the Movent; and these we call *Fluid* or *Soft* Bodies. For the words *Fluid*, *Soft*, *Tough* and *Hard* (in the same manner as *Great* and *Little*) are used onely comparatively; and are not different kinds, but different degrees of Quality.

3 To *Do*, and to *Suffer* is to Move and to be moved; and nothing is moved, but by that which toucheth it, and is also moved, (as has been formerly shewn). And how great sover the distance be, we say the first Movent moveth the last moved Body; but mediately; namely so, as that the first moveth the second, the second the third, and so on, till the last of all be touched. When therefore one Body having opposite Endeavour to another Body, moveth the same, and that moveth a third, and so on, I call that action *Propagation of Motion*.

4 When two fluid Bodies which are in a free and open Space, press one another, their parts will endeavour, or be moved towards the sides, not onely those parts which are there where the mutual contact is, but all the other parts. For in the first contact, the parts which are pressed by both the endeavouring Bodies, have no place either forwards or backwards in which they can be moved; and therefore they are pressed out towards the sides. And this expressure, when the forces are equal, is in a line perpendicular to the Bodies pressing. But whensoever the formost parts of both the Bodies are pressed, the hindermost also must be pressed at the same time; for the motion of the hindermost parts cannot in an instant be stopped by the resistance of the formost parts, but proceeds for some time; and therefore seeing they must have some place in which they may be moved, and that there is no place at all for them forwards, it is necessary that they be moved into the places which are towards the sides every way. And this effect followes of necessity, not onely in Fluid, but in Consistent and Hard Bodies, though it be not alwayes manifest to sense. For though from the compression of two stones we cannot with our eyes discerne any swelling outwards towards the sides, (as we perceive in two Bodies of wax;) yet we know well enough by reason, that some tumor must needs be there, though it be but little.

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5 But when the Space is enclosed, and both the Bodies be fluid, they will (if they be pressed together) penetrate one another, though differently according to their different endeavours. For suppose a hollow Cylinder of hard matter, well stopped at both ends, but filled first, below with some heavy fluid Body, as Quicksilver; and above with Water or Aire. If now the bottome of the Cylinder be turned upwards, the heaviest fluid Body which is now at the top, having the greatest endeavour downwards, and being by the hard sides of the vessel hindered from extending it selfe sideways, must of necessity either be received by the lighter Body, that it may sink through it, or else it must open a passage through it selfe, by which the lighter Body may ascend. For of the two Bodies, that whose parts are most easily separated, will the first be divided; which being done, it is not necessary that the parts of the other, suffer any separation at all. And therefore when two Liquours which are enclosed in the same vessel, change their places, there is no need that their smallest parts should be mingled with one another; for a way being opened through one of them, the parts of the other need not be separated.

Now if a fluid Body which is not enclosed press a hard Body, its endeavour will indeed be towards the internal parts of that hard Body; but (being excluded by the resistance of it) the parts of the fluid Body will be moved every way according to the Superficies of the hard Body, and that equally, if the pressure be perpendicular; for when all the parts of the Cause are equal, the Effects will be equal also. But if the pressure be not perpendicular, then the angles of Incidence being unequal, the expansion also will be unequal, namely, greater on that side where the angle is greater, because that motion is most direct which proceeds by the directest Line.

6 If a Body, pressing another Body do not penetrate it, it will nevertheless give to the part it presseth, an endeavour to yeild and recede in a straight line perpendicular to its Superficies in that point in which it is pressed.

Let A B C D (in the first figure) be a hard Body; and let another Body, falling upon it in the straight line E A, with any inclination, or without inclination, press it in the point A. I say the Body so

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pressing, & not penetrating it, will give to the part A an endeavour to yeild or recede in a straight Line perpendicular to the line AD.

For let A B be perpendicular to A D; and let B A be produced to F. If therefore A F be coincident with A E, it is of it self manifest that the motion in E A will make A to endeavour in the line A B. Let now E A be oblique to A D; and from the point E let the straight line E C be drawn, cutting A D at right angles in D; and let the rectangles A B C D and A D E F be completed. I have shewn (in the 8th Article of the 16th Chapter) that the Body will be carried from E to A by the concourse of two Uniform Motions, the one in E F and its parallels, the other in E D and its parallels. But the motion in E F and its parallels (whereof D A is one) contributes nothing to the Body in A, to make it endeavour or press towards B; and therefore the whole endeavour which the Body hath in the inclined line E A, to pass, or press the Straight line AD, it hath it all from the perpendicular motion or endeavour in F A. Wherefore the Body E after it is in A, will have onely that perpendicular endeavour which proceeds from the motion in F A, that is, in A B; which was to be proved.

7 If a hard Body falling upon, or pressing another Body, penetrate the same, its endeavour after its first penetration will be neither in the inclined line produced, nor in the perpendicular, but sometimes betwixt both, sometimes without them.

Let E A G (in the same 1 figure) be the inclined line produced; and First, let the passage through the Medium in which E A is, be easier then the passage through the Medium in which A G is. As soon therefore as the Body is within the Medium in which is A G, it will finde greater resistance to its motion in D A and its parallels, then it did whilest it was above A D; and therefore below A D it will proceed with slower motion in the parallels of D A, then above it. Wherefore the motion which is compounded of the two motions in E F and E D will be slower below AD, then above it; and therefore also, the Body will not proceed from A in E A produced, but below it. Seeing therefore the endeavour in A B is generated by the endeavour in F A; if to the endeavour in F A there be added the endeavour in D A, (which is not all taken away by the immersion of the point A into the lower Medium) the Body will not proceed from A in the perpendicular A B, but beyond.

yond it, namely, in some straight line between AB and AG, as in the line AH.

Secondly, let the passage through the Medium E A, be less easie then that through A G. The motion therefore which is made by the concourse of the motions in E F and F B, is slower above A D then below it; aud consequently, the endeavour will not proceed from A in E A produced, but beyond it, as in A I. Wherefore, If a hard Body falling, &c; which was to be proved.

This Divergency of the Straight line A H from the straight line AG, is that which the Writers of *Opticks* commonly call *Refraction*; which, when the passage is easier in the first then in the second Medium, is made by diverging from the line of Inclination towards the perpendicular; and contrarily, when the passage is not so easie in the first Medium, by departing farther from the perpendicular.

8 By the 6th *Theoreme* it is manifest, that the force of the Movent may be so placed, as that the Body moved by it, may proceed in a way almost directly contrary to that of the Movent; as we see in the motion of Ships.

For let A B (in the 2d figure) represent a Ship, whose length from the prow to the poop is A B; and let the winde lie upon it in the straight parallel lines C B, D E and F G, and let D E and F G be cut in E and G by a straight Line drawn from B perpendicular to A B; also let B E and E G be equal, and the angle A B C any angle how small soever. Then between B C and B A let the straight line B I be drawn; and let the Sail be conceived to be spred in the same line B I, and the winde to fall upon it in the points L, M and B; from which points, perpendicular to B I, let B K, M Q and L P be drawn. Lastly, let E N and G O be drawn perpendicular to B G, and cutting B K in H and K, and let H N and K O be made equal to one another, and severally equal to B A. I say the Ship B A by the winde falling upon it in C B, D E, F G, and other lines parallel to them, will be carried forwards almost opposite to the winde, that is to say, in a way almost contrary to the way of the Movent.

For the Winde that blowes in the Line C B, will (as hath been shewn in the 6th Article) give to the point B an endeavour to proceed in a straight line perpendicular to the straight line B I, that

is, in the ſtraight line B K; and to the points M and L an endeavour to proceed in the ſtraight lines M Q and L P, which are parallel to B K. Let now the meaſure of the time be B G, which is divided in the middle in E; & let the point B be carried to H in the time B E. In the ſame time therefore by the wind blowing in D M & F L (and as many other lines as may be drawn parallel to them) the whole Ship will be applyed to the ſtraight line H N. Alſo at the end of the ſecond time E G, it will be applyed to the ſtraight line K O. Wherefore the Ship will always go forwards; and the angle it makes with the winde will be equal to the angle A B C, how ſmall ſoever that angle be; and the way it makes will in every time be equal to the ſtraight line E H. I ſay thus it would be, if the Ship might be moved with as great celerity ſidewayes from B A towards K O, as it may be moved forwards in the line B A. But this is impoſſible, by reaſon of the reſiſtance made by the great quantity of water which preſſeth the ſide, much exceeding the reſiſtance made by the much ſmaller quantity which preſſeth the prow of the Ship; ſo that the way the Ship makes ſidewayes is ſcarce ſenſible; and therefore the point B will proceed almoſt in the very line B A, making with the winde the angle A B C, how acute ſoever, that is to ſay, it will proceed almoſt in the ſtraight line B C, that is, in a way almoſt contrary to the way of the Movent; which was to be demonſtrated.

But the Sayl in B I muſt be ſo ſtretched, as that there be left in it no boſome at all; for otherwiſe the ſtraight lines L P, M Q & BK will not be perpendicular to the plain of the Sayl, but falling below P, Q and K will drive the Ship backwards. But by making uſe of a ſmall Board for a Sayl, a little Waggon with wheels for the Ship, and of a ſmooth Pavement for the Sea, I have by experience found this to be ſo true, that I could ſcarce oppoſe the board to the winde in any obliquity though never ſo ſmall, but the Waggon was carried forwards by it.

By the ſame 6th. *Theoreme*, it may be found, how much a ſtroke which falls obliquely, is weaker then a ſtroke falling perpendicularly, they being like and equal in all other reſpects.

Let a ſtroke fall upon the Wall A B obliquely, as (for example) in the ſtraight line C A (in the 3d figure). Let C E be drawn parallel

rallel to AB, & DA perpendicular to the ſame AB & equal to CA; & let both the velocity & time of the motion in CA be equal to the velocity & time of the motion in DA. I ſay the ſtroke in CA will be weaker then that in DA in the proportion of E A to DA. For producing D A howſoever to F, the endeavour of both the ſtrokes will (by the 6th Art.) proceed from A in the perpendicular AF. But the ſtroke in C A is made by the concourſe of two motions in C E and E A; of which that in C E contributes nothing to the ſtroke in A, becauſe C E and B A are parallels; and therefore the ſtroke in C A is made by the motion which is in E A onely. But the velocity or force of the perpendicular ſtroke in E A, to the velocity or force of the ſtroke in D A, is as E A to D A. Wherefore the oblique ſtroke in CA is weaker then the perpendicular ſtroke in DA, in the proportion of EA to DA or C A; Which was to be proved.

9 In a full Medium, all Endeavour proceeds as far as the Medium it ſelf reacheth; that is to ſay, if the Medium be infinite, the Endeavour will proceed infinitely.

For whatſoever Endeavoureth, is Moved, and therefore whatſoever ſtandeth in its way, it maketh it yeild, at leaſt a little, namely ſo far as the Movent it ſelf is moved forwards. But that which yeildeth is alſo moved, and conſequently maketh that to yeild which is in its way, and ſo on ſucceſſively as long as the Medium is full; that is to ſay, infinitely, if the full Medium be infinite; which was to be proved.

Now although Endeavour thus perpetually propagated, do not alwayes appear to the Senſes as Motion; yet it appears as Action, or as the efficient cauſe of ſome Mutation. For if there be placed before our Eyes ſome very little object, as (for example) a ſmall grain of ſand, which at a certain diſtance is viſible; it is manifeſt that it may be removed to ſuch a diſtance as not to be any longer ſeen, though by its action it ſtill work upon the organs of ſight, as is manifeſt from that (which was laſt proved) that all Endeavour proceeds infinitely. Let it be conceived therefore to be removed from our Eyes to any diſtance how great ſoever, and a ſufficient number of other grains of ſand of the ſame bigneſs added to it; it is evident that the aggregate of all thoſe ſands will be viſible; and though none of them can be ſeen when it is ſingle and ſevered

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from the rest, yet the whole heap or hill which they make wil manifestly appear to the sight; which would be impossible if some action did not proceed from each several part of the whole heap.

10 Between the degrees of *Hard* and *Soft*, are those things which we call *Tough*, *Tough* being that, which may be bended without being altered from what it was; and the Bending of a Line, is either the adduction or diduction of the extreme parts, that is, a motion from Straightness to Crookedness, or contrarily, whilest the line remains still the same it was; for by drawing out the extreme points of a line to their greatest distance, the line is made straight, which otherwise is Crooked. So also the Bending of a Superficies, is the diduction or adduction of its extreme lines, that is, their Dilatation and Contraction.

11 *Dilatation* and *Contraction*, as also all *Flexion* supposes necessarily that the internal parts of the Body bowed do either come neerer to the external parts, or go further from them. For though *Flexion* be considered onely in the length of a Body, yet when that Body is bowed, the line which is made on one side will be convex, and the line on the other side will be concave; of which the concave being the interiour line, will (unless something be taken from it and added to the convex line) be the more crooked, that is, the greater of the two. But they are equal; and therefore in *Flexion* there is an accession made from the interiour to the exteriour parts; and on the contrary, in *Tension*, from the exteriour to the interiour parts. And as for those things which do not easily suffer such transposition of their parts, they are called *Brittle*; and the great force they require to make them yeild, makes them also with sudden motion to leap asunder, and break in pieces.

12 Also Motion is distinguished into *Pulsion* and *Traction*. And *Pulsion*, as I have already defined it, is when that which is moved, goes before that which moveth it. But contrarily, in *Traction* the Movent goes before that which is moved. Neverthelesss, considering it with greater attention, it seemeth to be the same with *Pulsion*. For of two parts of a hard Body, when that which is foremost drives before it the Medium in which the motion is made, at the same time that which is thrust forwards, thrusteth the next, and this again the next, and so on successively. In which action, if we

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ſuppoſe that there is no place void, it muſt needs be, that by continual Pulſion, namely, when that action has gone round, the Movent will be behind that part which at the firſt ſeemed not to be thruſt forwards,but to be drawn; ſo that now the Body which was drawn,goes before the Body which gives it motion; and its motion is no longer Traction, but Pulſion.

13 Such things as are removed from their places by forcible Compreſſion or Extenſion, and as ſoon as the force is taken away, doe preſently return and reſtore themſelves to their former ſituation, have the beginning of their reſtitution within themſelves, namely, a certain motion in their internal parts, which was there, when, before the taking away of the force, they were compreſſed, or extended. For that Reſtitution is motion, and that which is at reſt cannot be moved, but by a moved and a Contiguous Movent. Nor doth the cauſe of their Reſtitution proceed from the taking away of the force by which they were compreſſed or extended; for the removing of impediments hath not the efficacy of a cauſe (as has been ſhewn at the end of the 3d Article of the 15th Chapter).The Cauſe therefore of their Reſtitution, is ſome motion either of the parts of the Ambient, or of the parts of the Body compreſſed or extended. But the parts of the Ambient have no endeavour which contributes to their Compreſſion or Extenſion, nor to the ſetting of them at liberty, or Reſtitution. It remayns therefore that from the time of their Compreſſion or Extenſion there be left ſome endeavour (or motion) by which, the impediment being removed,every part reſumes its former place; that is to ſay, the whole Reſtores it ſelf.

14 In the *Carriage* of Bodies if that Body which carries another, hit upon any obſtacle, or be by any means ſuddenly ſtopped, and that which is carried be not ſtopped, it will go on, till its motion be by ſome external impediment taken away.

For I have demonſtrated in the 8th Chapter at the 19th Article, that Motion, unleſs it be hindred by ſome external reſiſtance, will be continued eternally with the ſame celerity; and in the 7th Article of the 9th Chap. that the action of an external Agent is of no effect without contact. When therefore that which carrieth another thing, is ſtopped, that ſtop doth not preſently take away the motion of that which is carried. It will therefore proceed,

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till its motion be by little and little extinguished by some external resistance; Which was to be proved; Though experience alone had been sufficient to prove this.

In like manner, if that Body which carrieth another be put from rest into sudden motion; that which is carried will not be moved forwards together with it, but will be left behind. For the contiguous part of the Body carried, hath almost the same motion with the Body which carries it; and the remote parts will receive different Velocities according to their different distances from the Body that carries them; namely, the more remote the parts are, the less will be their degrees of Velocity. It is necessary therefore that the Body which is carried, be left accordingly more or less behind. And this also is manifest by experience, when at the starting forward of the Horse, the Rider falleth backwards.

15 In *Percussion* therefore, when one hard Body is in some small part of it stricken by another with great force, it is not necessary that the whole Body should yeild to the stroke with the same celerity with which the stricken part yeilds. For the rest of the parts receive their motion from the motion of the part stricken and yeilding, which motion is less propagated every way towards the sides then it is directly forwards. And hence it is, that sometimes very hard Bodies, which being erected can hardly be made to stand, are more easily broken, then thrown down by a violent stroke; when neverthelesse, if all their parts together were by any weak motion thrust forwards they would easily be cast down.

16 Though the difference between *Trusion* and *Percussion* consist onely in this, that in *Trusion* the motion both of the Movent and Moved Body begin both together in their very contact; and in *Percussion* the striking Body is first moved, and afterwards the Body stricken; Yet their Effects are so different, that it seems scarce possible to compare their forces with one another. I say, any effect of Percussion being propounded, as for example the stroke of a Beetle of any weight assigned, by which a Pile of any given length, is to be driven into earth of any tenacity given, it seems to me very hard if not impossible to define, with what weight, or with what stroke, and in what time, the same pile may be driven to a depth assigned into the same earth. The cause of which difficulty

ficulty is this, that the velocity of the Percutient is to be compared with the magnitude of the Ponderant. Now Velocity, ſeeing it is computed by the length of ſpace tranſmitted, is to be accounted but as one Dimenſion; but Waight, is as a ſolid thing, being meaſured by the dimenſion of the whole Body. And there is no compariſon to be made of a Solid Body with a Length, that is, with a Line.

17 If the internal parts of a Body be at reſt, or retain the ſame ſituation with one another for any time how little ſoever, there cannot in thoſe parts be generated any new motion, or endeavour, whereof the efficient cauſe is not without the Body of which they are parts. For if any ſmall part which is comprehended within the Superficies of the whole Body, be ſuppoſed to be now at reſt, and by and by to be moved, that part muſt of neceſſity receive its motion from ſome moved and contiguous Body. But (by ſuppoſition) there is no ſuch moved and contiguous part within the Body. Wherefore, if there be any Endeavour or Motion, or change of ſituation, in the internal parts of that Body, it muſt needs ariſe from ſome efficient cauſe that is without the Body which contains them; Which was to be proved.

18 In hard Bodies therefore which are compreſſed or extended, if that which compreſſeth or extendeth them being taken away, they reſtore themſelves to their former place or ſituation, it muſt needs be, that that Endeavour (or Motion) of their internal parts, by which they were able to recover their former places or ſituations, was not extinguiſhed when the force by which they were compreſſed or extended was taken away. Therefore when the Lath of a Croſs-bow bent, doth, as ſoon as it is at liberty, reſtore it ſelf, though to him that judges by Senſe, both it and all its parts ſeem to be at reſt; yet he that judging by Reaſon, doth not account the taking away of impediment for an efficient cauſe, nor conceives that without an efficient cauſe any thing can paſs from Reſt to Motion, will conclude, that the parts were already in motion before they began to reſtore themſelves.

19 *Action* and *Reaction* proceed in the ſame Line, but from oppoſite Terms. For ſeeing Reaction is nothing but Endeavour in the Patient to reſtore it ſelf to that ſituation from which it was for-

ced by the Agent; the endeavour or motion both of the Agent and Patient (or Reagent) will be propagated between the same terms, (yet so, as that in Action the Term *from which*, is in Reaction the Term *to which*). And seeing all Action proceeds in this manner, not onely between the opposite Terms of the whole line in which it is propagated, but also in all the parts of that line, the Terms *from which* and *to which*, both of the Action and Reaction, will be in the same line. Wherefore Action and Reaction proceed in the same line, &c.

20 To what has been said of Motion, I will add what I have to say concerning Habit. *Habit* therefore is a generation of Motion, not of Motion simply, but an easie conducting of the moved Body in a certain and designed way. And seeing it is attained by the weakning of such endeavours as divert its motion, therefore such endeavours are to be weakned by little and little. But this cannot be done but by the long continuance of action, or by actions often repeated; and therefore Custome begets that Facility, which is commonly and rightly called *Habit*; and it may be defined thus; HABIT *is Motion made more easie and ready by Custome; that is to say, by perpetual endeavour, or by iterated endevours in a way differing from that in which the Motion proceeded from the beginning, and opposing such endeavours as resist.* And to make this more perspicuous by example, We may observe, that when one that has no skill in Musique, first, puts his hand to an Instrument, he cannot after the first stroke carry to his hand to the place where he would make the second stroke, without taking it back by a new endeavour, and as it were beginning again, pass from the first to the second. Nor will he be able to go on to the third place without another new endeavour, but he will be forced to draw back his hand again, and so successively, by renewing his endeavour at every stroke, till at the last by doing this often, and by compounding many interrupted motions or endeavours into one equal endeavour, he be able to make his hand go readily on from stroke to stroke in that order and way which was at the first designed. Nor are *Habits* to be observed in living creatures only, but also in Bodies inanimate. For we find, that when the Lath of a Crossbow is strongly bent, and would if the impediment

ment were removed return again with great force, if it remain a long time bent, it will get such a *Habit*, that when it is loosed and left to its own freedome, it will not onely not restore it self, but will require as much force for the bringing of it back to its first posture, as it did for the bending of it at the first.

CHAP. XXIII.

Of the Center of Equiponderation of Bodies pressing downwards in straight Parallel Lines.

1 *Definitions and Suppositions. 2 Two Plains of Equiponderation are not parallel. 3 The Center of Equiponderation is in every Plain of Equiponderation. 4 The Moments of equal Ponderants are to one another as their distances from the center of the Scale. 5, 6. The Moments of unequal Ponderants have their proportion to one another compounded of the proportions of their Waights and distances from the center of the Scale reciprocally taken. 7. If two Ponderants have their Moments and Distances from the Center of the Scale in reciprocal proportion, they are equally poised; and contrarily. 8 If the parts of any Ponderant press the Beam of the Scale every where equally, all the parts cut out off reckoned from the Center of the Scale, will have their Moments in the same proportion with that of the parts of a Triangle cut off from the Vertex by straight Lines parallel to the base. 9 The Diameter of Equiponderation of Figures which are deficient according to commensurable proportions of their altitudes and bases, divides the Axis, so that the part taken next the Vertex is to the other part as the complete figure to the deficient figure. 10 The diameter of Equiponderation of the Complement of the half of any of the said deficient figures, divides that line which is drawn through the vertex parallel to the base, so, that the part next the vertex is to the other part as the complete figure to the Complement. 11 The Center of Equiponderation of the half of any of the deficient figures in the first row of the Table of the 3d. Article of the 17th Chapter, may be found out by the numbers of the second row. 12 The center of Equiponderation of the half of any of the figures in the second row of the same Table, may be found out by the numbers of the fourth row. 13 The Center of Equiponderation of the half of any of the figures in the same Table, being known, the Center of the Excess of the same figure above a Triangle of the same altitude and base is also known. 14 The Center of Equiponderation of a solid Sector, is in the*

the Axis, so divided, that the part next the Vertex be to the whole Axis want half the Axis of the portion of the Sphere, as 3 to 4.

1 *Definitions.*

1 A *Scale*, is a straight line, whose middle point is immoveable, all the rest of its points being at liberty; and that part of the Scale which reaches from the center to either of the waights, is called the *Beam.*

2 *Equiponderation* is, when the endeavour of one Body which presses one of the Beams, resists the endeavour of another Body pressing the other Beam, so, that neither of them is moved; and the Bodies when neither of them is moved, are said to be *Equally poised.*

3 *Waight*, is the aggregate of all the Endeavours, by which all the points of that Body which presses the Beam, tend downwards in lines parallel to one another; and the Body which presses, is called the *Ponderant.*

4 *Moment*, is the Power which the Ponderant has to move the Beam, by reason of a determined situation.

5 *The plain of Equiponderation*, is that, by which the Ponderant is so divided, that the Moments on both sides remain equal.

6 *The Diameter of Equiponderation*, is the common Section of the two Plains of Equiponderation; and is in the straight line by which the waight is hanged.

7 *The Center of Equiponderation*, is the common point of the two Diameters of Equiponderation.

Suppositions

1 When two Bodies are equally pois'd, if waight be added to one of them, and not to the other, their Equiponderation ceases.

2 When two Ponderants of equal magnitude, and of the same Species or matter, press the Beam on both sides at equal distances from the center of the Scale, their Moments are equal. Also when two Bodies endeavour at equal distances from the center of the Scale, if they be of equal magnitude and of the same Species, their Moments are equal.

2 No

2 No two Plains of Equiponderation are parallel.

Let A B C D (in the first figure) be any Ponderant whatsoever; and in it let E F be a Plain of Equiponderation, parallel to which, let any other Plain be drawn, as G H. I say G H is not a Plain of Equiponderation. For seeing the parts A E F D and E B C F of the Ponderant A B C D, are equally pois'd; and the weight E G H F is added to the part A E F D, and nothing is added to the part E B C F, but the weight E G H F is taken from it; therefore (by the first Supposition) the parts A G H D and G B C H will not be equally pois'd; and consequently G H is not a Plain of Equiponderation. Wherefore, No two Plains of Equiponderation, &c. Which was to be proved.

3 The Center of Equiponderation is in every Plain of Equiponderation.

For if another Plain of Equiponderation be taken, it will not (by the last Article) be parallel to the former Plain; and therefore both those Plains will cut one another. Now that Section (by the 6th Definition) is the Diameter of Equiponderation. Again, if another Diameter of Equiponderation be taken, it will cut that former Diameter; and in that Section (by the 7th Definition) is the Center of Equiponderation. Wherefore the Center of Equiponderation is in that Diameter which lies in the said Plain of Equiponderation.

4 The Moment of any Ponderant applyed to one point of the Beam, to the Moment of the same, or an equal Ponderant applyed to any other point of the Beam, is as the distance of the former point from the Center of the Scale, to the distance of the later point from the same Center. Or thus, Those Moments are to one another, as the Arches of Circles which are made upon the Center of the Scale through those points, in the same time. Or lastly thus; They are, as the parallel bases of two Triangles, which have a common angle at the Center of the Scale.

Let A (in the 2d figure) be the Center of the Scale, and let the equal Poderants D and E press the Beam A B in the points B and C; also let the straight lines B D and C E be Diameters of Equiponderation; and the points D and E in the Ponderants D and E be their Centers of Equiponderation. Let A G F be drawn howsoever,

soever, cutting D B produced in F, and E C in G; and lastly, upon the common Center A, let the two arches B H and C I be described, cutting A G F in H and I. I say the Moment of the Ponderant D to the Moment of the Ponderant E, is as A B to A C, or as B H to C I, or as B F to C G. For the effect of the Ponderant D in the point B, is circular motion in the arch B H; and the effect of the Ponderant E in the point C, circular motion in the arch C I; and by reason of the equality of the Ponderants D and E, these motions are to one another as the Quicknesses or Velocities with which the points B and C describe the arches B H and C I, that is, as the arches themselves B H and C I, or as the straight parallels B F and C G, or as the parts of the Beam A B and A C; for A B. A C: : B F. C G: : B H. C I. are proportionals; and therefore the effects, that is, (by the 4th Definition) the Moments of the equal Ponderants applyed to several points of the Beam, are to one another, as A B and A C; or as the distances of those points from the center of the Scale; or as the parallel bases of the Triangles which have a common angle at A; or as the concentrick arches B H and C I; which was to be demonstrated.

5 Unequal Ponderants, when they are applyed to several points of the Beam, and hang at liberty (that is, so as the line by which they hang be the Diameter of Equiponderation, whatsoever be the figure of the Ponderant), have their Moments to one another in proportion compounded of the proportions of their distances from the center of the Scale, and of their Waights.

Let A (in the 3d figure) be the center of the Scale, and A B the Beam, to which let the two Ponderants C & D be applied at the points B and E. I say the proportion of the Moment of the Ponderant C, to the Moment of the Ponderant D, is compounded of the proportions of A B to A E and of the Waight C to the Waight D; or (if C and D be of the same species) of the magnitude C to the magnitude D.

Let either of them, as C, be supposed to be bigger then the other D. If therefore by the addition of F, F and D together be as one Body equal to C, the Moment of C to the Moment of F + D will be (by the last article) as B G is to E H. Now as F + D is to D, so let E H be to another E I; and the moment of F + D, that is

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of C, to the moment of D, will be as B G to E I. But the proportion of B G to E I is compounded of the proportions (of B G to E H that is) of A B to A E, and (of E H to E I, that is) of the waight C to the waight D. Wherefore unequal Ponderants, when they are applied, &c. which was to be proved.

6 The same figure remaining, if I K be drawn parallel to the Beame A B, and cutting A G in K; and K L be drawn parallel to to B G, cutting A B in L, the distances A B and A L from the center, will be proportional to the moments of C and D. For the moment of C is B G, and the moment of D is E I, to which K L is equal. But as the distance A B from the center, is to the distance A L from the center, so is B G the moment of the Ponderant C, to L K, or E I the moment of the Ponderant D.

7 If two Ponderants have their waights and distances from the center in reciprocal proportion, and the center of the Scale be between the points to which the Ponderants are applied, they will be equally poised. And contrarily, if they be equally poised, their waights and distances from the center of the Scale will be in reciprocall proportion.

Let the center of the Scale (in the same 3d figure) be A, the Beam A B; and let any Ponderant C, having B G for its moment, be applied to the point B; also let any other Ponderant D, whose moment is E I, be applied to the point E. Through the point I, let I K be drawn parallel to the Beam A B, cutting A G in K; also let K L be drawn parallel to B G. K L will then be the Moment of the Ponderant D; and (by the last Article) it will be as B G the Moment of the Ponderant C in the point B, to L K the Moment of the Ponderant D in the point E; so A B to A L. On the other side of the center of the Scale, let A N be taken equal to A L; and to the point N let there be applyed the Ponderant O, having to the Ponderant C the proportion of A B to A N. I say the Ponderants in B and N will be equally poised. For the proportion of the Moment of the Ponderant O in the point N, to the Moment of the Ponderant C in the point B, is (by the 5th Article) cōpounded of the proportions of the waight O to the waight B, & of the distance (from the center of the Scale) A N or A L to the distãce (frō the center of the Scale) A B. But seeing we have supposed, that the distance A B to the distance A N, is in reciprocal propor-

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tion of the Waight O to the waight C, the proportion of the Moment of the Ponderant O in the point N, to the Moment of the Ponderant C in the point B, will be compounded of the proportions of A B to A N, and of A N to A B. Wherefore, setting in order A B, A N, A B, the Moment of O to the Moment of C will be as the first to the last, that is, as A B to A B. Their Moments therefore are equal; and consequently the Plain which passes through A, will (by the fifth Definition) be a Plain of Equiponderation. Wherefore they will be equally poised, as was to be proved.

Now the converse of this is manifest. For if there be Equiponderation, and the proportion of the Waights and Distances be not reciprocal, then both the Waights will alwayes have the same Moments, although one of them have more waight added to it, or its distance changed.

Corollary. When Ponderants are of the same Species, and their Moments be equal, their Magnitudes and Distances from the center of the Scale will be reciprocally proportional. For in Homogeneous Bodies, it is as Waight to Waight, so Magnitude to Magnitude.

8 If to the whole length of the Beam there be applyed a Parallelogram, or a *Parallelopipedum*, or a *Prisma*, or a Cylinder, or the Superficies of a Cylinder, or of a Sphere, or of any portion of a Sphere or *Prisma*; the parts of any of them cut off with plains parallel to the base, will have their Moments in the same proportion with the parts of a Triangle which has its Vertex in the center of the Scale, and for one of its sides the Beam it self, which parts are cut off by Plains parallel to the base.

First, let the rectangled Parallelogram A B C D (in the 4th figure) be applyed to the whole length of the Beam A B, and producing C B howsoever to E, let the Triangle A B E be described. Let now any part of the Parallelogram, as A F, be cut off by the plain F G, parallel to the base C B; and let F G be produced to A E in the point H. I say the Moment of the whole A B C D to the Moment of its part A F, is as the Triangle A B E to the Triangle A G H, that is, in proportion duplicate to that of the distances from the center of the Scale.

For, the Parallelogram A B C D being divided into equal parts in-

infinite in number, by straight lines drawn parallel to the base; and supposing the Moment of the straight line C B to be B E; the Moment of the straight line F G, will (by the 7th Article) be G H; and the Moments of all the straight lines of that Parallelogram, will be so many straight lines in the Triangle ABE drawn parallel to the base B E; all which parallels together taken are the Moment of the whole Parallelogram A B C D; and the same parallels do also constitute the superficies of the Triangle A B E. Wherefore the Moment of the Parallelogram A B C D, is the Triangle A B E, and for the same reason, the Moment of the Parallelogram A F, is the Triangle A G H; and therefore the Moment of the whole Parallelogram, to the Moment of a Parallelogram which is part of the same, is as the Triangle A B E, to the Triangle A G H, or in proportion duplicate to that of the Beams to which they are applyed. And what is here demonstrated in the case of a Parallelogram, may be understood to serve for that of a Cylinder, and of a *Prisma*, and their Superficies; as also for the Superficies of a Sphere, of an Hemisphere, or any portion of a Sphere, (for the parts of the Superficies of a Sphere, have the same proportion with that of the parts of the Axis cut off by the same parallels by which the parts of the Superficies are cut off, as *Archimedes* has demonstrated); and therefore when the parts of any of these figures are equal and at equal distances from the Center of the Scale, their Moments also are equal, in the same manner as they are in Parallelograms.

Secondly, let the Parallelogram A K I B not be rectangled; the straight line IB wil neverthelesse press the point B perpendicularly in the straight line B E; & the straight line L G wil press the point G perpendicularly in the straight line G H; and all the rest of the straight lines which are parallel to I B will do the like. Whatsoever therefore the Moment be which is assigned to the straight line I B, as here (for example) it is supposed to be B E, if A E be drawn, the Moment of the whole Parallelogram A I will be the Triangle A B E; and the Moment of the part A L will be the Triangle A G H. Wherefore the Moment of any Ponderant, which has its sides equally applyed to the Beam, (whether they be applyed perpendicularly or obliquely) will be always to the Moment of a part of the same, in such proportion, as the whole Triangle has to a part

of the same cut off by a plain which is parallel to the base.

9 The Center of Equiponderation of any figure which is deficient according to commensurable proportions of the altitude and base diminished, and whose complete figure is either a Parallelogram, or a Cylinder, or a *Parallelopipedum*, divides the Axis, so, that the part next the Vertex, to the other part, is as the complete figure to the deficient figure.

For let C I A P E (in the 5th figure) be a deficient figure, whose Axis is A B, and whose complete figure is C D F E; and let the Axis A B be so divided in Z, that A Z be to Z B as C D F E is to CIAP E. I say the center of Equiponderation of the figure CIAPE will be in the point Z.

First, that the Center of Equiponderation of the figure C I A-P E is somewhere in the Axis A B, is manifest of it self; and therefore A B is a Diameter of Equiponderation. Let A E be drawn, and let B E be put for the Moment of the straight line C E; the Triangle A B E will therefore (by the 3d Article) be the Moment of the complete figure C D F E. Let the Axis A B be equally divided in L, and let G L H be drawn parallel and equal to the straight line C E, cutting the crooked line C I A P E in I and P, and the straight lines A C and A E in K and M. Moreover, let Z O be drawn parallel to the same C E; and let it be, as L G to L I, so L M to another L N; and let the same be done in all the rest of the straight lines possible, parallel to the base; and through all the points N, let the line A N E be drawn; the three-sided figure A N E B will therefore be the Moment of the figure C I A P E. Now the Triangle A B E is (by the 9th Article of the 17th Chapter) to the three-sided figure A N E B, as A B C D + A I C B is to A I C B twice taken, that is, as C D F E + C I A P E is to C I A P E twice taken. But as C I A P E is to C D F E, that is, as the waight of the deficient figure, is to the waight of the complete figure, so is C I A P E twice taken, to C D F E twice taken. Wherefore, setting in order C D F E + C I A P E. 2 C I A P E. 2 C D F E; the proportion of C D F E + C I A P E to C D F E twice taken, will be compounded of the proportion of C D F E + C I A P E to C I A P E twice taken, that is, of the proportion of the Triangle A B E to the threesided figure A N E B, that is, of the Moment of the complete figure to the Moment of the

the deficient figure, and of the proportion of C I A P E twice taken, to C D F E twice taken, that is, to the proportion reciprocally taken of the waight of the deficient figure to the waight of the complete figure.

Again, seeing by supposition A Z. Z B : : C D F E. C I A P E are proportionals; A B. A Z : : CDFE+CIAPE. CDFE will also (by cōpounding) be proportionals. And seeing AL is the half of AB, A L. A Z : : C D F E+C I A P E. 2 C D F E will also be proportionals. But the proportion of CDFE+C I A PE to 2 CDFE is compounded (as was but now shewn) of the proportions of Moment to Moment &c. and therefore the proportion of A L to A Z is compounded of the proportion of the Moment of the complete figure C D F E to the Moment of the deficient figure C I A P E, and of the proportion of the waight of the deficient figure C I A P E, to the waight of the complete figure C D F E; But the proportion of A L to A Z is compounded of the proportions of A L to B Z and of B Z to A Z. Now the proportion of B Z to A Z is the proportion of the Waights reciprocally taken, that is to say, of the waight C I A P F to the waight C D F E. Therefore the remayning proportion of A L to B Z, that is, of L B to B Z is the proportion of the Moment of the waight CDFE to the Moment of the waight C I A P E. But the proportion of A L to B Z is compounded of the proportions of A L to A Z and of A Z to Z B; of which proportions that of A Z to Z B is the proportion of the waight C D F E to the waight C I A P E. Wherefore (by the 5th Article of this Chapter) the remayning proportion of A L to A Z is the proportion of the distances of the points Z and L from the center of the Scale, which is A. And therefore (by the 6th Article) the waight C I A P E shall hang from O in the straight line O Z. So that O Z is one Diameter of Equiponderation of the waight C I A P E. But the straight line A B is the other Diameter of Equiponderation of the same waight C I A P E. Wherefore (by the 7th Definition) the point Z is the center of the same Equiponderation; which point (by construction) divides the axis so, that the part A Z which is the part next the vertex, is to the other part Z B, as the complete figure C D F E is to the deficient figure C I A P E; which is that which was to be demonstrated.

Coroll.

Corollary. The Center of Equiponderation of any of those plain three-sided figures, which are compared with their complete figures in the Table of the third Article of the 17th Chapter, is to be found in the same Table, by taking the Denominator of the fraction for the part of the axis cut off next the vertex, and the Numerator for the other part next the base. For example, if it be required to find the Center of Equiponderation of the second three-sided figure of foure Meanes, there is in the concourse of the second columne with the row of three-sided figures of four Meanes this fraction $\frac{5}{7}$, which signifies that that figure is to its parallelogrã or compleat figure as $\frac{5}{7}$ to Unity, that is, as $\frac{5}{7}$ to $\frac{7}{7}$, or as 5 to 7; and therefore the Center of Equiponderation of that figure, divides the axis, so, that the part next the vertex is to the other part as 7 to 5.

2 *Corollary.* The Center of Equiponderation of any of the Solids of those figures which are contained in the Table of the 8th Article of the same 17th Chapter, is exhibited in the same Table. For example, if the Center of Equiponderation of a Cone be sought for; the Cone will be found to be $\frac{1}{3}$ of its Cylinder; and therefore the Center of its Equiponderation will so divide the axis, that the part next the vertex, to the other part, will be as 3 to 1. Also the Solid of a three-sided figure of one Meane, that is, a parabolical Solid, seeing it is $\frac{2}{4}$, that is $\frac{1}{2}$ of its Cylinder, will have its Center of Equiponderation in that point, which divides the axis, so, that the part towards the vertex be double to the part towards the base.

10 The Diameter of Equiponderation of the Complement of the ha'f of any of those figutes which are contained in the Table of the 3d article of the 17th Chapter, divides that line which is drawne through the Vertex parallel and equall to the base, so, that the part next the Vertex, will be to the other part, as the Complete figure to the Complement.

For let A I C B (in the same 5 fig.) be the halfe of a Parabola, or of any other of those three-sided figures which are in the Table of the 3d article of the 17th Chap. whose Axis is A B, and base B C; having A D drawn from the Vertex, equall and parallel to the base B C; and whose complete figure is the parallelogramme A B C D. Let I Q be drawne, at any distance from the side C D, but

but parallel to it, and let A D be the altitude of the Complement A I C D, and Q I a line ordinately applyed in it. Wherefore the altitude A L in the deficient figure A I C B, is equal to Q I the line ordinately applyed in its Complement; and contrarily, L I the line ordinately applyed in the figure A I C B, is equall to the altitude A Q in its Complement; and so in all the rest of the ordinate lines and altitudes, the mutation is such, that that line which is ordinately applyed in the figure, is the altitude of its Complement. And therefore the proportion of the altitudes decreasing, to that of the ordinate lines decreasing, being multiplicate according to any number in the deficient figure, is submultiplicate according to the same number in its Complement. For example, if A I C B be a Parabola, seeing the proportion of A B to A L is duplicate to that of B C to L I, the proportion of A D to A Q in the Complement A I C D (which is the same with that of B C to L I) will be subduplicate to that of C D to Q I (which is the same with that of A B to A L); and consequently, in a Parabola, the Complement will be to the Parallelogramme as 1 to 3; in a three-sided figure of two Meanes, as 1 to 4; in a three-sided figure of three Meanes, as 1 to 5, &c. But all the ordinate lines together in A I C D are its moment; and all the ordinate lines in A I C B are its moment. Wherefore the moments of the Complements of the halves of Deficient figures in the Table of the 3d article of the 17th Chap. being compared, are as the Deficient figures themselves; and therefore the Diameter of Equiponderation will divide the straight line A D in such proportion, that the part next the Vertex be to the other part, as the complete figure A B C D is to the Complement A I C D.

Coroll. The diameter of Equiponderation of these halves, may be found by the Table of the 3d article of the 17th Chapter in this manner. Let there be propounded any deficient figure, namely the second three-sided figure of two Meanes. This figure is to the complete figure as $\frac{3}{5}$ to 1, that is as 3 to 5. Wherefore the Complement to the same complete figure is as 2 to 5; and therefore the diameter of Equiponderation of this Complement will cut the straight line drawne from the Vertex parallel to the base, so, that the part next the Vertex will be to the other part as 5 to 2.

And

And in like manner, any other of the said three-sided figures being propounded, if the numerator of its fraction (found out in the Table) be taken from the denominator, the straight line drawn from the Vertex is to be divided, so, that the part next the Vertex be to the other part, as the denominator is to the remainder which that substraction leaves.

11 The center of Equiponderation of the halfe of any of those crooked-lined figures which are in the first row of the Table of the 3d article of the 17th chapter, is in that straight line, which being parallel to the Axis, divides the base according to the numbers of the fraction next below it in the second row, so, that the Numerator be answerable to that part which is towards the Axis.

For example, let the first figure of three Means be taken, whose half is A B C D (in the 6th figure), and let the rectangle A B E D be completed. The Complement therefore will be B C D E. And seeing A B E D is to the figure A B C D (by the Table) as 5 to 4, the same A B E D will be to the Complement B C D E as 5 to 1. Wherefore if F G be drawn parallel to the base D A, cutting the axis, so, that A G be to G B as 4 to 5, the center of Equiponderation of the figure A B C D, will (by the precedent article) be somewhere in the same F G. Again, seeing (by the same article) the complete figure A B E D, is to the Complement B C D E as 5 to 1, therefore if B E and A D be divided in H and I as 5 to 1, the center of Equiponderation of the Complement B C D E will be somewhere in the straight line which connects H and I. Let now the straight line L K be drawn through M the center of the complete figure, parallel to the base; and the straight line N O, through the same center M, perpendicular to it; and let the straight lines L K and F G cut the straight line H I in P and Q. Let P R be taken quadruple to P Q; and let R M be drawn and produced to F G in S. R M therefore will be to M S as 4 to 1, that is, as the figure A B C D to its Complement B C D E. Wherefore seeing M is the center of the Complete figure A B E D; and the distances of R and S from the center M be in proportion reciprocall to that of the waight of the Complement B C D E to the waight of the figure A B C D, R and S will either be the centers of Equiponderation of their own fi-

figures, or those centers will be in some other points of the diameters of Equiponderation H I and F G. But this last is impossible. For no other straight line can be drawn through the point M terminating in the straight lines H I and F G, and retaining the proportion of M R to M S, that is, of the figure A B C D to its complement B C D E. The center therefore of Equiponderation of the figure A B C D is in the point S. Now seeing P M hath the same proportion to Q S which R P hath to R Q, Q S will be 5 of those parts of which P M is 4, that is, of which I N is 4. But I N or P M is 2 of those parts of which E B or F G is 6; and therefore if it be, as 4 to 5, so 2 to a fourth, that fourth will be $2\frac{1}{2}$. Wherefore Q S is $2\frac{1}{2}$ of those parts of which F G is 6. But F Q is 1; and therefore F S is $3\frac{1}{2}$. Wherefore the remayning part G S is $2\frac{1}{2}$. So that F G is so divided in S, that the part towards the Axis, is in proportion to the other part as $2\frac{1}{2}$ to $3\frac{1}{2}$, that is, as 5 to 7; which answereth to the fraction $\frac{5}{7}$ in the second row, next under the fraction $\frac{4}{7}$ in the first row. Wherefore drawing S T parallel to the Axis, the base wil be divided in like manner.

By this Method it is manifest, that the base of a Semiparabola will be divided into 3 and 5; and the base of the first three-sided figure of two Means, into 4 and 6; and of the first three-sided figure of four Means, into 6 and 8. The fractions therefore of the second row denote the proportions into which the bases of the figures of the first row are divided by the diameters of Equiponderation. But the first row begins one place higher then the second row.

12 The center of Equiponderation of the half of any of the figures in the second row of the same Table of the 3d article of the 17th Chapter, is in a straight line parallel to the Axis, and dividing the base according to the nũbers of the fraction in the fourth row, two places lower, so, as that the Numerator be answerable to that part which is next the Axis.

Let the half of the second three-sided figure of two Means be taken; and let it be A B C D (in the 7th Figure); whose complement is B C D E, and the rectangle completed A B E D. Let this rectangle be divided by the two straight lines L K & N O, cutting one another in the center M at right angles; and because A B E D

is to A B C D as 5 to 3, let A B be divided in G, so, that A G to B G be as 3 to 5; and let F G be drawn parallel to the base. Also because A B E D is (by the 9th article) to B C D E as 5 to 2, let B E be divided in the point I, so, that B I be to I E as 5 to 2; and let I H be drawn parallel to the Axis, cutting L K and F G in P and Q. Let now P R be so taken, that it be to P Q as 3 to 2, and let R M be drawn and produced to F G in S. Seeing therefore R P is to R Q, that is, R M to M S, as A B C D is to its complement BCDE, and the centers of Equiponderation of A B C D and B C D E are in the straight lines F G and H I, and the center of Equiponderation of them both together in the point M; R will be the center of the Complement B C D E, and S the center of the Figure A B C D. And seeing P M, that is IN, is to Q S, as R P is to R Q; and I N, or P M is 3 of those parts, of which B E, that is, F G is 14; therefore Q S is 5 of the same parts; and E I, that is F Q, 4; and F S, 9; and G S, 5. Wherefore the straight line S T being drawn parallel to the Axis, will divide the base A D into 5 and 9. But the fraction $\frac{5}{9}$ is found in the fourth row of the Table, two places below the fraction $\frac{3}{5}$ in the second row.

By the same method, if in the same second row, there be taken the second three-sided Figure of three Meanes, the center of Equiponderation of the half of it, will be found to be in a straight line parallel to the Axis, dividing the base according to the numbers of the fraction $\frac{6}{10}$, two places below in the fourth row. And the same way serves for all the rest of the Figures in the second row. In like manner, the center of Equiponderation of the third three-sided Figure of three Means, will be found to be in a straight line parallel to the Axis, dividing the base, so, that the part next the Axis, be to the other part, as 7 to 13, &c.

Coroll. The Centers of Equiponderation of the halves of the said Figures are known, seeing they are in the intersection of the straight lines S T and F G, which are both known.

13 The center of Equiponderation of the half of any of the Figures, which (in the Table of the 3d Article of the 17th Chap.) are compared with their Parallelograms, being known; the center of Equiponderation of the excess of the same Figure above its triangle, is also known.

For

For example, let the Semiparabola A B C D (in the 8th Fig.) be taken, whose Axis is A B; whose complete Figure is A B E D; and whose excess above its triangle is B C D B. Its center of Equiponderation may be fonnd out in this manner. Let FG be drawn parallel to the base, so, that A F be a third part of the Axis; and let HI be drawn parallel to the Axis, so, that A H be a third part of the base. This being done, the center of Equiponderation of the triangle A B D, will be I. Again, let K L be drawn parallel to the base, so, that A K be to A B as 2 to 5; and M N parallel to the Axis, so, that A M be to A D as 3 to 8; and let M N terminate in the straight line K L. The center therefore of Equiponderation of the Parabola A B C D is N; and therefore we have the centers of Equiponderation of the Semiparabola A B C D, and of its part the triangle A B D. That we may now finde the Center of Equiponderation of the remayning part B C D B, let I N be drawn and produced to O, so, that N O be triple to I N; and O will be the center sought for. For seeing the waight of A B D, to the waight of B C D B is in proportion reciprocall to that of the straight line N O to the straight line I N; and N is the center of the whole, and I the center of the triangle A B D; O will be the center of the remaining part, namely, of the figure B D C B; which was to be found.

Coroll. The Center of Equiponderation of the figure B D C B, is in the concourse of two straight lines, whereof one is parallel to the base, and divides the Axis, so, that the part next the base be $\frac{2}{5}$ or $\frac{6}{15}$ of the whole Axis; the other is parallel to the Axis, and so divides the base, that the part towards the Axis be $\frac{1}{2}$ or $\frac{12}{24}$ of the whole base. For drawing O P parallel to the base, it will be as I N to N O, so F K to K P, that is, so 1 to 3, or 5 to 15. But A F is $\frac{5}{15}$ or $\frac{1}{3}$ of the whole A B, and A K is $\frac{6}{15}$ or $\frac{2}{5}$; and F K $\frac{1}{15}$; and K P $\frac{3}{15}$; and therefore A P is $\frac{9}{15}$ of the Axis A B. Also A H is $\frac{1}{3}$ or $\frac{8}{24}$; and A M $\frac{3}{8}$ or $\frac{9}{24}$ of the whole base; and therefore O Q being drawn parallel to the Axis, M Q (which is triple to H M) will be $\frac{3}{24}$. Wherefore A Q is $\frac{12}{24}$ or $\frac{1}{2}$ of the base A D.

The excesses of the rest of the three-sided figures in the first row of the Table of the 3d article of the 17th Chapter, have their centers of Equiponderation in two straight lines which divide the

Axis and base according to those fractions, which adde 4 to the numerators of the fractions of a Parabola $\frac{9}{15}$ and $\frac{12}{24}$; and 6 to the denominators, in this mannner,

In a Parabola, The Axis $\frac{9}{15}$, The Base $\frac{12}{24}$
In the first three-sided figure, The Axis $\frac{13}{21}$, The Base $\frac{16}{30}$
In the second three-sided figure, The Axis $\frac{17}{27}$, The Base $\frac{20}{36}$ &c.

And by the same method, any man (if it be worth the paines) may find out the centers of Equiponderation of the excesses above their triangles of the rest of the figures in the second & third row, &c.

14 The center of Equiponderation of the Sector of a Sphere (that is, of a figure compounded of a right Cone whose Vertex is the center of the Sphere, and the portion of the Sphere whose base is the same with that of the Cone), divides the straight line which is made of the Axis of the Cone and halfe the Axis of the portion together taken, so, that the part next the Vertex be triple to the other part, or to the whole straight line, as 3 to 4.

For let A B C (in the 9th fig.) be the Sector of a Sphere, whose Vertex is the center of the Sphere A; whose Axis is A D; and the circle upon B C is the common base of the portion of the Sphere and of the Cone whose Vertex is A; the Axis of which portion is E D, and the halfe thereof F D; and the Axis of the Cone, A E. Lastly let A G be $\frac{3}{4}$ of the straight line A F. I say G is the center of Equiponderation of the Sector A B C.

Let the straight line F H be drawne of any length, making right angles with A F at F; and drawing the straight line A H, let the triangle A F H be made. Then upon the same center A let any arch I K be drawne, cutting A D in L; and its chord, cutting A D in M; and dividing M L equally in N, let N O be drawne parallel to the straight line F H, and meeting with the straight line A H in O.

Seeing now BDC is the Spherical Superficies of the portion cut off with a plain passing through B C, and cutting the Axis at right angles; and seeing F H divides E D the Axis of the portion into two equal parts in F; the center of Equiponderation of the Superficies

ficies B D C will be in F (by the 8th article); and for the same reason the center of Equiponderation of the Superficies I L K (K being in the straight line A C) will be in N. And in like manner, if there were drawne between the center of the Sphere A and the outermost Spherical Superficies of the Sector, arches infinite in number, the centers of Equiponderation of the Sphericall Superficies in which those arches are,, would be found to be in that part of the Axis, which is intercepted between the Superficies it selfe and a plaine passing along by the chord of the arch, and cutting the Axis in the middle at right angles.

Let it now be supposed that the moment of the outermost sphericall Superficies B D C is F H. Seeing therefore the Superficies B D C is to the Superficies I L K in proportion duplicate to that of the arch B D C to the arch I L K, that is, of B E to I M, that is, of F H to N O; let it be as F H to N O, so N O to another N P; and again, as N O to N P, so N P to another N Q; and let this be done in all the straight lines parallel to the base F H that can possibly be drawn between the base and the vertex of the triangle A F H. If then through all the points Q there be drawn the crooked line A Q H, the figure A F H Q A will be the complement of the first three-siled figure of two Meanes; and the same will also be the moment of all the Sphericall Superficies of which the Solid Sector A B C D is compounded; and by consequent, the moment of the Sector it selfe. Let now F H be understood to be the semidiameter of the base of a right Cone, whose side is A H, and Axis A F. Wherfore seeing the bases of the Cones which passe through F and N and the rest of the points of the Axis, are in proportion duplicate to that of the straight lines F H and N O, &c. the moment of all the bases together, that is, of the whole Cone, will be the figure it self A F H Q A; and therefore the center of Equiponderation of the Cone AFH is the same with that of the solid Sector. Wherefore seeing A G is $\frac{3}{4}$ of the Axis A F, the center of Equiponderation of the Cone A F H is in G; and therefore the center of the solid Sector is in G also, and divides the part A F of the Axis, so, that A G is triple to G F; that is, A G is to A F as 3 to 4; which was to be demonstrated.

Note, that when the Sector is a Hemisphere, the Axis of the Cone

Cone vanisheth into that point which is the center of the Sphere; and therefore it addeth nothing to half the Axis of the portion. Wherefore, if in the Axis of the Hemisphere, there be taken from the center, $\frac{3}{4}$ of halfe the Axis, that is, $\frac{3}{8}$ of the Semidiameter of the Sphere, there will be the center of Equiponderation of the Hemisphere.

[illegible]

CHAP. XXIV.

Of Refraction *and* Reflection.

1 *Definitions. 2 In perpendicular Motion there is no Refraction. 3 Things thrown out of a thinner into a thicker Medium, are so refracted, that the Angle Refracted is greater then the Angle of Inclination. 4 Endeavour which from one point tendeth every way, will be so Refracted, as that the sine of the Angle Refracted, will be to the sine of the Angle of Inclination, as the Density of the first Medium is to the Density of the second Medium, reciprocally taken. 5 The sine of the Refracted Angle in one Inclination is to the sine of the Refracted Angle in another Inclination, as the sine of the Angle of that Inclination is to the sine of the Angle of this Inclination. 6 If two lines of Incidence, having equal Inclination, be the one in a thinner, the other in a thicker Medium, the sine of the angle of Inclination will be a Mean proportional between the two sines of the Refracted angles. 7 If the angle of Inclination be semirect, and the line of Inclination be in the thicker Medium, and the proportion of their Densities be the same with that of the Diagonal to the side of a Square, and the separating Superficies be plain, the Refracted line will be in the separating Superficies. 8 If a Body be carried in a straight line upon another Body, and do not penetrate the same, but be reflected from it, the angle of Reflexion will be equal to the Angle of Incidence. 9 The same happens in the generation of Motion in the line of Incidence.*

1 *Definitions.*

1 REFRACTION, is the *breaking* of that straight Line, in which a Body is moved, or its Action would proceed in one and the same Medium, into two straight lines, by reason of the different natures of the two Mediums.

2 The former of these is called the *Line of Incidence*; the later the *Refracted Line*.

3 The *Point of Refraction*, is the common point of the Line of Incidence and of the Refracted Line.

4 The

4 The *Refracting Superficies*, which alſo is the *Separating Superficies* of the two Mediums, is that in which is the point of Refraction.

5 The *Angle Refracted*, is that which the Refracted Line makes in the point of Refraction, with that Line which from the ſame point is drawn perpendicular to the ſeparating Superficies in a different Medium.

6 The *Angle of Refraction*, is that which the Refracted line makes with the Line of Incidence produced.

7 The *Angle of Inclination*, is that which the Line of Incidence makes with that Line which from the point of Refraction is drawn perpendicular to the ſeparating Superficies.

8 The *Angle of Incidence*, is the Complement to a right Angle of the Angle of Inclination.

And ſo, (in the firſt Figure) the *Refraction* is made in A B F. The *Refracted Line* is B F. The *Line of Incidence* is A B. The *Point of Incidence, and of Refraction* is B. The *Refracting* or *Separating Superficies* is D B E. The *Line of Incidence produced directly* is ABC The *Perpendicular* to the ſeparating Superficies is B H. The *Angle of Refraction* is C B F. The *Angle Refracted* is H B F. The *Angle of Inclination* is A B G or H B C. The *Angle of Incidence* is A B D.

9 Moreover the *Thinner Medium*, is underſtood to be that in which there is leſs reſiſtance to Motion or to the generation of Motion; & the *Thicker*, that wherin there is greater reſiſtance.

10 And that Medium in which there is equal reſiſtance every where, is a *Homogeneous Medium*. All other Mediums are *Heterogeneous*.

2 If a Body paſs, or there be generation of Motion, from one Medium to another of different Denſity, in a line perpendicular to the Separating Superficies; there will be no Refraction.

For ſeeing on every ſide of the perdendicular all things in the Mediums are ſuppoſed to be like and equal; if the Motion it ſelf be ſuppoſed to be perpendicular, the Inclinations alſo will be equal, or rather none at all; and therefore there can be no cauſe, from which Refraction may be inferred to be on one ſide of the per-

perpendicular, which wil not cõclude the same Refraction to be on the other side. Which being so, Refraction on one side will destroy Refraction on the other side; and consequently, either the Refracted line will be every where, (which is absurd), or there will be no Refracted line at all; which was to be demonstrated.

Corol. It is manifest from hence, that the cause of Refraction consisteth onely in the obliquity of the line of Incidence, whether the Incident Body penetrate both the Mediums, or without penetrating, propagate motion by Pressure onely.

3 If a Body, without any change of situation of its internal parts, as a stone, be moved obliquely out of the thinner Medium, and proceed penetrating the thicker Medium; and the thicker Medium be such, as that its internal parts being moved, restore themselves to their former situation; the angle Refracted will be greater then the angle of Inclination.

For let D B E (in the same first figure) be the separating Superficies of two Mediums; and let a Body, as a stone thrown, be understood to be moved as is supposed in the straight line A B C; and let A B be in the thinner Medium, as in the Aire; and B C in the thicker, as in the Water. I say the stone, wch being thrown, is moved in the line A B, will not proceed in the line B C, but in some other line, namely that, with which the perpendicular B H makes the Refracted angle H B F greater then the angle of Inclination HBC.

For seeing the stone coming from A, and falling upon B, makes that which is at B proceed towards H, and that the like is done in all the straight lines which are parallel to B H; and seeing the parts moved restore themselves by contrary motion in the same line; there will be contrary motion generated in H B, and in all the straight lines which are parallel to it. Wherefore the motion of the stone will be made by the concourse of the motions in A G, that is, in D B, and in G B, that is, in B H, and lastly, in H B, that is, by the concourse of three motions. But by the concourse of the motions in A G and B H, the stone will be carried to C; and therefore by adding the motion in H B, it will be carried higher in some other line, as in B F, and make the angle H B F greater then the angle H B C.

And from hence may be derived the cause, why Bodies which

are thrown in a very oblique line, if either they be any thing flat, or be thrown with great force, will when they fall upon the water, be cast up again from the water into the aire.

For let A B (in the 2d figure) be the superficies of the water; into which from the point C, let a stone be thrown in the straight line C A, making with the line BA produced a very little angle C A D; and producing B A indefinitely to D, let C D be drawn perpendicular to it, and A E parallel to C D. The stone therefore will be moved in CA by the concourse of two motions in C D and DA, whose velocities are as the lines themselves C D and D A. And from the motion in C D and all its parallels downwards, as soon as the stone falls upon A, there will be Reaction upwards, because the water restores it self to its former situation. If now the stone be thrown with sufficient obliquity, that is, if the straight line C D be short enough, that is, if the endeavour of the stone downwards be less then the Reaction of the water upwards, that is, less then the endeavour it hath from its own gravity, (for that may be), the stone will (by reason of the excess of the endeavour which the water hath to restore it self, above that which the stone hath downwards) be raised again above the Superficies A B, and be carried higher, being reflected in a line which goes higher, as the line A G.

4 If from a point, whatsoever the Medium be, Endeavour be propagated every way into all the parts of that Medium; and to the same Endeavour there be obliquely opposed another Medium of a different nature, that is, either thinner or thicker; that Endeavour will be so refracted, that the sine of the angle Refracted, to the sine of the angle of Inclination, will be as the density of the first Medium to the density of the second Medium, reciprocally taken.

First, let a Body be in the thinner Medium in A (Figure 3d.); and let it be understood to have endeavour every way, and consequently that its endeavour proceed in the lines A B and A *b*; to which let B *b* the superficies of the thicker Medium be obliquely opposed in B and *b*, so that A B and A *b* be equal; and let the straight line B *b* be produced both wayes. From the points B and *b* let the perpendiculars B C and *b c* be drawn; and upon the centers

B

B and *b*, and at the equal distances B A and *b* A, let the Circles A C and A *c* be described, cutting B C and *bc* in C and *c*, and the same C B and *c b* produced in D and *d*, as also A B and A *b* produced in E and *e*. Then from the point A to the straight lines B C and *b c* let the perpendiculars A F and A *f* be drawn. A F therefore will be the sine of the angle of Inclination of the straight line A B, and A *f* the sine of the angle of Inclination of the straight line A *b*, which two Inclinations are by construction made equal. I say, as the density of the Medium in which are B C and *b c*, is to the density of the Medium in which are B D and *b d*, so is the sine of the angle Refracted, to the sine of the angle of Inclination.

Let the straight line F G be drawn parallel to the straight line A B, meeting with the straight line *b* B produced in G.

Seeing therefore A F and B G are also parallels, they will be equal; and consequently, the endeavour in A F is propagated in the same time, in which the endeavour in B G would be propagated if the Medium were of the same density. But because B G is in a thicker Medium, that is, in a Medium which resists the endeavour more then the Medium in which A F is, the endeavour will be propagated less in B G then in A F, according to the proportion which the density of the Medium in which A F is, hath to the density of the Medium in which B G is. Let therefore the density of the Medium in which B G is, be to the density of the Medium in which A F is, as B G is to B H; and let the measure of the time be the Radius of the Circle. Let H I be drawn parallel to B D, meeting with the circumference in I; and from the point I let I K be drawn perpendicular to B D; which being done, B H and I K will be equal; and I K will be to A F, as the density of the Medium in which is A F, is to the density of the Medium in which is I K. Seeing therefore in the time A B (which is the Radius of the Circle) the endeavour is propagated in A F in the thinner Medium, it will be propagated in the same time, that is, in the time B I in the thicker Medium from K to I. Therefore B I is the Refracted line of the line of Incidence A B; and I K is the sine of the angle Refracted; and A F, the sine of the angle of Inclination. Wherefore seeing I K is to A F, as the density of the Medium in which is A F to the density of the Medium in which is I K; it will be as the density of the Medium in which is A F, (or

BC) to the density of the Medium in which is I K (or B D), so the sine of the angle Refracted to the sine of the angle of Inclination. And by the same reason it may be shewn, that as the density of the thinner Medium is to the density of the thicker Medium, so will KI the sine of the angle Refracted be to A F the sine of the Angle of Inclination.

Secondly, let the Body which endeavoureth every way, be in the thicker Medium at I. If therefore both the Mediums were of the same density, the endeavour of the Body in I B would tend directly to L; and the sine of the angle of Inclination L M would be equal to I K or B H. But because the density of the Medium in which is IK, to the density of the Medium in which is L M, is as BH to B G, that is, to A F, the endeavour will be propagated further in the Medium in which L M is, then in the Medium in which I K is, in the proportion of density to density, that is, of M L to A F. Wherefore B A being drawn, the angle Refracted will be C B A, and its sine A F. But L M is the sine of the angle of Inclination; and therefore again, as the density of one Medium is to the density of the different Medium, so reciprocally is the sine of the angle Refracted to the sine of the angle of Inclination, which was to be demonstrated.

In this Demonstration, I have made the separating Superficies B b plain by construction. But though it were concave or convex, the Theoreme would neverthelesse be true. For the Refraction being made in the point B of the plain separating Superficies, if a crooked line, as P Q be drawn, touching the separating line in the point B; neither the Refracted line B I, nor the perpendicular B D will be altered; and the Refracted angle K B I, as also its sine K I will be still the same they were.

5 The sine of the angle Refracted in one Inclination, is to the sine of the angle Refracted in another Inclination, as the sine of the angle of that Inclination to the sine of the angle of this Inclination.

For seeing the sine of the Refracted angle is to the sine of the angle of Inclination, (whatsoever that Inclination be) as the density of one Medium, to the density of the other Medium; the proportion of the sine of the Refracted angle, to the sine of the angle of Inclination, will be compounded of the proportions of density to density,

sity,

sity; and of the sine of the angle of one Inclination to the sine of the angle of the other Inclination. But the proportions of the densities in the same Homogeneous Body, are supposed to be the same. Wherefore Refracted angles in different Inclinations, are as the sines of the angles of those Inclinations; which was to be demonstrated.

6 If two lines of Incidence having equal inclination, be the one in a thinner, the other in a thicker Medium; the sine of the angle of their Inclination, will be a mean proportional between the two sines of their angles Refracted.

For let the straight line AB (in the same 3d figure) have its Inclination in the thinner Medium, and be refracted in the thicker Medium in B I; and let E B have as much Inclination in the thicker Medium, and be refracted in the thinner Medium in B S; and let R S the sine of the angle Refracted be drawn. I say the straight lines R S, A F and I K are in continual proportion. For it is, as the density of the thicker Medium to the density of the thinner Medium, so R S to A F. But it is also, as the density of the same thicker Medium, to that of the same thinner Medium, so A F to I K. Wherefore R S. A F : : A F. I K are propoortionals; that is, R S, A F and I K are in continual proportion, and A F is the Mean proportional; which was to be proved.

7 If the angle of Inclination be semirect, and the line of Inclination be in the thicker Medium, and the proportion of the Densities be as that of a Diagonal to the side of its Square, and the separating Superficies be plain, the Refracted line will be in that separating Superficies.

For in the Circle A C (in the 4th figure) let the angle of Inclination A B C be an angle of 45 degrees. Let C B be produced to the Circumference in D; & let C E (the sine of the angle E B C) be drawn, to which, let B F be taken equal in the separating line B G. B C E F will therefore be a Parallelogram, & F E & B C, that is, F E and B G equal. Let AG be drawn, namely, the Diagonal of the Square whose side is B G; and it will be, as A G to E F, so B G to B F; & so (by supposition) the density of the Medium in which C is, to the density of the Medium in which D is; and so also the sine of the angle Refracted to the sine of the angle of Inclination.

Draw-

Drawing therefore F D, & from D the line D H perpendicular to A B produced, DH will be the sine of the angle of Inclination. And seeing the sine of the angle Refracted is to the sine of the angle of Inclination, as the density of the Medium in which is C, is to the density of the Medium in which is D, that is, (by supposition) as A G is to F E, that is, as D H is to B G; and seeing D H is the sine of the angle of Inclination, B G will therefore be the sine of the angle Refracted. Wherefore B G will be the Refracted line, and lye in the plain separating Superficies; which was to be demonstrated.

Coroll. It is therefore manifest, that when the Inclination is greater then 45 degrees, as also when it is less, provided the density be greater, it may happen that the Refraction will not enter the thinner Medium at all.

8 If a Body fall in a straight line upon another Body, and do not penetrate it, but be reflected from it, the angle of Reflexion will be equal to the angle of Incidence.

Let there be a Body at A (in the 5th figure), which falling with straight motion in the line A C upon another Body at C, passeth no further, but is reflected; and let the angle of Incidence be any angle, as A C D. Let the straight line C E be drawn, making with D C produced the angle E C F equall to the angle A C D; and let A D be drawn perpendicular to the straight line D F. Also in the same straight line D F let C G be taken equall to C D; and let the perpendicular G E be raised, cutting C E in E. This being done, the triangles A C D and E C G will be equall and like. Let C H be drawn equal and parallel to the straight line A D; and let H C be produced indefinitely to I. Lastly let E A be drawn, which will passe through H, and be parallel and equall to G D. I say the motion from A to C in the straight line of Incidence AC, will be reflected in the straight line C E.

For the motion from A to C is made by two coefficient or concurrent motions, the one in A H parallel to D G, the other in A D perpendicular to the same D G; of which two motions, that in A H workes nothing upon the Body A after it has been moved as farre as C, because (by supposition) it doth not passe the straight line D G; whereas the endeavour in A D, that is in H C, worketh further towards I. But seeing it doth onely presse and not penetrate

netrate; there will be reaction in H, which causeth motion from C towards H; and in the mean time the motion in H E remaines the same it was in A H; and therefore the Body will now be moved by the concourse of two motions in C H and H E, which are equall to the two motions it had formerly in A H and H C. Wherefore it will be carried on in C E. The angle therefore of Reflection will be E C G, equall (by construction) to the angle A C D; which was to be demonstrated.

Now when the Body is considered but as a point, it is all one, whether the Superficies or line in which the Reflection is made, be straight or crooked; for the point of Incidence and Reflexion C, is as well in the crooked line which toucheth D G in C, as in D G it selfe.

9 But if we suppose that not a Body be moved, but some Endeavour onely be propagated from A to C, the Demonstration will neverthelesse be the same. For all Endeavour is motion; and when it hath reached the Solid Body in C, it presseth it, and endeavoureth further in C I. Wherefore the reaction will proceed in C H; and the endeavour in C H concurring with the endeavour in H E, will generate the endeavour in C E, in the same manner as in the repercussion of Bodies moved.

If therefore Endeavour be propagated from any point to the concave Superficies of a Spherical Body, the Reflected line with the circumference of a great circle in the same Sphere, will make an angle equall to the angle of Incidence.

For if Endeavour be propagated from A (in the 6 fig.) to the circumference in B, and the center of the Sphere be C, and the line C B be drawne, as also the Tangent D B E; and lastly if the angle F B D be made equall to the angle A B E, the Reflexion will be made in the line B F, as hath been newly shewn. Wherefore the angles which the straight lines A B and F B make with the circumference, will also be equall. But it is here to be noted that if C B be produced howsoever to G, the endeavour in the line G B C will proceed onely from the perpendicular reaction in G B; and that therefore there will be no other endeavour in the point B towards the parts which are within the Sphere, besides that which tends towards the center.

And

And here I put an end to the third part of this Discourse; in which I have considered Motion and Magnitude by themselves in the abstract. The fourth and last part, concerning the *Phænomena of Nature*, that is to say, concerning the Motions and Magnitudes of the Bodies which are parts of the World, reall and existent, is that which followes.

PHYSIQVES, or the PHÆNOMENA of NATVRE.

CHAP. XXV.

Of Senſe *and* Animal Motion.

1 *The connexion of what hath been ſaid with that which followeth.* 2 *The inveſtigation of the nature of Senſe, and the Definition of Senſe.* 3 *The Subject and Object of Senſe.* 4 *The Organ of Senſe.* 5 *All Bodies are not indued with Senſe.* 6 *But one Phantaſme at one and the ſame time.* 7 *Imagination, the Remayns of paſt Senſe, (which alſo is Memory). Of Sleep.* 8 *How Phantaſmes ſucceed one another.* 9 *Dreames, whence they proceed.* 10 *Of the Senſes, their kindes, their Organs, and Phantaſmes proper and common.* 11 *The Magnitude of Images, how and by what it is determined.* 12 *Pleaſure, Pain, Appetite, and Averſion, what they are.* 13 *Deliberation and Will, what.*

1 I Have (in the firſt Chapter) defined *Philoſophy* to be *Knowledge of Effects acquired by true Ratiocination, from knowledge firſt had of their Cauſes and Generation; and of ſuch Cauſes or Generations as may be, from former knowledge of their Effects or Appearances.* There are therefore two Methods of Philoſophy, One from the Generation of things to their poſſible Effects, and the other from their Effects or Appearances to ſome poſſible Generation of the ſame. In the former of theſe, the Truth of the firſt Principles of our ratiocination (namely Definitions) is made and conſtituted by our ſelves, whileſt we conſent and agree about the Appellations of things. And this part I have finiſhed in the foregoing Chapters; in which (if I am not deceived) I have affirmed nothing (ſaving

the Definitions themselves) which hath not good coherence with the Definitions I have given; that is to say, which is not sufficiently demonstrated to all those that agree with me in the use of Words and Appellations; for whose sake onely I have written the same. I now enter upon the other part; which is the finding out by the Appearances or Effects of Nature which we know by Sense, some wayes and means by which they may be (I do not say, they are) generated. The Principles therefore, upon which the following discourse depends, are not such as we our selves make and pronounce in general terms, as Definitions; but such, as being placed in the things themselves by the Authour of Nature, are by us observed in them; and we make use of them in single and particular, not universal propositions. Nor do they impose upon us any necessity of constituting Theoremes; their use being onely (though not without such general Propositions as have been already demonstrated) to shew us the possibility of some production or generation. Seeing therefore the Science which is here taught, hath its Principles in the Appearances of Nature, and endeth in the attayning of some knowledge of Natural causes, I have given to this Part, the title of PHYSIQUES, or the PHÆNOMENA of NATURE. Now such things as *appear*, or are *shewn* to us by Nature, we call *Phænomena* or Appearances.

Of all the *Phænomena*, or *Appearances* which are neer us, the most admirable is *Apparition* it self, τὸ φαίνεσθαι; namely, that some Natural Bodies have in themselves the patterns almost of all things, and others of none at all. So that if the *Appearances* be the Principles by which we know all other things, we must needs acknowledge *Sense* to be the Principle by which we know those Principles; & that all the knowledge we have is derived from it. And as for the causes of *Sense*, we cannot begin our search of them from any other *Phænomenon* then that of *Sense* it self. But you will say, by what Sense shall we take notice of Sense? I answer, by Sense it self, namely, by the Memory which for some time remains in us of things sensible, though they themselves pass away. For he that perceives that he hath perceived, *remembers*.

In the first place therefore the causes of our Perception, that is, the causes of those Ideas and Phantasmes which are perpetually gene-

generated within us whilest we make use of our Senses, are to be enquired into; and in what manner their generation proceeds. To help which Inquisition, we may observe first of all, that our Phantasmes or Ideas are not alwayes the same; but that new ones appear to us, and old ones vanish, according as we apply our Organs of Sense, now to one Object, now to another. Wherefore they are generated, and perish. And from hence it is manifest, that they are some change or mutation in the Sentient.

2 Now that all Mutation or Alteration is Motion, or Endeavour (and Endeavour also is Motion) in the internal parts of the thing that is altered, hath been proved (in the 9th Article of the 8th Chapter) from this, that whilest even the least parts of any Body remain in the same situation in respect of one another, it cannot be said that any alteration (unless perhaps that the whole Body together hath been moved) hath hapned to it; but that it both appeareth and is the same it appeared & was before. Sense therefore in the Sentient, can be nothing else but motion in some of the internal parts of the Sentient; and the parts so moved, are parts of the Organs of Sense. For the parts of our Body by which we perceive any thing, are those we commonly call the *Organs of Sense*. And so we find what is the Subject of our Sense, namely, that in which are the Phantasmes; and partly also we have discovered the nature of Sense, namely, that it is some internal Motion in the Sentient.

I have shewn besides (in the 8th Chap. at the 7th Article) that no Motion is generated but by a Body contiguous and Moved. From whence it is manifest, that the immediate cause of Sense or Perception consists in this, that the first Organ of Sense is touched and pressed. For when the uttermost part of the Organ is pressed, it no sooner yeilds, but the part next within it, is pressed also; and in this manner, the pressure or Motion is propagated through all the parts of the Organ to the innermost. And thus also the pressure of the uttermost part proceeds from the pressure of some more remote Body, and so continually, till we come to that from which, as from its fountain we derive the Phantasme or Idea that is made in us by our Sense. And this, whatsoever it be, is that we commonly call the *Object*. Sense therefore is some internal Motion in

in the Sentient, generated by ſome internal Motion of the parts of the Object, and propagated through all the *Media* to the innermoſt part of the Organ. By which words I have almoſt defined what Senſe is.

Moreover, I have ſhewn (in the 2d Article of the 15 Chapter) that all Reſiſtance is Endeavour oppoſite to another Endeavour, that is to ſay, Reaction. Seeing therefore there is in the whole Organ by reaſon of its own internal natural Motion; ſome Reſiſtance or Reaction againſt the Motion which is propagated from the Object to the innermoſt part of the Organ, there is alſo in the ſame Organ an Endeavour oppoſite to the Endeavour which proceeds from the Object; ſo that when that Endeavour inwards is the laſt action in the act of Senſe, then from the Reaction, how little ſoever the duration of it be, a Phantaſme or Idea hath its being; which by reaſon the Endeavour is now outwards, doth alwayes appear as ſomething ſituate without the Organ. So that now I ſhall give you the whole Definition of Senſe, as it is drawn from the explication of the cauſes thereof, and the order of its generation, thus; SENSE *is a Phantaſme, made by the Reaction and endeavour outwards in the Organ of Senſe, cauſed by an Endeavour inwards from the Object, remayning for ſome time more or leſs.*

3 The *Subject* of Senſe, is the *Sentient* it ſelf, namely, ſome living Creature; and we ſpeak more correctly, when we ſay a *Living Creature* ſeeth, then when we ſay the *Eye* ſeeth. The *Object*, is the thing *Perceived*, and it is more accurately ſaid, that we ſee the Sun, then that we ſee the Light. For Light & Colour & Heat & Sound, and other qualities which are commonly called *Senſible*, are not *Objects*, but Phantaſms in the Sentients. For a Phantaſm is the act of Senſe, and differs no otherwiſe from Senſe then *fieri* (that is, Being a doing) differs from *Factum eſſe*, (that is, Being done;) which difference, in things that are done in an Inſtant, is none at all; and a Phantaſme is made in an Inſtant. For in all Motion which proceeds by perpetual propagation, the firſt part being moved moves the ſecond, the ſecond the third, and ſo on to the laſt, and that to any diſtance how great ſoever. And in what point of time the firſt or formoſt part proceeded to the place of the ſecond, which is thruſt on; in the ſame point of time the laſt ſave

one

one proceeded into the place of the last yeilding part; which by reaction, in the same instant, if the reaction be strong enough, makes a Phantasme; and a Phantasme being made, Perception is made together with it,

4 The *Organs* of Sense, which are in the Sentient, are such parts thereof, that if they be hurt, the very generation of Phantasmes is thereby destroyed, though all the rest of the parts remain intire. Now these parts in the most of Living Creatures are found to be certain Spirits and Membranes, which proceeding from the *Pia Mater*, involve the Brain and all the Nerves; also the Brain it self, and the Arteries which are in the Brain; and such other parts, as being stirred, the Hart also, which is the fountain of all Sense is stirred together with them. For whensoever the action of the Object reacheth the Body of the Sentient, that action is by some Nerve propagated to the Brain; and if the Nerve leading thither be so hurt or obstructed, that the Motion can be propagated no further, no Sense follows. Also if the motion be intercepted between the Brain and the Heart by the defect of the Organ by which the action is propagated, there will be no perception of the Object.

5 But though all Sense, as I have said, be made by Reaction, nevertheless it is not necessary, that every thing that Reacteth should have Sense. I know there have been Philosophers, & those learned men, who have maintained that all Bodies are endued with Sense, Nor do I see how they can be refuted, if the nature of Sense be placed in Reaction onely. And, though by the Reaction of Bodies inanimate a Phantasme might be made, it would nevertheless cease, as soon as ever the Object were removed. For unless those Bodies had Organs, (as living Creatures have) fit for the retaining of such Motion as is made in them, their Sense would be such, as that they should never remēber the same. And therefore this hath nothing to do with that Sense which is the subject of my discourse. For by Sense we commonly understand the judgement we make of Objects by their Phantasmes; namely, by comparing and distinguishing those Phantasmes; which we could never do, if that motion in the Organ, by which the Phantasme is made, did not remain there for some time, and make the same Phantasme return

turn. Wherefore Sense, as I here understand it, and which is commonly so called, hath necessarily some memory adhering to it, by which former and later Phantasmes may be compared together, and distinguished from one another.

Sense therefore properly so called, must necessarily have in it a perpetual variety of Phantasmes, that they may be discerned one from another. For if we should suppose a man to be made with cleer Eyes, and all the rest of his Organs of Sight well disposed, but endued with no other Sense; and that he should look onely upon one thing, which is alwayes of the same colour and figure without the least appearance of variety, he would seem to me, whatsoever others may say, to see, no more then I seem to my self to feel the Bones of my own Limbs by my Organs of Feeling; and yet those Bones are alwayes, and on all sides touched by a most sensible Membrane. I might perhaps say he were astonished, and looked upon it; but I should not say he saw it; it being almost all one for a man to be alwayes sensible of one and the same thing, and not to be sensible at all of any thing.

6 And yet such is the nature of Sense, that it does not permit a man to discern many things at once. For seeing the nature of Sense consists in Motion; as long as the Organs are employed about one Object, they cannot be so Moved by another at the same time, as to make by both their Motions one sincere Phantasme of each of them at once. And therefore two several Phantasmes will not be made by two Objects working together, but onely one Phantasme compounded from the action of both.

Besides, as when we divide a Body, we divide its place; and when we reckon many Bodies, we must necessarily reckon as many places; and contrarily, as I have shewn in the first Article of the 7th Chapter; so what number soever we say there be of Times, we must understand the same number of Motions also; and as oft as we count many Motions, so oft we reckon many times. For though the object we looke upon be of diverse colours, yet with those diverse colours it is but one varied Object, and not variety of Objects.

Moreover, whilest those Organs which are common to all the Senses (such as are those parts of every Organ which proceed (in Men)

Men) from the root of the Nerves, to the Hart) are vehemently stirred by a strong action from some one Object, they are (by reason of the contumacy which the motion they have already gives them against the reception of all other motion) made the lesse fit to receive any other impression from whatsoever other Objects, to what sense soever those Objects belong. And hence it is, that an earnest studying of one Object, takes away the Sense of all other Objects for the present. For *Study* is nothing else but a possession of the mind, that is to say, a vehement motion made by some one Object in the Organs of Sense, which are stupid to all other motions as long as this lasteth; according to what was said by *Terence*, *Populus studio stupidus in funambulo animum occuparat*. For what is *Stupor* but that which the Greekes call ἀναισθησία, that is, a cessation from the Sense of other things? Wherefore at one and the same time, we cannot by Sense perceive more then one single Object; as in reading, we see the letters successively one by one, and not all together, though the whole Page be presented to our eye; and though every severall letter be distinctly written there, yet when we looke upon the whole page at once, we read nothing.

From hence it is manifest, that every endeavour of the Organ outwards, is not to be called Sense, but that onely which at severall times is by Vehemence made stronger and more prædominant than the rest; which deprives us of the Sense of other Phantasmes, no otherwise then the Sun deprives the rest of the starres of light, not by hindering their action, but by obscuring and hiding them with his excesse of brightnesse.

7. But the motion of the Organ, by which a Phantasme is made, is not commonly called Sense, except the Object be present. And the Phantasme remaining after the Object is removed or past by, is called *Fancy*, and in latine *Imaginatio*; which word (because all Phantasmes are not Images) doth not fully answer the signification of the word *Fancy* in its generall acceptation. Neverthelesse I may use it safely enough, by understanding it for the Greek φαντασία.

IMAGINATION therefore is nothing else but *Sense decaying*, or *weakned*, by the absence of the Object. But what may be the cause of this decay or weakning? Is the Motion the weaker because the Ob-

Object is taken away? If it were, then Phantasmes would alwayes and necessarily be less cleare in the Imagination, then they are in Sense; which is not true. For in Dreams (which are the Imaginations of those that sleep) they are no less clear then in Sense it self. But the reason why in men Waking the Phantasms of things past are more obscure then those of things present, is this, that their Organs being at the same time moved by other present Objects, those Phantasmes are the lesse prædominant. Whereas in Sleep, the passages being shut up, externall action doth not at all disturbe or hinder internall motion.

If this be true, the next thing to be considered, will be, whether any cause may be found out, from the supposition whereof it will follow, that the passage is shut up from the externall Objects of Sense to the internall Organ. I suppose therefore, that by the continuall action of Objects, (to which a Reaction of the Organ, and more esqecially of the Spirits, is necessarily consequent) the Organ is wearied, that is, its parts are no longer moved by the Spirits without some pain; and consequently the Nerves being abandoned and grown slack, they retire to their fountain which is the cavity either of the Brain or of the Heart; by which means the action which proceeded by the Nerves is necessarily intercepted. For Action, upon a Patient that retires from it, makes but little Impression at the first; and at last, when the Nerves are by little and little slackned, none at all. And therefore there is no more Reaction, that is, no more Sense, till the Organ being refreshed by Rest, and by a supply of new Spirits recovering strength and motion, the Sentient awaketh. And thus it seems to be alwayes, unless some other præternatural cause intervene; as Heat in the internal parts from lassitude, or from some disease stirring the Spirits and other parts of the Organ in some extraordinary manner.

8 Now it is not without cause, nor so casual a thing as many perhaps think it, that Phantasmes in this their great variety, proceed from one another; and that the same Phantasmes sometimes bring into the mind other Phantasmes like themselves, and at other times extreamly unlike. For in the motion of any continued Body, one part followes another by cohæsion; and therefore, whilst we turne our Eies and other Organs successively to many

Objects

Objects, the motion which was made by every one of them remayning, the Phantasmes are renewed as often as any of those motions comes to be prædominant above the rest; and they become prædominant in the same order, in which at any time formerly they were generated by Sense. So that when by length of time very many Phantasmes have been generated within us by Sense, then allmost any thought may arise from any other thought; in so much that it may seeme to be a thing indifferent and casuall, which thought shall follow which. But for the most part this is not so uncertain a thing to waking as to sleeping men. For the thought or Phantasme of the desired End, brings in all the Phantasmes that are meanes conducing to that end, and that in order backewards from the last to the first, and againe forwards from the beginning to the End. But this supposes both Appetite, and Judgement to discerne what meanes conduce to the end; which is gotten by Experience; and Experience is store of Phantasmes, arising from the sense of very many things. For φαντάζεσθαι and *Meminisse*, *Fancy* and *Memory* differ onely in this, that Memory supposeth the time past, which Fancy doth not. In Memory, the Phantasmes we consider are as if they were worne out with time; but in our Fancy we consider them as they are; which distinction is not of the things themselves, but of the considerations of the Sentient. For there is in Memory, something like that which happens in looking upon things at a great distance; in which, as the small parts of the Object are not discerned by reason of their remotenesse; so in Memory, many accidents and places and parts of things, which were formerly perceived by Sense, are by length of time decayed and lost.

The perpetuall arising of Phantasmes, both in Sense and Imagination, is that which we commonly call *Discourse of the Mind*, and is common to men with other living Creatures. For he thta thinketh, compareth the Phantasmes that passe, that is, taketh notice of their likenesse or unlikenesse to one another. And as he that observes readily the likenesses of things of different natures, or that are very remote from one another, is said to have a good Fancy; so he is said to have a good Judgement, that finds out the unlikenesses or differences of things that are like one another.

Now this observation of differences, is not perception made by a common Organ of Sense, distinct from Sense or Perception properly so called; but is Memory of the differences of particular Phantasmes remaining for some time; as the distinction between Hot and Lucid, is nothing else but the Memory both of a Heating, and of an Enlightning Object.

9 The Phantasmes of men that sleep, are DREAMS. Concerning which we are taught by experience these five things. First, that for the most part there is neither order nor coherence in them. Secondly, that we dream of nothing but what is compounded and made up of the Phantasmes of Sense past. Thirdly, that somtimes they proceed (as in those that are drowsy) from the interruption of their Phantasmes by little and little broken and altered through sleepiness; and sometimes also they begin in the midst of sleep. Fourthly, that they are clearer then the Imaginations of waking men, except such as are made by Sense it self, to which they are equal in clearness. Fifthly, that when we dream, we admire neither the places nor the looks of the things that appear to us. Now from what hath been said, it is not hard to shew what may be the causes of these *Phænomena*. For as for the first, seeing all Order and Coherence proceeds from frequent looking back to the End, that is, from Consultation; it must needs be, that seeing in sleep we lose all thought of the End, our Phantasmes succeed one another, not in that order which tends to any End, but as it happeneth, and in such manner, as Objects present themselves to our Eyes when we look indifferently upon all things before us, and see them, not because we would see them, but because we do not shut our Eyes; for then they appear to us without any order at all. The second proceeds from this, that in the silence of Sense, there is no new motion from the Objects, and therefore no new Phantasme, unless we call that new, which is compounded of old ones, as a *Chimæra*, a golden Mountain, and the like. As for the third, why a Dream is sometimes as it were the continuation of Sense, made up of broken Phantasmes, as in men distempered with sickness, the reason is manifestly this, that in some of the Organs Sense remains, and in others it faileth. But how some Phantasmes may be revived, when all the exteriour Organs are benummed with sleep, is not so easily shewn. Neverthelesss, that which hath

hath already been ſaid, contains the reaſon of this alſo. For whatſoever ſtrikes the *Pia Mater*, reviveth ſome of thoſe Phantaſmes that are ſtill in motion in the Brain; and when any internal motion of the Heart reacheth that Membrane, then the prædominant motion in the Brain makes the Phantaſme. Now the Motions of the Heart are Appetites and Averſions, of which I ſhall preſently ſpeak further. And as Appetites and Averſions are generated by Phantaſmes, ſo reciprocally Phantaſmes are generated by Appetites and Averſions. For example, Heat in the Heart proceeds from Anger and Fighting; and again from Heat in the Heart, (whatſoever be the cauſe of it) is generated Anger, and the Image of an Enemy in Sleep. And as Love and Beauty ſtirre up heat in certain Organs; ſo Heat in the ſame Organs, from whatſoever it proceeds, often cauſeth Deſire, and the Image of an unreſiſting Beauty. Laſtly, Cold doth in the ſame manner generate Feare in thoſe that ſleep, and cauſeth them to dream of Ghoſts, and to have Phantaſmes of horrour and danger; as Fear alſo cauſeth Cold in thoſe that wake; ſo reciprocal are the motions of the Heart and Brain. The fourth, namely, that the things we ſeem to ſee and feel in ſleep, are as clear as in ſenſe it ſelf, proceeds from two cauſes; one, that having then no ſenſe of things without us, that internal motion which makes the Phantaſme, in the abſence of all other impreſſions, is prædominant; and the other, that the parts of our Phantaſms which are decayed and worn out by time, are made up with other fictitious parts. To conclude, when we dream, we do not wonder at ſtrange places, and the appearances of things unknown to us, becauſe Admiration requires that the things appearing be new and unuſual, which can happen to none but thoſe that remember former appearances; whereas in ſleep, all things appear as preſent.

But it is here to be obſerved, that certain Dreams, eſpecially ſuch as ſome men have when they are between ſleeping and waking, and ſuch as happen to thoſe that have no knowledge of the nature of Dreams, and are with all ſuperſtitious, were not heretofore, nor are now accounted Dreams. For the Apparitions men thought they ſaw, and the Voices they thought they heard in ſleep, were not believed to be Phantaſmes, but things ſubſiſting of them-

selves, and Objects without those that dreamed. For to some men, as well sleeping as waking, but especially to guilty men, and in the night, and in hallowed places, Feare alone, helped a little with the stories of such Apparitions, hath raised in their minds terrible Phantasmes, which have been, and are still deceiptfully received for things really true, under the names of *Ghosts* and *Incorporeal Substances*.

10 In most living Creatures there are observed five kinds of Senses, which are distinguished by their Organs, and by their different kinds of Phantasmes; namely, *Sight*, *Hearing*, *Smell*, *Tast* and *Touch*; and these have their Organs partly peculiar to each of them severally, and partly common to them all. The Organ of Sight is partly *animate*, and partly *inanimate*. The inanimate parts, are the three *Humours*; namely, the *Watry Humour*, which by the interposition of the Membrane called *Uvea*, (the perforation whereof is called the *Apple* of the *Eye*) is contained on one side by the first concave superficies of the Eye, and on the other side by the *Ciliary processes* and the Coat of the *Cristalline humour*; the *Cristalline*, which (hanging in the midst between the Ciliary processes, and being almost of Spherical figure, and of a thick consistence) is inclosed on all sides with its own transparent Coat; and the *Vitreous* or *Glassie Humour*, which filleth all the rest of the Cavity of the Eye, and is somewhat thicker then the Watry Humour, but thinner then the Cristalline. The *animate* part of the Organ is, first, the Membrane *Choroeides*, which is a part of the *Pia Mater*, saving that it is covered with a Coat derived from the marrow of the Optique Nerve, which is called the *Retina*; and this *Choroeides*, seeing it is part of the *Pia Mater*, is continued to the beginning of the *Medulla Spinalis* within the Scull, in which all the Nerves which are within the Head have their roots. Wherefore all the *Animal Spirits* that the Nerves receive, enter into them there; for it is not imaginable that they can enter into them any where else. Seeing therefore Sense is nothing else but the action of Objects propagated to the furthest part of the Organ; and seeing also that *Animal Spirits* are nothing but *Vital Spirits* purified by the Hart and carried from it by the Arteries; it follows necessarily, that the action is derived from the Heart by some of the Arteries to the roots of the Nerves

which

which are in the Head, whether those Arteries be the *Plexus Retiformis*, or whether they be other Arteries which are inserted into the substance of the Brain. And therefore those Arteries are the Complement, or the remaying part of the whole Organ of Sight. And this last part is a common Organ to all the Senses; wheras that which reacheth from the Eie to the roots of the Nerves is proper onely to Sight. The proper Organ of Hearing is the *Tympanum* of the Eare, and its own Nerve; from which to the Heart the Organ is Common. So the proper Organs of Smel & Tast are Nervous Membranes, in the Palate and Tongue for the Taste, and in the Nostrils for the Smell; and from the roots of those Nerves to the Heart all is common. Lastly, the proper Organ of Touch are Nerves and Membranes dispersed through the whole Body; which Membranes are derived from the root of the Nerves. And all things else belonging alike to all the Senses seem to be administred by the Arteries, and not by the Nerves.

The proper Phantasme of Sight is *Light*; and under this name of Light, *Colour* also (which is nothing but perturbed Light) is comprehended. Wherefore the Phantasme of a Lucid Body, is Light; and of a coloured Body, Colour. But the Object of Sight, properly so called is neither Light nor Colour, but the Body it self which is lucid, or enlightned, or coloured. For Light and Colour being Phantasmes of the Sentient, cannot be Accidents of the Object. Which is manifest enough from this, that Visible things appear oftentimes in places, in which we know assuredly they are not, and that in different places they are of different colours, and may at one and the same time appear in divers places. Motion, Rest, Magnitude and Figure are common both to the Sight and Touch; and the whole appearance together of *Figure, and Light or Colour*, is by the Greeks commonly called εἶδος, & εἴδωλον, & ἰδέα; and by the Latines, *Species* and *Imago*; all which names signifie no more but *Appearance*.

The phantasme which is made by Hearing, is *Sound*; by Smell, *Odour*; by Tast, *Savour*; and by Touch, *Hardness* and *Softness*, *Heat* and *Cold*, *Wetness*, *Oiliness*, and many more, which are easier to be distinguished by sense then words. *Smoothness*, *Roughness*, *Rarity* and *Density* refer to Figure, and are therefore common both to

Touch

Touch and Sight. And as for the Objects of Hearing, Smel, Tast, and Touch, they are not Sound, Odour, Savour, Hardness, &c. but the Bodies themselves from which Sound, Odour, Savour, Hardness, &c. proceed; Of the causes of which, and of the manner how they are produced, I shall speak hereafter.

But these Phantasmes, though they be effects in the Sentient, (as Subject) produced by Objects working upon the Organs; yet there are allso other effects besides these, produced by the same Objects, in the same Organs; namely, certain Motions proceeding from Sense, which are called *Animal Motions*. For seeing in all Sense of external things, there is mutual Action and Reaction, that is, two Endeavours opposing one another, it is manifest, that the motion of both of them together will be continued every way, especially to the confines of both the Bodies. And when this happens in the internal Organ, the Endeavour outwards will proceed in a solid Angle, which will be greater, and consequently the Idea greater, then it would have been if the impression had been weaker.

11 From hence the Natural cause is manifest, First, why those things seem to be greater, which (*cæteris paribus*) are seen in a greater Angle. Secondly, why in a serene cold night when the Moon doth not shine, more of the fixed Stars appear, then at another time. For their action is less hindred by the serenity of the Aire, and not obscured by the greater Light of the Moon, which is then absent; and the Cold making the Air more pressing, helpeth or strengtheneth the action of the Stars upon our Eies, in so much as Stars may then be seen which are seen at no other time. And this may suffice to be said in general concerning Sense made by the Reaction of the Organ. For, as for the place of the Image, the deceptions of Sight, and other things of which we have experience in our selves by Sense, being they depend for the most part upon the Fabrick it self of the Eie of Man, I shall speak of them then when I come to speak of Man.

12 But, there is another kind of Sense, of which I will say somthing in this place, namely, the Sense of *Pleasure* and *Pain*, proceeding, not from the Reaction of the Heart outwards, but from continual action from the outermost part of the Organ towards the

Heart.

Heart. For the original of Life being in the Heart, that motion in the Sentient which is propagated to the Heart, must necessarily make some alteration or diversion of Vital Motion, namely, by quickning or slackening, helping or hindering the same. Now when it helpeth, it is Pleasure, and when it hindereth, it is Pain, Trouble, Grief, &c. And as Phantasmes seem to be without, by reason of the Endeavour outwards; so Pleasure and Pain by reason of the Endeavour of the Organ inwards seem to be within, namely, there where the first Cause of the Pleasure or Pain is; as when the Pain proceeds from a Wound, we think the Pain and the Wound are both in the same place.

Now Vital Motion, is the Motion of the Bloud, perpetually circulating (as hath been shewn from many infallible signes and marks by Doctor *Harvey*, the first Observer of it) in the Veins and Arteries. Which Motion, when it is hindered by some other Motion made by the action of sensible Objects, may be restored again either by bending or setting straight the parts of the Body; which is done when the Spirits are carried now into these, now into other Nerves, till the Pain (as farre as is possible) be quite taken away. But if Vital Motion be helped by Motion made by Sense, then the parts of the Organ will be disposed to guide the Spirits in such manner, as conduceth most to the preservation and augmentation of that motion by the help of the Nerves. And in animal motion this is the very first Endeavour, and found even in the *Embrio*; which while it is in the wombe, moveth its limbes with voluntary motion, for the avoiding of whatsoever troubleth it, or for the pursuing of what pleaseth it. And this first Endeavour, when it tends towards such things as are known by experience to be pleasant, is called APPETITE, (that is, an Approaching;) and when it shuns what is troublesome, AVERSION, or Flying from it. And little Infants, at the beginning, and as soon as they are born, have appetite to very few things, as also they avoid very few, by reason of their want of Experience and Memory; & therefore they have not so great a variety of animal Motion as we see in those that are more grown. For it is not possible, without such knowledge as is derived from Sense, that is, without Experience and Memory, to know what will prove pleasant, or hurtful; onely there is some

place-

place for conjecture from the looks or aspects of things. And hence it is, that though they do not know what may do them good or harm, yet sometimes they approach, and sometimes retire from the same thing as their doubt prompts them. But afterwards by accustoming themselves by little and little, they come to know readily what is to be pursued, and what to be avoided; and also to have a ready use of their Nerves and other Organs in the pursuing and avoiding of good and bad. Wherefore *Appetite* and *Aversion* are the first Endeavours of *Animal Motion*.

Consequent to this first Endeavour, is the Impulsion into the Nerves, and Retraction again of Animal Spirits, of which it is necessary there be some Receptacle or place neer the original of the Nerves; and this Motion or Endeavour is followed by a swelling and Relaxation of the Muscles, and lastly, these are followed by Contraction and Extension of the limbes, which is Animal Motion.

13. The Considerations of Appetites and Aversions are divers. For seeing Living Creatures have sometimes Appetite, and sometimes Aversion to the same thing, as they think it will either be for their good or their hurt, while that vicissitude of Appetites and Aversions remains in them, they have that series of Thoughts which is called DELIBERATION, which lasteth as long as they have it in their power to obtain that which pleaseth, or to avoid that which displeaseth them. *Appetite* therefore and *Aversion* are simply so called as long as they follow not Deliberation. But if Deliberation have gone before, then the last act of it, if it be Appetite, is called WILL; if Aversion, UNWILLINGNESSE; so that the same thing is called both *Will* and *Appetite*; but the consideration of them (namely, before and after Deliberation) is divers. Nor is that which is done within a Man whilest he Willeth any thing, different from that which is done in other living Creatures, whilest (Deliberation having preceded) they have Appetite.

Neither is the freedome of Willing, or not willing, greater in Man, then in other living Creatures. For where there is Appetite, the entire cause of Appetite hath preceded; and consequently the act of Appetite could not choose but follow, that is, hath of necessity followed, (as is shewn Chapt. 9th Article 5.) And therefore

fore such a *Liberty* as is free from *Necessity*, is not to be found in the *Will* either of Men or Beasts. But if by *Liberty* we understand the faculty or power, not of *Willing*, but of *Doing* what they Will, then certainly that Liberty is to be allowed to both; and both may equally have it, whensover it is to be had.

Again, when Appetite and Aversion do with celerity succeed one another, the whole series made by them, hath its name sometimes from one, sometimes from the other. For the same Deliberation (whilest it inclines sometimes to one, sometimes to the other) is from Appetite called HOPE, and from Aversion FEAR. For where there is no Hope, it is not to be called Fear, but HATE; and where no Fear, not Hope, but DESIRE. To conclude, all the Passions, called *Passions of the Minde*, consist of Appetite and Aversion (except pure Pleasure and Pain, which are a certain *Fruition* of good or Evil;) as *Anger*, is Aversion from some imminent evil, but such as is joyned with Appetite of avoiding that evil by force. But because the Passions and Perturbations of the Minde are innumerable; and many of them not to be discerned in any Creatures besides Men; I will speak of them more at large in that Section which is concerning *Man*. As for those Objects (if there be any such) which do not at all stir the Mind, we are said to *Contemn* them.

And thus much of Sense in general. In the next place I shall speak of Sensible Objects.

CHAP. XXVI.

Of the World, and of the Starres.

1 *The Magnitude and Duration of the World, inscrutable.* 2 *No place in the World Empty.* 3 *The arguments of Lucretius for Vacuum, invalid.* 4 *Other arguments for the establishing of Vacuum, invalid.* 5 *Six suppositions for the salving of the Phænomena of Nature.* 6 *Possible causes of the Motions Annual and Diurnal; and of the apparent Direction, Station and Retrogradation of the Planets.* 7 *The supposition of Simple Motion, why likely.* 8 *The cause of the Excentricity of the annual motion of the Earth.* 9 *The cause why the Moon hath alwayes one and the same face turned towards the Earth.* 10 *The cause of the Tides of the Ocean.* 11 *The cause of the Præcession of the Equinoxes.*

1 Consequent to the Contemplation of *Sense* is the contemplation of *Bodies*; which are the efficient causes or Objects of Sense. Now every *Object* is either a part of the whole World, or an Aggregate of parts. The greatest of all Bodies, or sensible Objects, is the *World* it self, which we behold when we look round about us from this point of the same which we call the Earth. Concerning the World, as it is one Aggregate of many parts, the things that fall under inquiry are but few; and those we can determine, none. Of the whole World we may inquire what is its *Magnitude*, what its *Duration*, and how many there be; but nothing else. For as for *Place* and *Time*, that is to say, Magnitude and Duration, they are only our own fancies of a Body simply so called, that is to say of a Body indefinitely taken, as I have shewne before in the 7 chapter. All other Phantasmes, are of Bodies or Objects as they are distinguished from one another; as Colour, the Phantasme of coloured Bodies; Sound, of Bodies that move the Sense of Hearing, &c. The questions concerning the Magnitude of the World, are, whether it be *Finite* or *Infinite*, *Full* or *not Full*; Concerning its Duration, whether it had a *Beginning*, or be *Eternall*; and concerning the num-

number, whether there be *One* or *Many* (though as concerning the Number, if it were of infinite Magnitude, there could be no controversy at all.) Also if it had a beginning, then by what *Cause* and of what *Matter* it was made; and againe, from whence that Cause and that Matter had their being, will be new questions; till at last we come to one or many eternall Cause or Causes. And the determination of all these things belongeth to him that professeth the universal doctrine of Philosophy, in case as much could be known as can be sought. But the knowledge of what is Infinite can never be attained by a finite Inquirer. Whatsoever we know that are Men, we learn it from our Phantasmes; and of *Infinite* (whether Magnitude or Time) there is no Phantasme at all; so that it is impossible either for a man, or any other creature to have any conception of *Infinite*. And though a man may from some Effect proceed to the immediate Cause thereof, & frō that to a more remote Cause, and so ascend continually by right ratiocination from Cause to Cause; yet he will not be able to proceed eternally; but wearied, will at last give over, without knowing whether it were possible for him to proceed to an end, or not. But whether we suppose the World to be Finite, or Infinite, no absurdity will follow. For the same things which now appear, might appear, whether the Creator had pleased it should be Finite or Infinite. Besides, though from this, that nothing can move it self, it may rightly be inferred that there was some first eternal Movent; yet it can never be inferred (though some use to make such inference) that that Movent was eternally Immoveable, but rather eternally Moved. For as it is true, that nothing is moved by it self, so it is true also that nothing is moved but by that which is already moved. The questions therefore about the *Magnitude* and *Beginning* of the World, are not to be determined by Philosophers, but by those that are lawfully authorised to order the Worship of God. For as Almighty God, when he had brought his People into *Judæa*, allowed the Priests the first fruits reserved to himself; so when he had delivered up the World to the disputations of Men, it was his pleasure, that all Opinions concerning the nature of *Infinite* and *Eternal*, known onely to himself, should (as the first-fruits of Wisdom) be judged by those whose Ministery he meant to use in the ordering of Religion. I cannot therefore

commend those that boast they have demonstrated by reasons drawn from natural things, that the World had a Beginning. They are contemned by Idiots, because they understand them not; and by the Learned, because they understand them; by both deservedly. For who can commend him that demonstrates thus? If the World be Eternal, then an infinite number of dayes (or other measures of Time) preceded the birth of *Abraham*. But the birth of *Abraham* preceded the birth of *Isaac*; and therefore one Infinite is greater then another Infinite, or one Eternal then another Eternal; which (he sayes) is absurd. This Demonstration is like his, who from this, that the number of even Numbers is infinite, would conclude, that there are as many even Numbers, as there are Numbers simply, that is to say, the even Numbers are as many, as all the even and od together. They which in this manner take away Eternity from the World, do they not by the same means take away Eternity from the Creator of the World? From this absurdity therefore they run into another, being forced to call Eternity *Nunc stans*, a standing still of the present Time, or *an abiding Now*; and (which is much more absurd) to give to the infinite number of Numbers, the name of *Unity*. But why should Eternity be called an *Abiding Now*, rather then an *Abiding Then*? Wherefore there must either be many Eternities, or *Now* and *Then* must signifie the same. With such Demonstrators as these, that speak in another language, it is impossible to enter into disputation. And the men that reason thus absurdly, are not Idiots, but (which makes the absurdity unpardonable) Geometricians, and such as take upon them to be Judges (impertinent, but severe Judges) of other mens Demonstrations. The reason is this, that as soon as they are entangled in the Words *Infinite* and *Eternal*, (of which we have in our mind no Idea, but that of our own insufficiency to comprehend them) they are forced either to speak something absurd, or (which they love worse) to hold their peace. For Geometry hath in it somewhat like Wine; which when new, is windy; but afterwards though less pleasant, yet more wholsome. Whatsoever therefore is true, young Geometricians think Demonstrable; but elder not. Wherefore I purposely pass over the Questions of Infinite and Eternal; contenting my self with that Doctrine con-

concerning the Beginning and Magnitude of the World, which I have been perswaded to by the holy Scriptures, and fame of the Miracles which confirm them; and by the Custome of my Country, and reverence due to the Laws. And so I pass on to such things as it is not unlawful to dispute of.

2 Concerning the World it is further questioned, whether the parts thereof be contiguous to one another, in such manner, as not to admit of the least empty space between; and the disputation both for & against it, is carried on with probability enough. For the taking away of *Vacuum*, I will instance in onely one experiment, a common one, but (I think) unanswerable.

Let A B (in the first fig.) represent a vessel (such as Gardiners use to water their Gardens withal;) whose bottom B is ful of litle holes; & whose mouth A may be stopt with ones finger when there shall be need. If now this vessel be filled with water, the hole at the top A being stopt, the water will not flow out at any of the holes in the bottom B. But if the finger be removed to let in the air above, it will run out at them all, and as soon as the finger is applyed to it again, the water wil suddenly & totally be stayed again from running out. The cause whereof seems to be no other but this, that the Water cannot by its natural endeavour to descend, drive down the aire below it, because there is no place for it to go into, unless either by thrusting away the next contiguous aire it proceed by continual endeavour to the hole A, where it may enter and succeed into the place of the water that floweth out; or else by resisting the endeavour of the water Downwards, penetrate the same, and pass up through it. By the first of these wayes (while the hole at A remains stopped) there is no possible passage; nor by the second, unless the holes be so great, that the water flowing out at them, can by its own waight force the Air at the same time to ascend into the vessel by the same holes; as we see it does in a vessel whose mouth is wide enough, when we turn suddenly the bottom upwards to poure out the water; for then the Aire being forced by the waight of the water, enters (as is evident by the sobbing and resistance of the water) at the sides or circumference of the orifice. And this I take for a sign that all Space is full; for without this the naturall motion of the water (which is a heavy Body) downwards, would not be hindered.

3 On

3 On the contrary, for the eſtabliſhing of *Vacuum*, many & ſpecious arguments and experiments have been brought. Neverthelesſe there ſeemes to be ſomething wanting in all of them to conclude it firmely. Theſe arguments for *Vacuum* are partly made by the followers of the doctrine of *Epicurus*; who taught that the World conſiſts of very ſmall Spaces not filled by any Bodie, and of very ſmall Bodies that have within them no empty Space, (which by reaſon of their hardneſſe he calls *Atomes*); and that theſe ſmall Bodies and Spaces are every where intermingled. Their arguments are thus delivered by *Lucretius*.

And firſt he ſayes that unleſſe it were ſo, there could be no motion; For the office and property of Bodies is to withſtand and hinder motion. If therfore the Univerſe were filled with Body, motion would every where be hindered, ſo, as to have no beginning any where; & conſequently there would be no motion at all. It is true that in whatſoever is full, and at reſt in all its parts, it is not poſſible motion ſhould have beginning. But nothing is drawn from hence for the proving of *Vacuum*. For though it ſhould be granted that there is *Vacuum*; yet if the Bodies which are intermingled with it, ſhould all at once and together be at reſt, they would never be moved again. For it has been demonſtrated above (in the 9th Chapter 7th Article) that nothing can be moved but by that which is contiguous and already moved. But ſuppoſing that all things are at reſt together, there can be nothing contiguous and moved; and therefore no beginning of motion. Now the denying of the beginning of motion, doth not take away preſent motion, unleſs beginning be taken away from Body alſo. For motion may be either coeternal, or concreated with Body. Nor doth it ſeem more neceſſary that Bodies were firſt at reſt, and afterwards moved, then that they were firſt moved, and reſted (if ever they reſted at all) afterwards. Neither doth there appear any cauſe, why the matter of the World ſhould for the admiſſion of motion, be intermingled with empty ſpaces, rather then full; I ſay full, but withall fluid. Nor laſtly, is there any reaſon why thoſe hard Atomes may not alſo by the motion of intermingled fluid matter be congregated & brought together into compounded Bodies of ſuch bigneſs as we ſee. Wherefore nothing can by this argument be con-

concluded, but that motion was either coeternal, or of the ſame duration with that which is moved; neither of which concluſions conſiſteth with the doctrine of *Epicurus*, who allows neither to the World nor to Motion any Beginning at all. The neceſſity therefore of *Vacuum* is not hitherto demonſtrated. And the cauſe (as far as I underſtand from them that have diſcourſed with me of *Vacuum*) is this, that whileſt they contemplate the nature of *Fluid*, they conceive it to conſiſt as it were of ſmall grains of hard matter, in ſuch manner as meal is fluid, made ſo by grinding of the Corn; when neverthleſs it is poſſible to conceive *Fluid* to be of its own nature as homogeneous, as either an Atome, or as *Vacuum*, it ſelf.

The ſecond of their arguments is taken from waight, and is contained in theſe Verſes of *Lucretius*,

Corporis officium eſt quoniam premere omnia deorſum;
Contrà autem natura manet ſine Pondere Inanis;
Ergo quod magnum eſt æque, Leviuſque videtur,
Nimirum plus eſſe ſibi declarat Inanis.

That is to ſay, *Seeing the office and property of Body is to preſs all things downwards; and on the contrary, ſeeing the nature of* Vacuum *is to have no waight at all; Therefore when of two Bodies of equal magnitude, one is lighter then the other, it is manifeſt that the lighter Body hath in it more* Vacuum *then the other.*

To ſay nothing of the Aſſumption concerning the endeavour of Bodies downwards, which is not rightly aſſumed, becauſe the World hath nothing to do with *Downwards*, which is a mere fiction of ours; Nor of this, that if al things tended to the ſame loweſt part of the World, either there would be no coaleſcence at all of Bodies, or they would all be gathered together into the ſame place. This onely is ſufficient to take away the force of the argument, that Aire intermingled with thoſe his Atomes, had ſerved as well for his purpoſe, as his intermingled *Vacuum*.

The third argument is drawn from this, That Lightning, Sound, Heat and Cold do penetrate all Bodies (except Atomes) how ſolid ſoever they be. But this reaſon, except it be firſt demonſtrated that the ſame things cannot happen (without *Vacuum*) by perpetual

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generation of Motion, is altogether invalid. But that all the same things may so happen, shall in due place be demonstrated.

Lastly, the fourth argument is set down by the same *Lucretius* in these Verses.

Duo de concursu corpora lata
Si citò dissiliant, nempe aer omne necesse est
Inter corpora quod fuerat, possidat Inane.
Is porro quamvis circum celerantibus auris
Confluat, haud poterit tamen uno tempore totum
Compleri spatium; nam primum quemque necesse est
Occupet ille locum, deinde omnia possideantur.

That is, *If two flat Bodies be suddenly pulled asunder, of necessity the Air must come between them to fill all the space they left empty. But with what celerity soever the Air flow in, yet it cannot in one instant of time fill the whole space, but first one part of it, then successively all.*

Which neverthelesse is more repugnant to the opinion of *Epicurus*, then of those that deny *Vacuum*. For though it be true, that if two Bodies were of infinite hardness, and were joyned together by their Superficies which were most exactly plain, it would be impossible to pull them asunder, in regard it could not be done but by Motion in an instant; yet, if as the greatest of all Magnitudes cannot be given, nor the swiftest of all Motions, so neither the hardest of all hard Bodies; it might be, that by the application of very great force, there might be place made for a successive flowing in of the Aire, namely by separating the parts of the joyned Bodies by succession, beginning at the outermost and ending at the innermost part. He ought therefore first to have proved, that there are some Bodies extreamly hard, not relatively, as compared with softer Bodies, but absolutely, that is to say, infinitely hard; which is not true. But if we suppose (as *Epicurus* doth) that Atomes are indivisible, and yet have small superficies of their own; then if two Bodies should be joyned together by many, or but one onely small superficies of either of them, then I say this argument of *Lucretius* would be a firme demonstration, that no two Bodies made up of Atomes (as he supposes) could ever possibly be pulled asunder by any

any force whatsoever. But this is repugnant to daily experience.

And thus much of the arguments of *Lucretius*. Let us now consider the arguments which are drawn from the experiments of later Writers.

4 The first experiment is this, That if a hollow vessel be thrust into water with the bottom upwards, the water will ascend into it; which they say it could not do, unless the Aire within were thrust together into a narrower place; and that this were also impossible except there were little empty places in the Aire. Also that when the Aire is compressed to a certain degree, it can receive no further compression, its small particles not suffering themselves to be pent into less room. This reason, if the Aire could not pass through the Water as it ascends within the vessel, might seem valid. But it is sufficiently known, that Aire will penetrate Water by the application of a force equal to the gravity of the Water. If therefore the force by which the Vessel is thrust down, be greater, or equal to the endeavour by which the water naturally tendeth downwards, the Aire will go out that way where the resistance is made, namely, towards the edges of the Vessel. For, by how much the deeper is the water which is to be penetrated, so much greater must be the depressing force. But after the Vessel is quite under water, the force by which it is depressed, that is to say, the force by which the water riseth up, is no longer encreased. There is therefore such an equilibration between them, as that the natural endeavour of the water downwards, is equal to the endeavour by which the same water is to be penetrated to the encreased depth.

The second experiment is, That if a concave Cylinder of sufficient length (made of Glass, that the experiment may be the better seen) having one end open, and the other close shut, be filled with Quicksilver, and the open end being stopped with ones finger, be together with the finger dipped into a dish or other vessel in which also there is Quicksilver, and the Cylinder be set upright, we shall, the finger being taken away (to make way for the descent of the Quicksilver) see it descend into the Vessel under it, till there be onely so much remayning within the Cylinder as may fill about 26 Inches of the same; and thus it will alwayes happen whatsoever be the Cylinder, provided that the length be

not leſs then 26 Inches. From whence they conclude that the cavity of the Cylinder above the Quickſilver remayns empty of all Body. But in this experiment I finde no neceſſity at all of *Vacuum*. For when the Quickſilver which is in the Cylinder deſcends, the Veſſel under it muſt needs be filled to a greater height, and conſequently ſo much of the contiguous Air muſt be thruſt away as may make place for the Quickſilver which is deſcended. Now if it be asked whether that Aire goes, what can be anſwered but this, that it thruſteth away the next Aire, & that the next, & ſo ſucceſſively, till there be a return to the place where the propulſion firſt began? and there, the laſt Aire thus thruſt on will preſs the Quickſilver in the Veſſel with the ſame ſorce with which the firſt Aire was thruſt away; and if the force with which the Quickſilver deſcends be great enough (which is greater or leſs, as it deſcends from a place of greater or leſs height) it will make the Aire penetrate the Quickſilver in the veſſel, and go up into the Cylinder to fill the place which they thought was left empty. But becauſe the Quickſilver hath not in every degree of height force enough to cauſe ſuch penetration, therefore in deſcending it muſt of neceſſity ſtay ſomewhere, namely there, where its endeavour downwards, and the reſiſtance of the ſame to the penetration of the Aire come to an *æquilibrium*. And by this experiment it is manifeſt, that this *æquilibrium* will be at the height of 26 Inches, or thereabouts.

The third experiment is, That when a Veſſel hath as much Air in it as it can naturally contain, there may neverthel eſs be forced into it as much water as will fill three quarters of the ſame Veſſel. And the experiment is made in this manner. Into the glaſs bottle, repreſented (in the 2d figure) by the Sphere F G, whoſe center is A, let the pipe B A C be ſo fitted, that it may preciſely fill the mouth of the bottle; and let the end B, be ſo neer the bottom, that there may be onely ſpace enough left for the free paſſage of the water which is thruſt in above. Let the upper end of this pipe have a Cover at D, with a ſpout at E, by which the water (when it aſcends in the pipe) may run out. Alſo let H C be a Cock, for the opening or ſhutting of the paſſage of the water between B and D, as there ſhall be occaſion. Let the Cover D E be taken off; and the Cock H C being opened, let a Syringe full of water be forced

ced in; and before the Syringe be taken away, let the Cock be turned to hinder the going out of the Aire. And in this manner let the injection of water be repeated as often as it ſhall be requiſite, till the water riſe within the bottle, for example, to G F. Laſtly, the Cover being faſtened on again, and the Cock H C opened, the water will run ſwiftly out at E, and ſink by little and little from G F to the bottom of the pipe B.

From this *Phænomenon* they argue for the neceſſity of *Vacuum* in this manner. The Bottle from the beginning was full of Aire; which Aire could neither go out by penetrating ſo great a length of water as was injected by the pipe, nor by any other way. Of neceſſity therefore all the water as high as F G, as alſo all the Aire that was in the bottle before the water was forced in, muſt now be in the ſame place, which at firſt was filled by the Aire alone; which were impoſſible, if all the ſpace within the bottle were formerly filled with Aire preciſely, that is, without any *Vacuum*. Beſides, though ſome man perhaps may think the Air, being a thinne Body, may paſs through the Body of the water contained in the pipe, yet from that other *Phænomenon*, (namely, that all the water which is in the ſpace B F G, is caſt out again by the ſpout at E, for which it ſeems impoſſible that any other reaſon can be given beſides the force by which the Aire frees it ſelf from compreſſion) it follows, that either there was in the bottle ſome ſpace empty, or that many Bodies may be together in the ſame place. But this laſt is abſurd; and therefore the former is true, namely, that there was *Vacuum*.

This argument is infirm in two places. For firſt that is aſſumed which is not to be granted; and in the ſecond place an experiment is brought, which I think is repugnant to *Vacuum*. That which is aſſumed is, that the Aire can have no paſſage out through the pipe. Nevertheleſs we ſee daily that Aire eaſily aſcends from the bottom to the Superficies of a River (as is manifeſt by the bubbles that riſe); nor doth it need any other cauſe to give it this motion, then the natural endeavour downwards of the Water. Why therefore may not the endeavour upwards of the ſame Water acquired by the injection (which endeavour upwards is greater then the natural endeavour of the water downwards) cauſe the aire in the bottle to penetrate in like manner the water that preſſeth

it downwards; especially seeing the water as it riseth in the bottle, doth so press the Aire that is above it, as that it generateth in every part thereof an endeavour towards the external superficies of the pipe, and consequently maketh all the parts of the enclosed aire to tend directly towards the passage at B? I say this is no less manifest, then that he aire which riseth up from the bottom of a River should penetrate the water, how deep soever it be. Werefore I do not yet see any cause why the force by which the water is injected, should not at the same time eject the aire.

And as for their arguing the necessity of *Vacuum* from the rejection of the water; In the first place, supposing there is *Vacuum*, I demand by what principle of motion that ejection is made. Certainly, seeing this motion is from within outwards, it must needs be caused by some Agent within the bottle; that is to say, by the aire it self. Now the motion of that aire, being caused by the rising of the water, begins at the bottom, and tends upwards; whereas the motion by which it ejecteth the water ought to begin above, and tend downwards. From whence therefore hath the enclosed aire this endeavour towards the bottom. To this question I know not what answer can be given, unless it be said, that the aire descends of its own accord to expel the water. Which because it is absurd, and that the aire after the water is forced in, hath as much room as its magnitude requires, there will remain no cause at all why the water should be forced out. Wherefore the assertion of *Vacuum* is repugnant to the very experiment which is here brought to establish it.

Many other *Phænomena* are usually brought for *Vacuum*, as those of *Weather-glasses*, *Æolipiles*, *Wind-guns*, &c. Which would all be very hard to be salved, unless water be penetrable by aire, without the intermixture of empty space. But now, seeing aire may with no great endeavour pass through, not onely water, but any other fluid Body, though never so stubborn, as Quicksilver, these *Phænomena* prove nothing. Neverthelesss, it might in reason be expected, that he that would take away *Vacuum*, should without *Vacuum* shew us such causes of these *Phænomena*, as should be at least of equal, if not greater probability. This therefore shall be done in the following discourse, when I come to speak of these *Phænomena* in their

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proper places. But first the most general *Hypotheses* of natural Philosophy are to be premised.

And seeing that Suppositions are put for the true Causes of apparent Effects, every Supposition, except such as be absurd, must of necessity consist of some supposed possible Motion (for Rest can never be the Efficient Cause of any thing), & Motion supposeth Bodies Moveable; of which there are three kinds, *Fluid*, *Consistent*, and *mixt of both*. *Fluid* are those, whose parts may by very weak endeavour be separated from one another; and *Consistent* those, for the separation of whose parts greater force is to be applyed. There are therefore degrees of Consistency; which degrees, by comparison with more or less Consistent, have the names of *Hardness*, or *Softness*. Wherefore a Fluid Body is alwayes divisible into Bodies equally Fluid, as Quantity into Quantities; and Soft Bodies, of whatsoever degree of Softness, into Soft Bodies of the same degree. And though many men seem to conceive no other difference of *Fluidity*, but such as ariseth from the different magnitudes of the parts (in which Sense, Dust, though of Diamonds, may be called Fluid); Yet I understand by *Fluidity*, that which is made such by Nature equally in every part of the Fluid Body; not as Dust is Fluid, for so a House which is falling in pieces may be called Fluid; but in such manner as Water seems Fluid, and to divide it self into parts perpetually Fluid. And this being well understood, I come to my Suppositions.

5 First, therefore I suppose, That the Immense Space which we call the World, is the Aggregate of all Bodies; which are either Consistent & Visible, as the Earth and the Starres; or Invisible, as the small Atomes which are disseminated through the whole space between the Earth and the Stars; and lastly, that most Fluid *Æther*, which so fils all the rest of the Universe, as that it leaves in it no empty place at all.

Secondly, I suppose with *Copernicus*, That the greater Bodies of the World, which are both consistent and permanent, have such order amongst themselves, as that the *Sunne* hath the first place, *Mercury* the second, *Venus* the third, *The Earth* (with the *Moon* going about it) the fourth, *Mars* the fifth, *Jupiter* (with his Attendants) the sixth, *Saturne* the seventh, and after these the *Fixed Starres* have their several distances from the Sunne.

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Thirdly, I suppose, That in the Sunne & the rest of the Planets, there is and alwayes has been a Simple Circular Motion.

Fourthly, I suppose, That in the Body of the Aire there are certain other Bodies intermingled, which are not Fluid; but withal that they are so small, that they are not preceptible by Sense, and that these also have their proper Simple Motion; and are some of them more, some less hard or consistent.

Fifthly, I suppose with *Kepler*, That as the distance between the Sunne and the Earth, is to the distance between the Moon and the Earth; so the distance between the Moon and the Earth, is to the Semidiameter of the Earth.

As for the Magnitude of the Circles, and the Times in which they are described by the Bodies which are in them, I will suppose them to be such as shall seem most agreeable to the *Phænomena* in question,

6 The causes of the different Seasons of the Year, and of the several variations of Dayes and Nights in all the parts of the superficies of the Earth, have been demonstrated first by *Copernicus*, and since by *Kepler*, *Galilæus* and others, from the supposition of the Earths diurnal revolution about its own Axis, together with its Annual motion about the Sunne, in the Ecliptick according to the order of the Signes, and thirdly, by the annual revolution of the same Earth about its own center contrary to the order of the Signs. I suppose with *Copernicus*, That the diurnal revolution is from the motion of the Earth, by which the Æquinoctial Circle is described about it. And as for the other two annual motions, they are the efficient cause of the Earths being carried about in the Ecliptick in such manner, as that its Axis is alwayes kept parallel to it self. Which parallelisme was for this reason introduced, lest by the Earths annual revolution, its Poles should seem to be necessarily carried about the Sunne, contrary to experience. I have (in the 10th Artic. of the 21th Chap.) demonstrated from the supposition of Simple Circular Motion in the Sun, that the Earth is so carried about the Sunne, as that its Axis is thereby kept always parallel to it self. Wherefore, from these two supposed motions in the Sunne, the one Simple Circular Motion; the other Circular Motion about its owns Center, it may be demonstrated, that the Year hath both the same variations of Dayes and Nights, as have been demonstrated by *Copernicus*.

For

For if the Circle *abcd* (in the 3d Figure) be the Ecliptick, whose Center is *e*, and Diameter *aec*; and the Earth be placed in *a*, & the Sunne be moved in the little Circle *fghi*, namely, according to the order *f*, *g*, *h* & *i*, it hath been demonstrated, that a Body placed in *a*, will be moved in the same order through the points of the Ecliptick *a*, *b*, *c* & *d*, and will alwayes keep its Axis parallel to its self.

But if (as I have supposed) the Earth also be moved with Simple Circular Motion in a plain that passeth through *a*, cutting the plain of the Ecliptick so, as that the common section of both the plains be in *ac*, thus also the Axis of the Earth will be kept alwayes parallel to it self. For let the Center of the Earth be moved about in the Circumference of the Epicycle whose Diameter is *lak*, which is a part of the straight line *lac*. Therefore *lak* the Diameter of the Epicycle, passing through the Center of the Earth, will be in the plain of the Ecliptick. Wherefore seeing that by reason of the Earths Simple Motion both in the Ecliptick and in its Epicycle, the straight line *lak* is kept alwayes parallel to it self, every other straight line also taken in the Body of the Earth, and consequently its Axis, will in like manner be kept alwayes parallel to it self; so that in what part soever of the Ecliptick the Center of the Epicycle be found, and in what part soever of the Epicycle the Center of the Earth be found at the same time, the Axis of the Earth will be parallel to the place where the same Axis would have been, if the Center of the Earth had never gone out of the Ecliptick.

Now as I have demonstrated the simple annual motion of the Earth from the supposition of Simple Motion in the Sunne; so from the supposition of Simple Motion in the Earth may be demonstrated the monethly Simple Motion of the Moon. For if the names be but changed the Demonstration will be the same, and therefore need not be repeated.

7 That which makes this supposition of the Sunnes Simple Motion in the Epicycle *fghi* probable, is First, that the Periods of all the Planets are not onely described about the Sunne, but so described, as that they are al contained within the Zodiack, that is to say, within the latitude of about 16 degrees; for the cause of this

seems

ſeems to depend upon ſome power in the Sunne, eſpecially in that part of the Sunne which reſpects the Zodiack. Secondly, that in the whole compaſſe of the heavens there appears no other Body, from which the cauſe of this *Phænomenon* can in probability be derived. Beſides, I could not imagine, that ſo many and ſuch various motions of the Planets ſhould have no dependance at all upon one another. But by ſuppoſing motive power in the Sunne, we ſuppoſe motion alſo; for power to move, without motion, is no power at all. I have therefore ſuppoſed that there is in the Sunne for the governing of the primary Planets, and in the Earth for the governing of the Moon, ſuch motion, as being received by the primary Planets and by the Moon, makes them neceſſarily appear to us in ſuch manner as we ſee them; Whereas, that circular motion (which is commonly attributed to them) about a fixed Axis, (which is called Converſion) being a motion of their parts onely, and not of their whole Bodies, is inſufficient to ſalve their Appearances. For ſeeing whatſoever is ſo moved, hath no endeavour at all towards thoſe parts which are without the circle, they have no power to propagate any endeavour to ſuch Bodies as are placed without it. And as for them that ſuppoſe this may be done by Magnetical Virtue, or by incorporeall and immateriall Species, they ſuppoſe no naturall cauſe; nay no cauſe at all. For there is no ſuch thing as an Incorporeal Movent; and Magnetical Virtue is a thing altogether unknown; and whenſoever it ſhall be known, it will be found to be a motion of Body. It remaines therefore, that if the primary Planets be carried about by the Sunne, and the Moon by the Earth, they have the ſimple circular motions of the Sunne and the Earth for the cauſes of their circulations. Otherwiſe, if they be not carried about by the Sunne and the Earth, but that every Planet hath been moved as it is now moved ever ſince it was made, there will be of their motions no cauſe naturall. For either theſe motions were concreated with their Bodies, and their cauſe is ſupernatural; or they are coeternal with them, and ſo they have no cauſe at all. For whatſoever is Eternall was never generated.

I may add beſides, to confirme the probability of this ſimple motion, that allmoſt all learned men are now of the ſame opinion with

with *Copernicus* concerning the parallelisme of the Axis of the Earth, it seemed to me to be more agreeable to truth, or at least more handsome, that it should be caused by simple Circular Motion alone, than by two motions, one in the Ecliptick, and the other about the Earths own Axis the contrary way, neither of them Simple, nor either of them such as might be produced by any motion of the Sunne. I thought best therefore to retain this Hypothesis of Simple Motion; and from it to derive the causes of as many of the *Phænomena* as I could, and to let such alone as I could not deduce frm thence.

It will perhaps be objected, that although by this supposition the reason may be given of the Parallelisme of the Axis of the Earth, and of many other Appearances; neverthelesse, seeing it is done by placing the Body of the Sunne in the Center of that Orbe which the Earth describes with its annual motion, the supposition it self is false, because this annual Orbe is excentrique to the Sunne. In the first place therefore let us examine what that Excentricity is, and whence it proceeds.

8 Let the annual Circle of the Earth *a b c d* (in the same 3d figure) be divided into four equal parts by the straight lines *a c* & *b d*, cutting one another in the Center *e*; and let *a* be the beginning of *Libra*, *b* of *Capricorn*, *c* of *Aries*, and *d* of *Cancer*; and let the whole Orbe *a b c d* be understood (according to *Copernicus*) to have every way so great distance from the Zodiack of the fixed Starres, that it be in comparison with it but as a point. Let the Earth be now supposed to be in the beginning of *Libra* at *a*. The Sunne therefore will appear in the beginning of *Aries* at *c*. Wherefore if the Earth be moved from *a* to *b*, the apparent motion of the Snnne will be from *c* to the beginning of *Cancer* in *d*; and the Earth being moved forwards from *b* to *c*, the Sunne also will appear to be moved forwards to the beginning of *Libra* in *a*; Wherefore *c d a* will be the Summer Arch, and the Winter Arch will be *a b c*. Now in the time of the Suns apparent motion in the Summer Arch, there are numbred $186\frac{3}{4}$ dayes; and consequently the Earth makes in the same time the same number of diurnal conversions in the Arch *a b c*; and therefore the Earth in its motion through the Arch *c d a* will make onely $178\frac{1}{2}$ diurnal conversions, Wherefore

the Arch *abc* ought to be greater then the Arch *cda* by 8¼ dayes, that is to say, by almost so many degrees. Let the Arch *ar*, as also *cs*, be each of them an Arch of two degrees and $\frac{1}{16}$. Wherefore the Arch *rbs* will be greater then the Semicircle *abc* by 4 degrees and ½, and greater then the Arch *sdr* by 8 degrees and ¼. The Equinoxes therefore will be in the points *r* & *s*; and therefore also when the Earth is in *r*, the Sunne will appear in *s*. Wherefore the true place of the Sunne will be in *t*, that is to say, without the Center of the Earths annual motion by the quantity of the Sine of the Arch *ar*, or the Sine of two degrees and 16 minutes. Now this Sine, putting 100000 for the Radius, will be neer 3580 parts thereof. And so munh is the Excentricity of the Earths annual motion, provided that that motion be in a perfect circle; and *s* & *r* are the Equinoctial points; and the straight lines *sr* & *ca* produced both wayes till they reach the Zodiack of the fixed Starres, wil fall stil upon the same fixed Starres, because the whole Orbe *abcd* is supposed to have no magnitude at all in respect of the great distance of the fixed Starres.

Supposing now the Sun to be in *c*, it remains that I shew the cause why the Earth is neerer to the Sunne, when in its annual motion it is found to be in *d*, then when it is in *b*. And I take the cause to be this. When the Earth is in the beginning of *Capricorn* at *b*, the Sunne appears in the beginning of *Cancer* at *d*; & then is the midst of Summer. But in the midst of Summer, the Northern parts of the Earth are towards the Sunne, which is almost all dry land, containing all *Europe*, and much the greatest part of *Asia* and *America*. But when the Earth is in the beginning of *Cancer* at *d*, it is the midst of Winter, and that part of the Earth is towards the Sunne, which contains those great Seas called the South Sea and the Indian Sea, which are of farre greater extent then all the dry Land in that Hemisphere. Wherefore (by the last Article of the 21 Chapter) when the Earth is in *d*, it will come neerer to its first Movent, that is, to the Sunne which is in *t*; that is to say, the Earth is neerer to the Sunne in the midst of Winter when it is in *d*, then in the midst of Summer when it is *b*; and therefore during the Winter the Sunne is in its *Perigæum*, and in its *Apogæum* during the Summer. And thus I have shewn a possible cause of the

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Excentricity of the Earth; which was to be done.

I am therefore of *Keplers* opinion, in this, that he attributes the Excentricity of the Earth to the difference of the parts thereof, and supposes one part to be affected, and another disaffected to the Sunne. And I dissent from him in this, that he thinks it to be by Magnetick virtue, and that this Magnetick virtue, or attraction and thrusting back of the Earth is wrought by immateriate Species; which cannot be; because nothing can give motion, but a Body moved and contiguous. For if those Bodies be not moved which are contiguous to a Body unmoved, how this Body should begin to be moved is not imaginable; as has been demonstrated in the 7th Article of the 9th Chapter, and often inculcated in other places, to the end that Philosophers might at last abstain from the use of such unconceiveable connexions of words. I dissent also from him in this, that he says the similitude of Bodies is the cause of their mutual attraction, For if it were so, I see no reason why one Egg should not be attracted by another. If therefore one part of the Earth be more affected by the Sunne, then another part, it proceeds from this, that one part hath more water, the other more dry land. And from hence it is, (as I shewed above) that the Earth comes neerer to the Sunne when it shines upon that part where there is more water, then when it shines upon that where there is more dry Land.

9 This Excentricity of the Earth is the cause why the way of its annual motion is not a perfect Circle, but either an Elliptical, or almost an Elliptical line; as also why the Axis of the Earth is not kept exactly parallel to it self in all places, but onely in the Equinoctial points.

Now seeing I have said that the Moon is carried about by the Earth, in the same manner that the Earth is by the Sunne, and that the Earth goeth about the Sunne in such manner as that it shews sometimes one Hemisphere, sometimes the other to the Sunne; it remains to be enquired, why the Moon has alwayes one and the same face turned towards the Earth.

Suppose therefore the Sunne to be moved with Simple Motion in the little Circle *f g h i*, (in the fourth figure) whose Center is *t*; and let ♈ ♋ ♎ ♑ be the annuall Circle of the Earth; and *a* the beginning,

ginning of *Libra*. About the point *a* let the little Circle *l k* be described; and in it let the Center of the Earth be understood to be moved with Simple motion; and both the Sunne & the Earth to be moved according to the order of the Signes. Upon the Center *a* let the way of the Moon *m n o p* be described; and let *q r* be the Diameter of a Circle cutting the Globe of the Moon into two Hemispheres, whereof one is seen by us when the Moon is at the full, and the other is turned from us.

The Diameter therefore of the Moon *q o r* will be perpendicular to the Straight Line *t a*. Wherefore the Moon is carried by reason of the Motion of the Earth from *o* towards *p*. But by reason of the motion of the Sunne, if it were in *p* it would at the same time be carried from *p* towards *o*; and by these two contrary Movents the straight line *q r* will be turned about; and in a Quadrant of the Circle *m n o p* it will be turned so much as makes the fourth part of its whole conversion. Wherefore when the Moon is in *p*, *q r* will be parallel to the straight line *m o*. Secondly, when the Moon is in *m*, the straight line *q r* will by reason of the motion of the Earth be in *m o*. But by the working of the Suns motion upon it in the quadrant *p m*, toe same *q r* will be turned so much as makes another quarter of its whole conversion. When therefore the Moon is in *m*, *q r* will be perpendicular to the straight line *o m*. By the same reason, when the Moon is in *n*, *q r* will be parallel to the straight line *m o*; and the Moon returning to *o*, the same *q r* will return to its first place; and the Body of the Moon will in one entire period make also one entire conversion upon her own Axis. In the making of which, it is manifest, that one and the same face of the Moon is always turned towards the Earth. And if any Diameter were taken in that little Circle, in which the Moon were supposed to be carried about with simple motion, the same effect would follow; for if there were no action from the Sun, every Diameter of the Moon would be carried about always parallel to it self. Wherefore I have given a possible cause, why one and the same face of the Moon is alwayes turned towards the Earth.

But it is to be noted, that when the Moon is without the Eclip-tick, we do not alwayes see the same face precisely. For we see onely that part which is illuminated. But when the Moon is without the

the Ecliptick, that part which is towards us, is not exactly the same with that which is illuminated.

10 To these three simple motions, one of the Sunne, another of the Moon, and the third of the Earth, in their own little Circles *f g h i*, *l k*, & *q r*, together with the Diurnal conversion of the Earth (by which conversion all things that adhere to its superficies are necessarily carried about with it) may be referred the three *Phænomena* concerning the Tides of the Ocean. Whereof the first, is the alternate elevation and depression of the Water at the Shores, twice in the space of 24 houres and neer upon 52 minutes; for so it has constantly continued in all ages. The second, that at the New and Full Moons the elevations of the Water are greater, then at other times between. And the third, that when the Sunne is in the Equinoctial, they are yet greater then at any other time. For the salving of which *Phænomena*, we have already the foure above-mentioned Motions; to which I assume also this, that the part of the Earth which is called *America*, being higher then the Water, and extended almost the space of a whole Semicircle from North to South, gives a stop to the motion of the Water. This being granted,

In the same 4th figure, where *l b k c* is supposed to be in the plain of the Moons monethly motion, let the little Circle *l d k e* be described about the same Center *a* in the plain of the Equinoctial. This Circle therefore will decline from the Circle *l b k c* in an angle of almost 28 degrees and ½ (for the greatest declination of the Ecliptick is 23½, to which adding 5 for the greatest declination of the Moon from the Ecliptick, the summe wil be 28 degrees and ½). Seeing now the Waters which are under the Circle of the Moons course, are (by reason of the Earths Simple Motion in the plain of the same Circle) moved together with the Earth, (that is to say, together with their own bottoms) neither out-going nor out-gone; if we add the Diurnal motion, by which the other Waters which are under the Equinoctial are moved in the same order; and consider withall that the Circles of the Moon and of the Equinoctial intersect one another; it will be manifest, that both those Waters, which are under the Circle of the Moon, and under the Equinoctiall, will runne together under the Equinoctial; and consequent-

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ly, that their Motion will not onely be swifter then the ground that carries them; but allso that the waters themselves will have greater elevation whensoever the Earth is in the Equinoctial. Wherefore, whatsoever the cause of the Tides may be, this may be the cause of their augmentation at that time.

Againe, seeing I have supposed the Moon to be carried about by the simple motion of the Earth in the little circle *lbkc*; and demonstrated (at the 4 article of the 21 chapter) that whatsoever is moved by a Movent that hath simple motion, will be moved allwayes with the same velocity; it follows, that the center of the Earth will be carried in the circumference *lbkc* with the same velocity, with which the Moon is carried in the circumference *mnop*. Wherefore the time in which the Moon is carried about in *mnop*, is to the time in which the Earth is carried about in *lbkc*, as one circumference to the other, that is, as *ao* to *ak*. But *ao* is observed to be to the Semidiameter of the Earth as 59 to 1; and therefore the Earth (if *ak* be put for its Semidiameter) will make 59 revolutions in *lbkc*, in the time that the Moon makes one monthly circuit in *mnop*. But the Moon makes her monthly circuit in little more then 29 dayes. Wherefore the Earth shal make its circuit in the circumference *lbkc* in 12 hours and a little more, namely about 26 minutes more; that is to say, it shall make two circuits in 24 hours and allmost 52 minutes; which is observed to be the time between the high water of one day and the high water of the day following. Now the course of the waters being hindered by the southern part of *America*, their motion will be interrupted there; and consequently, they will be elevated in those places, and sink down again by their own waight, twice in the space of 24 hours and 52 minutes. And thus I have given a possible cause of the diurnall reciprocation of the Ocean.

Now from this swelling of the Ocean in those parts of the Earth, proceed the Flowings and Ebbings in the Atlantick, Spanish, Brittish and German Seas; which though they have their set times, yet upon severall Shores they happen at severall hours of the day; and they receive some augmentation from the North, by reason that the shores of *China* and *Tartaria*, hindering

ing the generall course of the waters, makes them swell there, and discharge themselves in part through the straight of *Anian* into the Northern Ocean, and so into the German Sea.

As for the Spring Tides which happen at the time of the New & Full Moons, they are caused by that simple motion which at the beginning I supposed to be allwayes in the Moone. For as, when I shewed the cause of the Excentricity of the Earth, I derived the elevation of the waters from the simple motion of the Sunne; so the same may here be derived from the simple motion of the Moon. For though from the generation of Clouds, there appeare in the Sunne a more manifest power of elevating the waters, then in the Moon; yet the power of encreasing moisture in Vegetables and living creatures appears more manifestly in the Moon then in the Sunne; which may perhaps proceed from this, that the Sunne raiseth up greater, and the Moon lesser drops of water. Neverthelesse, it is more likely, and more agreeable to common observation, that Raine is raised not only by the Sunne but allso by the Moon; for allmost all men expect change of weather at the time of the Conjunctions of the Sunne and Moon with one another, and with the Earth, more then in the time of their Quarters.

In the last place, the cause why the Spring Tides are greater at the time of the Æquinoxes, hath been already sufficiently declared in this Article, where I have demonstrated, that the two Motions of the Earth, namely, its Simple Motion in the little Circle *l b k c*, and its Diurnal motion in *l d k e*, cause necessarily a greater elevation of Waters when the Sunne is about the Æquinoxes, then when he is in other places. I have therefore given possible causes of the *Phænomenon* of the Flowing and Ebbing of the Ocean.

11 As for the explication of the yearly *Præcession of the Æquinoctial points*, we must remember, that (as I have already shewn) the annual motion of the Earth is not in the Circumference of a Circle, but of an Ellipsis, or a line not considerably different from that of an Ellipsis. In the first place therefore this Elliptical line is to be described.

Let the Ecliptick ♎ ♑ ♈ ♋ (in the 5th figure) be divided into four equal parts by the two straight lines *a b* and ♑ ♋, cutting one

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another at right angles in the Center *c*; and taking the Arch *b d* of 2 degrees and 16 minutes, let the straight line *d e* be drawn parallel to *a b* and cutting ♑ ♋ in *f*; which being done, the Excentricity of the Earth will be *c f*. Seeing therefore the annual motion of the Earth is in the Circumference of an Ellipsis, of which ♑ ♋ is the greater Axis, *a b* cannot be the lesser Axis; for *a b* and ♑ ♋ are equal. Wherefore the Earth passing through *a* & *b*, will either pass above ♑, as through *g*, or passing through ♑, will fall between *c* and *a*; it is no matter which. Let it pass therefore through *g*; and let *g l* be taken equal to the straight line ♑♋; and dividing *g l* equally in *i*, *g i* will be equal to ♑♋, & *i l* equal to *f* ♋; and consequently the point *i* will cut the Excentricity *c f* into two equal parts; and taking *i h* equal to *i f*, *h i* will be the whole Excentricity. If now a straight line (namely, the line ♎ *i* ♈) be drawn through *i* parallel to the straight lines *a b* and *e d*, the way of the Sunne in Summer (namely, the Arch ♎ *g* ♈) will be greater then his way in Winter by 8 degrees and $\frac{1}{4}$. Wherefore the true Æquinoxes wil be in the straight line ♎ *i* ♈, and therefore the Ellipsis of the Earths annual motion will not pass through *a*, *g*, *b* & *l*; but through ♎, *g*, ♈ & *l*. Wherfore the annual motion of the Earth is in the Ellipsis ♎ *g* ♈ *l*, and cannot be (the Excentricity being salved) in any other line. And this perhaps is the reason, why *Kepler*, against the opinion of all the Astronomers of former time, thought fit to bisect the Excentricity of the Earth, or (according to the Ancients) of the Sunne, not by diminishing the quantity of the same Excentricity, because the true measure of that quantity, is the difference by which the Summer Arch exceeds the Winter Arch, but by taking for the Center of the Ecliptick of the great Orbe the point *c* neerer to *f*, & so placing the whole great Orbe as much neerer to the Ecliptick of the fixed Stars towards ♋, as is the distance between *c* & *i*. For seeing the whole great Orbe is but as a point in respect of the immense distance of the fixed Starres, the two straight lines ♎ ♈ and *a b* being produced both wayes to the beginnings of *Aries* and *Libra*, will fall upon the same points of the Sphere of the fixed Stars. Let therefore the Diameter of the Earth *m n* be in the plain of the Earths annual motion. If now the Earth be moved by the Sunnes simple motion in the Circumference of the

the Ecliptick about the Center *i*, this Diameter will bee kept alwayes parallel to it ſelf and to the ſtraight line *gl*. But ſeeing the Earth is moved in the Circumference of an Ellipſis without the Ecliptick, the point *n*, whilſt it paſſeth through ♎ ♑ ♈ will go in a leſſer Circumference then the point *m*; and conſequently as ſoon as ever it begins to be moved, it will loſe its paralleliſme with the ſtraight line ♑ ♋, ſo that *m n* produced will at laſt cut the ſtraight line *gl* produced. And contrarily, as ſoon as *m n* is paſt ♈, (the Earth making its way in the internal Ellipticall line ♈ *l* ♎) the ſame *m n* produced towards *m*, will cut *l g* produced. And when the Earth hath allmoſt finiſhed its whole circumference, the ſame *m n* ſhall againe make a right angle with a line drawn from the center *i*, a little ſhort of the point from which the Earth began its motion. And there the next yeare ſhall be one of the Æquinoctial points, namely, neer the end of ♍; the other ſhall be oppoſite to it neer the end of ♓. And thus the points in which the Days and Nights are made equall, doe every year fall back; but with ſo ſlow a motion, that in a whole year, it makes but 51 firſt minutes. And this relapſe being contrary to the order of the Signes, is commonly called the *Præceſſion of the Æquinoxes*. Of which I have from my former Suppoſitions deduced a poſſible cauſe; which was to be done.

According to what I have ſaid concerning the cauſe of the Excentricity of the Earth; and according to *Kepler*, who for the cauſe thereof ſuppoſeth one part of the Earth to be affected to the Sunne the other part to be diſaffected, the *Apogæum* & *Perigæum* of the Sunne ſhould be moved every year in the ſame order, and with the ſame velocity, with which the Æquinoctiall points are moved; and their diſtance from them ſhould allwayes be the quadrant of a circle; which ſeems to be otherwiſe. For Aſtronomers ſay, that the Æquinoxes are now, the one about 28 degrees gone back from the firſt Star of *Aries*, the other as much from the beginning of *Libra*. So that the *Apogæum* of the Sunne, or the *Aphelium* of the Earth ought to be about the 28th degree of *Cancer*; but it is reckoned to be in the 7th degree. Seeing therefore we have not ſufficient evidence of the ὅτι *(that ſo it is)*, it is in vaine to ſeek for the διότι *(why it is ſo)*. Wherefore, as long as the motion of the

Apogæum is not obſervable by reaſon of the ſlowneſſe thereof; and as long as it remaines doubtful whether their diſtance from the Æquinoctiall points be more or leſſe then a quadrant preciſely; ſo long it may be lawfull for me to thinke they proceed both of them with equall velocity.

Alſo, I doe not at all meddle with the cauſes of the Excentricities of *Saturne*, *Jupiter*, *Mars*, and *Mercury*. Nevertheleſſe, ſeeing the Excentricity of the Earth may (as I have ſhewne) be cauſed by the unlike conſtitution of the ſeveral parts of the Earth which are alternately turned towards the Sunne, it is credible alſo that like effects may be produced in theſe other Planets from their having their Superficies of unlike parts.

And this is all I ſhall ſay concerning *Sidereal Philoſophy*. And though the cauſes I have here ſuppoſed be not the true cauſes of theſe *Phænomena*, yet I have demonſtrated that they are ſufficient to produce them, according to what I at firſt propounded.

CHAP. XXVII.

Of Light, Heat, and of Colours.

1 Of the immense Magnitude of some Bodies, and the unspeakable Littleness of others. 2 Of the cause of the Light of the Sun. 3 How Light heateth. 4 The generation of Fire from the Sunne. 5 The generation of Fire from Collision. 6 The cause of Light in Glow-wormes, Rotten Wood, and the Bolonian *Stone. 7 The cause of Light in the concussion of Sea-water. 8 The cause of Flame, Sparks, and Colliquation. 9 The cause why wet Hay sometimes burns of its own accord. Also the cause of Lightning. 10 The cause of the force of Gunpowder; and what is to be ascribed to the Coals, what to the Brimstone, and what to the Nitre. 11 How Heat is caused by Attrition. 12 The distinction of Light into First, Second, &c. 13 The causes of the Colours we see in looking through a Prisma of Glass, namely, of Red, Yellow, Blue and Violet Colour. 14 Why the Moon and the Starres appear redder in the Horizon then in the midst of the Heaven. 15 The cause of Whiteness. 16 The cause of Blackness.*

1 BEsides the Starres (of which I have spoken in the last Chapt.), whatsoever other Bodies there be in the World, they may be all comprehended under the name of *Intersidereal* Bodies. And these I have already supposed to be either the most fluid *Æther*, or such Bodies whose parts have some degree of cohæsion. Now these differ from one another in their several *Consistencies, Magnitudes, Motions* and *Figures*. In Consistency, I suppose some Bodies to be Harder, others Softer through all the several degrees of *Tenacity*. In Magnitude, some to be Greater, others Less, and many unspeakably Little. For we must remember that by the Understanding, Quantity is divisible into divisibles perpetually. And therefore if a man could do as much with his hands as he can with his Understanding, he would be able to take from any given magnitude, a part which should be less then any other magnitude given. But the Omnipotent Creator of the World can actually from a part of any thing take another part, as farre as we by

by our Understanding can conceive the same to be divisible. Wherefore there is no impossible Smalness of Bodies. And what hinders but that we may think this likely ? For we know there are some living Creatures so small, that we can scarce see their whole Bodies. Yet even these have their young ones, their little Veins, and other Vessels, and their Eyes so smal, as that no *Microscope* can make them visible. So that we cannot suppose any magnitude so little, but that our very supposition is actually exceeded by Nature. Besides, there are now such *Microscopes* commonly made, that the things we see with them appear a hundred thousand times bigger, then they would do if we looked upon them with our bare Eyes. Nor is there any doubt but that by augmenting the power of these *Microscopes* (for it may be augmented as long as neither Matter nor the hands of Workmen are wanting) every one of those hundred thousandth parts might yet appear a hundred thousand times geater then they did before. Neither is the Smalness of some Bodies to be more admired, then the vast Greatness of others. For it belongs to the same infinite Power, as well to augment infinitely, as infinitely to diminish; To make the great Orbe, (namely, that whose Radius reacheth from the Earth to the Sunne) but as a point in respect of the distance between the Sunne and the fixed Starres; and on the contrary, to make a Body so little, as to be in the same proportion less then any other visible Body, proceeds equally from one and the same Authour of Nature. But this of the immense distance of the fixed Starres (which for a long time was accounted an incredible thing) is now believed by almost all the Learned. Why then should not that other of the smalness of some Bodies, become credible at some time or other ? For the Majesty of God appears no less in small things then in great; and as it exceedeth humane sense in the immense greatness of the Universe; so also it doth in the smalness of the parts thereof. Nor are the first Elements of Compositions, nor the first Beginnings of Actions, nor the first Moments of Times more credible, then that which is now believed of the vast distance of the fixed Starres.

Some things are acknowledged by mortal men to be very Great, though Finite, as seeing them to be such. They acknowledge also, that some things which they do not see, may be of infinite magnitude.

nitude. But they are not presently, nor without great study perswaded that there is any Mean between Infinite & the Greatest of those things which either they see or imagine. Neverthelesse, when after meditation & contemplation, many things which we wondred at before are now grown more familiar to us, we then believe them, and transferre our admiration from the Creatures to the Creator. But how little soever some Bodies may be; yet I will not suppose their quantity to be less, then is requisite for the salving of the *Phænomena*. And in like manner I shall suppose their motion, namely, their Velocity and Slowness, and the Variety of their Figures, to be onely such as the explication of their natural causes requires. And lastly, I suppose, that the parts of the pure *Æther* (as if it were the First Matter) have no motion at all, but what they receive from Bodies which float in them, and are not themselves fluid.

2 Having laid these Grounds, let us come to speake of Causes, and in the first place let us inquire what may be the cause of the Light of the Sunne. Seeing therefore the Body of the Sunne doth by its simple circular motion thrust away the ambient æthereall substance sometimes one way sometimes another, so that those parts which are next the Sunne being moved by it, doe propagate that motion to the next remote parts, and these to the next, and so on continually; it must needs be, that notwithstanding any distance, the foremost part of the Eie will at last be pressed; and by the pressure of that part, the motion will be propagated to the innermost part of the Organ of Sight, namely to the Heart; and from the reaction of the Heart, there will proceed an endeavour back by the same way, ending in the endeavour outwards of the Coat of the Eie called the *Retina*. But this endeavour outwards (as has been defined in the 25 chapter) is the thing which is called Light, or the Phantasme of a Lucid Body. For it is by reason of this Phantasme that an Object is called Lucid. Wherefore we have a possible cause of the Light of the Sunne; which I undertook to find.

3 The generation of the Light of the Sunne is accompanied with the generation of Heat. Now every man knowes what Heat is in himselfe, by feeling it when he growes Hot; but what it is in other things he knowes onely by ratiocination. For it is one thing to

grow

grow Hot, and another thing to Heat, or make Hot. And therefore though we perceive that the Fire or the Sunne Heateth, yet we doe not perceive that it is it selfe Hot. That other living creatures whilest they make other things Hot, are Hot themselves, we inferre by reasoning from the like sense in our selves. But this is not a necessary inference. For though it may truly be said of living Creatures, that *They Heat, therefore they are themselves Hot*; yet it cannot from hence be truly inferred, that *Fire Heateth, therefore it is it selfe Hot*; no more then this, *Fire causeth Pain, therefore it is it self in Pain.* Wherefore that is onely and properly called Hot, which when we feel, we are necessarily Hot.

Now when we grow Hot, we find that our Spirits and Blood, and whatsoever is fluid within us, is called out from the internall to the externall parts of our Bodies, more or lesse, according to the degree of the Heat; and that our Skin swelleth. He therefore that can give a possible cause of this Evocation and Swelling, and such as agreeth with the rest of the *Phænomena* of Heat; may be thought to have given the cause of the Heat of the Sunne.

It hath been shewn (in the 5 article of the 21 chapter) that the fluid Medium which we call the Aire, is so moved by the simple circular motion of the Sunne, as that all its parts, even the least, do perpetually change places with one another; which change of places is that which there I called Fermentation. From this Fermentation of the Aire, I have (in the 8 article of the last chapter) demonstrated, that the water may be drawn up into the clouds.

And I shall now shew, that the fluid parts may in like manner by the same Fermentation be drawn out from the internall to the externall parts of our Bodies. For seeing that wheresoever the fluid Medium is contiguous to the Body of any living creature, there the parts of that Medium are by perpetuall change of place separated from one another; the contiguous parts of the living creature must of necessity endeavour to enter into the spaces of the separated parts. For otherwise those parts (supposing there is no *Vacuum*) would have no place to go into. And therefore that which is most fluid and separable in the parts of the living creature which are contiguous to the Medium, will go first out; and into the place thereof will succeed such other parts as can most

easily transpire through the pores of the skin. And from hence it is necessary, that the rest of the parts which are not separated, must all together be moved outwards, for the keeping of all places full. But this motion outwards of all parts together must of necessity press those parts of the ambient Aire which are ready to leave their places, and therefore all the parts of the Body endeavouring at once that way, makes the Body swell. Wherefore a possible cause is given of Heat from the Sunne; which was to be done.

4 We have now seen how Light and Heat are generated; Heat by the simple motion of the Medium, making the parts perpetually change places with one another; and Light by this, that by the same simple motion Action is propagated in a straight line. But when a Body hath its parts so moved, that it sensibly both Heats and Shines at the same time, then it is, that we say Fire is generated.

Now by Fire I do not understand a Body distinct from matter combustible or glowing, as Wood or Iron, but the matter it self, not simply and always, but then onely when it shineth and heateth. He therefore that renders a cause possible, and agreeable to the rest of the *Phænomena*, namely, whence and from what action both the *Shining* and *Heating* proceed, may be thought to have given a possible cause of the generation of *Fire*.

Let therefore A B C (in the first Figure) be a Sphere, or the portion of a Sphere, whose Center is D; and let it be transparent and homogeneous, as Cristal, Glass, or Water, and objected to the Sunne. Wherefore the foremost part A B C, will by the simple motion of the Sunne, by which it thrusts forwards the Medium, be wrought upon by the Sun-beams in the straight lines E A, F B and G C; which straight lines may in respect of the great distance of the Sunne be taken for parallels. And seeing the Medium within the Sphere is thicker then the Medium without it, those Beams will be refracted towards their perpendiculars. Let the straight lines E A and G C be produced till they cut the Sphere in H and I; and drawing the perpendiculars A D and C D, the refracted Beams E A and G C will of necessity fall, the one between A H and A D, the other between C I and C D. Let those refracted

Beams

Beams be A K and C L. And again, let the lines D K M & D L N be drawn perdendicular to the Sphere; and let A K and C L be produced till they meet with the ſtraight line B D produced in O. Seeing therefore the Medium within the Sphere is thicker then that without it, the refracted line A K will recede further from the perpendicular K M, then K O will recede from the ſame. Wherefore K O will fall between the refracted line and the perpendicular. Let therefore the refracted line be K P, cutting F O in P; and for the ſame reaſon the ſtraight line L P will be the refracted line of the ſtraight line C L. Wherfore, ſeeing the Beams are nothing elſe but the Wayes in which the motion is propagated, the motion about P will be ſo much more vehement then the motion about A B C, by how much the baſe of the portion A B C is greater then the baſe of a like portion in the Sphere whoſe Center is P, and whoſe magnitude is equal to that of the little Circle about P, which comprehendeth all the Beams that are propagated from A B C; and this Sphere being much leſs then the Sphere A B C, the parts of the Medium, that is, of the Aire about P, will change places with one another with much greater celerity, then thoſe about A B C. If therefore any matter Combuſtible, that is to ſay, ſuch as may be eaſily diſſipated, be placed in P, the parts of that matter (if the proportion be great enough between A C and a like portion of the little circle about P) wil be freed from their mutual cohæſion, and being ſeparated will acquire ſimple motion. But vehement ſimple motion generates in the beholder a Phantaſm of Lucid and Hot, as I have before demonſtrated of the ſimple motion of the Sunne; and therefore the combuſtible matter which is placed in P will be made Lucid and Hot, that is to ſay, will be Fire. Wherefore I have rendered a poſſible cauſe of Fire; which was to be done.

5 From the manner by which the Sunne generateth Fire, it is eaſy to explaine the manner by which Fire may be generated by the colliſion of two Flints. For by that Colliſion, ſome of thoſe particles of which the ſtone is compacted, are violently ſeparated and thrown off; and being withall ſwiftly turned round, the Eie is moved by them, as it is in the generation of Light by the Sunne. Wherefore they ſhine; and falling upon matter which

is

is already halfe dissipated, such as is Tinder, they throughly dissipate the parts thereof, and make them turn round. From whence (as I have newly shewn) Light and Heat, that is to say, Fire is generated.

6 The shining of Glow-worms, some kinds of Rotten Wood, and of a kinde of stone made at *Bolognia*, may have one common cause, namely the exposing of them to the hot Sunne. We finde by experience that the *Bolonian* stone shines not unless it be so exposed; and after it has been exposed it shines but for a little time, namely, as long as it retains a certain degree of heat. And the cause may be, that the parts of which it is made, may together with heat have Simple Motion imprinted in them by the Sunne. Which if it be so, it is necessary, that it shine in the dark, as long as there is sufficient heat in it; but this ceasing, it will shine no longer. Also we find by expetience, that in the Glow-worm there is a certain thick humour, like the Cristalline humour of the Eie; which if it be taken out, and held long enough in ones fingers, and then be carried into the dark, it will shine by reason of the warmth it received from the fingers; but as soon as it is cold, it will cease shining. From whence therefore can these creatures have their Light, but from lying all day in the Sun-shine, in the hottest time of Summer? In the same manner, Rotten Wood, except it grow rotten in the Sunshine, or be afterwards long enough exposed to the Sunne, will not shine. That this doth not happen in every Worm, nor in all kinds of Rotten Wood, nor in all Calcined Stones, the cause may be, that the parts of which those Bodies are made, are different both in motion and figure from the parts of Bodies of other kinds.

7 Also the Sea-water shineth when it is either dashed with the strokes of Oares, or when a Ship in its course breaks strongly through it; but more or less according as the Winde blows from different points. The cause whereof may be this, that the particles of salt (though they never shine in the Salt-pits, where they are but slowly drawn up by the Sunne) being here beaten up into the aire in greater quantities, and with more force, are withall made to turn round, and consequently to shine, though weakly. I have therefore given a possible cause of this *Phænomenon*.

8 If such matter as is compounded of hard little Bodies, be set

 on

on fire, it must needs be, that as they flye out in greater or lesse quantities, the Flame which is made by them, will be greater or less. And if the æthereal or fluid part of that matter fly out together with them, their motion will be the Swifter, as it is in Wood, and other things which flame with a manifest mixture of Winde. When therefore these hard particles by their flying out, move the Eye strongly, they shine bright; and a great quantity of them flying out together, they make a great shining Body. For Flame being nothing but an aggregate of shining particles, the greater the aggregate is, the greater and more manifest will be the Flame. I have therefore shewn a possible cause of Flame. And from hence the cause appears evidently, why Glass is so easily and quickly melted by the small Flame of a candle blown, which will not be melted without blowing, but by a very strong Fire.

Now if from the same matter, there be a part broken off (namely such a part as consisteth of many of the small particles), of this is made a Spark. For from the breaking off, it hath a violent turning round; and from hence it shines. But though from this matter, there fly neither Flame nor Sparks; yet some of the smallest parts of it may be carried out as farre as to the Superficies, and remain there, as Ashes; the parts whereof are so extremely small, that it cannot any longer be doubted how farre Nature may proceed in Dividing.

Lastly, though by the application of fire to this matter, there fly little or nothing from it, yet there will be in the parts an endeavour to Simple motion; by which the whole Body will either be Melted, or (which is a degree of Melting) Softned. For all Motion has some effect upon all Matter whatsoever (as has been shewn at the 3d Article of the 16th Chapter). Now if it be softned to such a degree, as that the stubbornness of the parts be exceeded by their gravity, then we say it is Melted; otherwise, Softned, and made Pliant and *Ductile*.

Again, the matter having in it some particles hard, others æthereal or watery, if by the application of fire these later be called out, the former will thereby come to a more full contact with one another; and consequently will not be so easily separated; that is to say, the whole Body will be made Harder. And this may

be

be the cause why the same Fire makes some things Soft, others Hard.

9 It is known by experience, that if Hay be laid wet together in a heap, it will after a time begin to smoke, and then burn as it were of it self. The cause whereof seems to be this, that in the Aire which is enclosed within the Hay, there are those little Bodies, which (as I have supposed) are moved freely with simple Motion. But this Motion being by degrees hindred more and more by the descending moisture, which at the last fils and stops all the passages, the thinner parts of the Aire ascend by penetrating the water; and those hard little Bodies being so thrust together that they touch and press one another, acquire stronger motion, till at last by the increased strength of this motion the watery parts are first driven outwards, from whence appears Vapour; and by the continued increase of this motion, the smallest particles of the dryed Hay are forced out, and recovering their natural simple Motion, they grow Hot and Shine, that is to say, they are set on Fire.

The same also may be the cause of Lightning; which happens in the hottest time of the yeare, when the water is raised up in greatest quantity, and carried highest. For after the first Clouds are raised, others after others follow them; and being congeled above, they happen (whilest some of them ascend and others descend) to fall upon another in such manner, as that in some places all their parts are joyned together, in others they leave hollow Spaces between them; aud into these spaces (the æthereall parts being forced out by the compressure of the Clouds) many of the harder little Bodies are so pent together, as that they have not the liberty of such motion as is naturall to the Aire. Wherefore their endeavour growes more vehement, till at last they force their way through the Clouds, sometimes in one place, sometimes in another; and breaking through with great noise, they move the aire violently, & striking our Eies generate Light, that is to say, they Shine. And this Shining is that we call Lightning.

10 The most common *Phænomenon* proceeding from Fire, and yet the most admirable of all others, is the force of Gunpowder fired; which being compounded of Niter, Brimstone and Coles beaten small, hath from the Coles its first taking fire; from the

Brimstone its nourishment and flame, that is to say, Light and motion; and from the Niter the vehemence of both. Now if a piece of Niter, (before it is beaten) be laid upon a burning Cole, first it melts, and like water quencheth that part of the Cole it toucheth. Then Vapor or Aire flying out where the Cole and Niter joyne, bloweth the Cole with great swiftnesse and vehemence on all sides. And from hence it comes to passe, that by two contrary motions (the one, of the particles which go out of the burning Cole, the other, of those of the æthereall and watery substance of the Niter) is generated that vehement Motion and Inflammation. And lastly, when there is no more action from the Niter (that is to say, when the volatile parts of the Niter are flown out) there is found about the sides a certain white substance, which being thrown again into the fire, will grow red hot again, but will not be dissipated, at least, unlesse the fire be augmented. If now a possible cause of this be found out, the same will also be a possible cause why a grain of Gunpowder set on fire doth expand it selfe with such vehement motion, and Shine. And it may be caused in this manner.

Let the particles of which Niter consisteth, be supposed to be some of them hard, others watery, and the rest æthereall. Also let the hard particles be supposed to be spherically hollow, like small bubbles, so that many of them growing together may constitute a Body whose little cavernes are filled with a substance which is either watery, or æthereal, or both. As soon therefore as the hard particles are dissipated, the watery and æthereal particles will necessarily fly out; and as they fly, of necessity blow strongly the burning Coles and Brimstone which are mingled together; whereupon there will follow a great expansion of Light, with vehement flame, and a violent dissipation of the particles of the Niter, the Brimstone and the Coles. Wherefore I have given a possible cause of the force of fired Gunpowder.

It is manifest from hence, that for the rendering of the cause why a bullet of lead or iron shot from a peece of Ordnance flies with so great velocity, there is no necessity to introduce such Rarefaction, as (by the common definition of it) makes the same Matter to have sometimes more, sometimes lesse Quantity; which is un-

unconceiveable. For every thing is ſaid to be greater or leſſe, as it hath more or leſſe Quantity. The violence with which a bullet is thruſt out of a Gun, proceeds from the ſwiftneſſe of the ſmall particles of the fired Powder; at leaſt it may proceed from that cauſe, without the ſuppoſition of any Empty Space.

11 Beſides, by the attrition or rubbing of one Body againſt another, as of Wood againſt Wood, we find that not only a certaine degree of Heat, but Fire it ſelfe is ſometimes generated. For ſuch motion, is the reciprocation of preſſure, ſometimes one way ſometimes the other; and by this reciprocation, whatſoever is fluid in both the peeces of Wood, is forced hither and thither; and conſequently, to an endeavour of getting out; and at laſt by breaking out makes Fire.

12 Now Light is diſtinguiſhedinto, Firſt, Second, Third,and ſo on infinitely. And we call that Firſt Light, which is in the firſt Lucid Bodie; as the Sunne, Fire, &c. Second, that which is in ſuch Bodies as being not tranſparent are illuminated by the Sunne; as the Moon, a Wall, &c. and Third, that which is in Bodies not tranſparent but illuminated by Second Light, &c.

13 *Colour* is Light, but troubled Light; namely, ſuch as is generated by perturbed motion; as ſhall be made manifeſt by the Red, Yellow, Blew and Purple which are generated by the interpoſition of a *Diaphanous Priſma* (whoſe oppoſite baſes are triangular) between the Light and that which is enlightened.

For let there be a Priſma of Glaſſe, or of any other tranſparent matter which is of greater denſity then Aire; and let the triangle A B C be the baſe of this *Priſma*. Alſo let the ſtraight line D E be the diameter of the Sunnes Body, having oblique poſition to the ſtraight line A B; and let the Sunne-beames paſſe in the lines D A and E B C. And laſtly let the ſtraight lines D A and E C be produced indefinitely to F and G. Seeing therefore the ſtraight line D A, by reaſon of the denſity of the Glaſſe is refracted towards the perpendicular; let the line refracted at the point A be the ſtraight line A H. And againe, ſeeing the Medium below A C is thinner then that above it, the other refraction which will be made there, will diverge from the perpendicular. Let therefore this ſecond refracted line be A I. Alſo let the ſame be

be done at the point C, by making the first refracted line to be C K, and the second C L. Seeing therefore the cause of the refraction in the point A of the straight line of A B, is the excess of the resistance of the Medium in A B above the resistance of the Aire, there must of necessity be reaction from the point A towards the point B; and consequently the Medium at A within the triangle A B C, will have its motion troubled; that is to say, the straight motion in A F and A H, will be mixed with the transverse motion between the same A F and A H, represented by the short transverse lines in the triangle A F H. Againe, seeing at the point A of the straight line A C, there is a second refraction from A H in A I, the motion of the Medium will againe be perturbed by reason of the transverse reaction from A towards C, represented likewise by the short transverse lines in the triangle A H I. And in the same manner there is a double perturbation represented by the transverse lines in the triangles C G K and C K L. But as for the light between A I and C G, it will not be perturbed; because if there were in all the points of the straight lines A B and A C, the same action which is in the points A and C, then the plaine of the triangle C G K would be every where coincident with the plaine of the triangle A F H; by which meanes all would appear alike between A and C. Besides, it is to be observed, that all the reaction at A, tends towards the illuminated parts which are between A and C, and consequently perturbeth the First Light. And on the contrary, that all the reaction at C tends towards the parts without the triangle, or without the *Prisma* A B C, where there is none but Second Light; and that the triangle A F H shewes that perturbation of Light which is made in the Glasse it selfe; as the triangle A H I shewes that perturbation of Light which is made below the Glasse. In like manner, that C G K shewes the perturbation of Light within the Glasse; and C K L that which is below the Glasse. From whence there are four divers motions, or four different illuminations or Colours; whose differences appear most manifestly to the Sense in a *Prisma* (whose base is an equilaterall triangle) when the Sunne-beames that passe through it are received upon a white paper. For the triangle A F H appears Red to the Sense; the triangle A H I Yellow; the tri-

triangle C G K Green, and approaching to Blew; and lastly the triangle C K L appears Purple. It is therefore evident, that when weak but First Light passeth through a more resisting diaphanous Body, as Glasse, the beames which fall upon it tranversly, make Rednesse; and when the same First Light is stronger, as it is in the thinner Medium below the straight line A C, the transverse beames make Yellownesse. Also when Second Light is strong, as it is in the triangle C G K (which is neerest to the First Light) the transverse beames make Greenesse; and when the same Second Light is weaker, as in the triangle C K L, they make a Purple colour.

14 From hence may be deduced a cause why the Moon and Starres appear bigger and redder neer the Horizon then in the Mid-heaven. For between the Eie and the apparent Horizon, there is more impure aire, such as is mingled with Watery and Earthy little Bodies, then is between the same Eie and the more elevated part of Heaven. But Vision is made by Beames which constitute a Cone, whose base, if we look upon the Moon, is the Moons Face, and whose vertex is in the Eie; and therefore many beams from the Moon must needs fall upon little Bodies that are without the Visual Cone, and be by them reflected to the Eie. But these reflected beams tend all in lines which are transverse to the Visual Cone, and make at the Eie an angle which is greater then the angle of the Cone. Wherefore the Moon appeares greater in the Horizon, then when she is more elevated. And because those reflected beames go transversely, there will be generated (by the last article) Rednesse. A possible cause therefore is shewne, why the Moon, as also the Starres appear Greater and Redder in the Horizon, then in the midst of heaven. The same also may be the cause why the Sunne appears in the Horizon, Greater, and of a colour more degenerating to Yellow, then when he is higher elevated. For the reflection from the little Bodies between, and the transverse motion of the Medium are still the same. But the Light of the Sunne is much stronger then that of the Moon; and therefore (by the last article) his Splendor must needs by this perturbation degenerate into Yellownesse.

But for the generation of these four colours, it is not necessary that

that the figure of the Glaſs be a *Priſma*; for if it were Spherical it would doe the ſame. For in a Sphere the Sunne-beames are twice refracted and twice reflected. And this being obſerved by *Des Cartes*; and with all that a Rainebow never appeares but when it rains; as alſo, that the drops of raine have their figures almoſt Spherical; he hath ſhewne from thence the cauſe of the colours in the Rainbow; which therefore need not be repeated.

15 *Whiteneſs* is Light; but Light perturbed by the reflexions of many beams of Light comming to the Eye together within a little ſpace. For if Glaſs, or any other *Diaphanous* Body be reduced to very ſmall parts by contuſion or concuſſion; every one of thoſe parts (if the Beams of a lucid Body be from any one point of the ſame reflected to the Eye) will repreſent to the beholder an Idea or Image of the whole lucid Body, that is to ſay, a Phantaſme of White. For the ſtrongeſt Light is the moſt White; and therefore many ſuch parts will make many ſuch Images. Wherefore if thoſe parts lie thick and cloſe together, thoſe many Images will appear confuſedly, and will by reaſon of the confuſed Light repreſent a White Colour. So that from hence may be deduced a poſſible cauſe, why Glaſs beaten, that is, reduced to powder, looks White. Alſo why Water and Snow are White; they being nothing but a heap of very ſmall *Diaphanous* Bodies, namely, of little Bubbles, from whoſe ſeveral convex Superficies, there are by reflexion made ſeveral confuſed Phantaſmes of the whole lucid Body; that is to ſay, *Whiteneſs*. For the ſame reaſon, Salt and Nitre are White; as conſiſting of ſmall Bubbles which contain within them Water and Aire; as is manifeſt in Nitre, from this, that being thrown into the fire, it violently blowes the ſame; which Salt alſo doth, but with leſs violence. But if a White Body be expoſed, not to the Light of the Day, but to that of the Fire, or of a Candle, it will not at the firſt ſight be eaſily judged whether it be White or Yellow; the cauſe whereof may be this, that the light of thoſe things which burn and flame, is almoſt of a middle Colour between Whiteneſs and Yellowneſs.

16 As Whiteneſs is Light, ſo *Blackneſs* is the privation of Light, or Darkneſs. And from hence it is; Firſt, that all Holes, from which no light can be reflected to the Eie, appear Black.

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Secondly, that when a Body hath little eminent particles erected straight up from the Superficies (ſo that the Beams of Light which fall upon them are reflected, not to the Eie, but to the Body it ſelf) that Superficies appears Black, in the ſame manner as the Sea appears Black, when ruffled by the Wind. Thirdly, that any combuſtible matter is by the fire made to look Black before it ſhines. For the endeavour of the fire being to diſſipate the ſmalleſt parts of ſuch Bodies as are thrown into it, it muſt firſt raiſe and erect thoſe parts, before it can work their diſſipation. If therefore the fire be put out before the parts be totally diſſipated, the Cole will appear Black; for the parts being onely erected, the Beams of Light falling upon them will not be reflected to the Eie, but to the Cole it ſelf. Fourthly, that Burning Glaſſes do more eaſily burn Black things then White. For in a White Superficies, the eminent parts are convex, like little bubbles; and therefore the Beams of Light which fall upon them are reflected every way from the reflecting Body. But in a Black Superficies, where the eminent particles are more erected, the Beams of Light falling upon them, are all neceſſarily reflected towards the Body it ſelf; and therefore Bodies that are Black are more eaſily ſet on fire by the Sun-beams, then thoſe that are White. Fifthly, that all Colours that are made of the mixture of White and Black, proceed from the different poſition of the particles that riſe above the Superficies, and their different forms of aſperity. For according to theſe differences, more or fewer Beams of Light are reflected from ſeveral Bodies to the Eie. But in regard thoſe differences are innumerable, and the Bodies themſelves ſo ſmall, that we cannot perceive them, the explication and preciſe determination of the Cauſes of all Colours is a thing of ſo great difficulty, that I dare not undertake it.

Chap. XXVIII.

Of Cold, Wind, Hard, Ice, Restitution *of Bodies bent*, Diaphanous, Lightning *and* Thunder; *and of the* Heads *of Rivers*.

1 *Why Breath from the same mouth sometimes heats, and sometimes cools.* 2 *Wind, and the Inconstancy of Winds, whence.* 3 *Why there is a constant, though not a great Wind from East to West neer the Equator.* 4 *What is the effect of Aire pent in between the Clouds.* 5 *No change from Soft to Hard, but by motion.* 6 *What is the cause of Cold neer the Poles.* 7 *The cause of Ice, and why the Cold is more remiss in rainy then in clear weather. Why water doth not freeze in deep Wells, as it doth neer the Superficies of the Earth. Why Ice is not so heavy as Water; and why Wine is not so easily frozen as Water.* 8 *Another cause of Hardness from the fuller contact of Atomes. Also how Hard things are broken.* 9 *A third cause of Hardness from Heat.* 10 *A fourth cause of Hardness from the motion of Atomes enclosed in a narrow space.* 11 *How Hard things are Softned.* 12 *Whence proceeds the spontaneous Restitution of things Bent.* 13 *Diaphanous, and Opacous, what they are, and whence.* 14 *The cause of Lightning and Thunder.* 15 *Whence it proceeds that Clouds can fall again, after they are once elevated and frozen.* 16 *How it could be that the Moon was eclipsed, when she was not diametrally opposite to the Sunne.* 17 *By what means many Sunnes may appear at once.* 18 *Of the Heads of Rivers.*

1 AS, when the motion of the ambient æthereal substance makes the Spirits and fluid parts of our Bodies tend outwards, we acknowledge Heat; so, by the endeavour inwards of the same spirits and humours, we feel Cold. So that to *Cool*, is to make the exterior parts of the Body endeavour inwards, by a motion contrary to that of *Calefaction*, by which the internal parts are called out-

wards. He therefore that would know the cause of Cold must find by what motion or motions, the exterior parts of any Body endeavour to retire inwards. To begin with those *Phænomena* which are the most familiar; There is almost no man but knows, that breath blown strongly, and which comes from the mouth with violence, that is to say, the passage being straight, will Cool the hand; and that the same breath blown gently, that is to say, through a greater aperture, wil warm the same. The cause of which *Phænomenon* may be this. The breath going out, hath two motiōs; the one, of the whole and direct, by which the formost parts of the hand are driven inwards; the other, simple motion of the small particles of the same breath, which (as I have shewn in the 3d Article of the last Chapter) causeth Heat. According therefore as either of these Motions is predominant, so there is the sense sometimes of Cold, sometimes of Heat. Wherefore, when the breath is softly breathed out at a large passage, that simple Motion which causeth Heat prevaileth, and consequently Heat is felt; and when by compressing the lips the breath is more strongly blown out, then is the direct motion prevalent, which makes us feel Cold. For the direct motion of the breath or aire, is Wind; and all Wind Cools, or diminisheth former heat.

2 And seeing not onely great Wind, but almost any Ventilation and stirring of the Aire, doth refrigerate; the reason of many experiments concerning Cold cannot well be given, without finding first what are the causes of Wind. Now Wind is nothing else but the direct motion of the Aire thrust forwards; which neverthelesse, when many Winds concurre may be circular, or otherwise indirect, as it is in Whirle-winds. Wherefore in the first place we are to enquire into the Causes of Winds. *Wind* is Aire moved in a considerable quantity, and that either in the manner of Waves, which is both forwards & also up & down; or else forwards onely.

Supposing therefore the Aire both cleer and calm, for any time how little soever; yet the greater Bodies of the World, being so disposed and ordered as has been said, it will be necessary that a Wind presently arise some where. For seeing that motion of the parts of the Aire which is made by the Simple Motion of the Sunne in his own Epicycle, causeth an exhalation of the particles of

of water from the Seas and all other moist Bodies,and those particles make Clouds; it must needs follow, that whilest the particles of water pass upwards, the particles of Aire (for the keeping of all Spaces full)be justled out on every side,and urge the next particles, and these the next, till having made their circuit,there comes continually so much Aire to the hinder parts of the Earth, as there went water from before it. Wherefore the ascending Vapours move the Aire towards the sides every way; and all direct motion of the Aire being Wind, they make a Wind. And if this Wind meet often with other Vapours which arise in other places,it is manifest that the force thereof will be augmented,& the way or course of it changed.Besides, according as the Earth by its diurnal motion turns sometimes the drier, sometimes the moister part towards the Sunne, so sometimes a greater, sometimes a less quantity of Vapours will be raised, that is to say, sometimes there will be a less, sometimes a greater Wind. Wherefore I have rendred a possible cause of such Winds, as are generated by Vapours; and also of their Inconstancy.

From hence it follows,that these Winds cannot be made in any place which is higher then that to which Vapours may ascend. Nor is that incredible which is reported of the highest Mountains, as the *Pique of Tenariffe* and the *Andes of Peru*, namely, that they are not at all troubled with these inconstant Winds.And if it were certain, that neither Rain nor Snow were ever seen in the highest tops of those Mountains, it could not be doubted but that they are higher then any place to which Vapours use to ascend.

3 Neverthelefs, there may be Wind there, though not that which is made by the ascent of Vapours,yet a less & more constant Wind (like the continued blast of a pair of bellows) blowing from the East.And this may have a double cause,the one,the diurnal motion of the Earth; the other, its simple motion in its own Epicycle. For these Mountains being (by reason of their height) more eminent then all the rest of the parts of the Earth, do by both these Motions drive the Aire from the West Eastwards. To which though the diurnal Motion contribute but little; yet seeing I have supposed that the simple Motion of the Earth in its own Epicycle, makes two revolutions in the same time in which the diurnal Mo-

tion makes but one; and that the Semidiameter of the Epicycle is double to the Semidiameter of the diurnal Conversion, the Motion of every point of the Earth in its own Epicycle will have its velocity quadruple to that of the diurnal Motion; so that by both these Motions together, the tops of those Hils will sensibly be moved against the Aire; and consequently a Wind will be felt. For whether the Air strike the Sentient, or the Sentient the Air, the perception of Motion will be the same. But this Wind, seeing it is not caused by the ascent of Vapours, must necessarily be very Constant.

4 When one Cloud is already ascended into the Aire, if another Cloud ascend towards it, that part of the Aire which is intercepted between them both, must of necessity be pressed out every way. Also when both of them, whilest the one ascends, and the other either stayes, or descends, come to be joyned in such manner as that the æthereal substance be shut within them on every side, it will by this compression also go out by penetrating the Water. But in the mean time, the hard particles which are mingled with the Aire, and are agitated (as I have supposed) with Simple Motion, wil not pass through the water of the clouds, but be more straightly compressed within their cavities. And this I have demonstrated at the 4th and 5th Articles of the 22th Chapter. Besides, seeing the Globe of the Earth floateth in the Aire which is agitated by the Sunnes Motion, the parts of the Aire resisted by the Earth, will spread themselves every way upon the Earths Superficies; as I have shewn at the 8th Article of the 21th Chapter.

5 We perceive a Body to be Hard, from this, that when touching it we would thrust forwards that part of the same which we touch, we cannot do it otherwise then by thrusting forwards the whole Body. We may indeed easily and sensibly thrust forwards any particle of the Aire or Water which we touch, whilst yet the rest of its parts remain (to sense) unmoved. But we cannot do so to any part of a stone. Wherfore I define a *Hard* Body to be that, whereof no part can be sensibly moved, unless the whole be moved. Whatsoever therefore is Soft or Fluid, the same can never be made Hard but by such motion, as makes many of the parts together stop the motion of some one part, by resisting the same.

6 These things premised, I shall shew a possible cause why there

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is greater Cold neer the Poles of the Earth, then further from them. The motion of the Sunne between the Tropicks, driving the Aire towards that part of the Earths Superficies which is perpendicularly under it, makes it spread it self every way; and the velocity of this expansion of the Aire grows greater and greater, as the Superficies of the Earth comes to be more and more straightned, that is to say, as the Circles which are parallel to the Æquator come to be less and less. Wherefore this expansive motion of the Aire, drives before it the parts of the Aire which are in its way continually towards the Poles more and more strongly, as its force comes to be more and more united, that is to say, as the Circles which are parallel to the Æquator are less and less; that is, so much the more, by how much they are neerer to the Poles of the Earth. In those places therefore which are neerer to the Poles, there is greater Cold, then in those which are more remote from them. Now this expansion of the Aire upon the Superficies of the Earth from East to West, doth by reason of the Sunnes perpetual accession to the places which are successively under it, make it Cold at the time of the Sunnes Rising and Setting; but as the Sunne comes to be continually more and more perpendicular to those cooled places, so by the Heat which is generated by the supervening Simple Motion of the Sunn, that Cold is again remitted; and can never be great, because the action by which it was generated, is not permanent. Wherefore I have rendred a possible cause of Cold in those places that are neer the Poles, or where the obliquity of the Sunne is great.

7 How Water may be congeled by Cold, may be explained in this manner. Let A (in the first figure) represent the Sunne, and B the Earth. A will therefore be much greater then B. Let E F be in the plain of the Equinoctial; to which let G H, I K and L C be parallel. Lastly, let C and D be the Poles of the Earth. The Aire therefore by its action in those parallels will rake the Superficies of the Earth; and that with motion so much the stronger, by how much the parallel Circles towards the Poles grow less and less. From whence must arise a Wind, which will force together the uppermost parts of the water, and withal raise them a little, weakning their endeavour towards the Center of the Earth. And from

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their endeavour towards the center of the Earth, joyned with the endeavour of the said Wind, the uppermost parts of the water will be pressed together, and coagulated, that is to say, the top of the water will be skinned over and hardned. And so againe the Water next the top will be hardned in the same manner, till at length the Ice be thick. And this Ice being now compacted of little hard Bodies, must also containe many particles of ayre received into it.

As Rivers and Seas, so also in the same manner may the Clouds be frozen. For when by the ascending and descending of severall Clouds at the same time, the Air intercepted between them is by compression forced out, it rakes, & by little & little hardens them. And though those smal drops (which usually make Clouds) be not yet united into greater Bodies, yet the same Wind will be made; & by it, as water is congeled into Ice, so will Vapours in the same manner be congeled into Snow. From the same cause it is that Ice may be made by art, and that not farre from the fire. For it is done by the mingling of Snow and Salt together, and by burying in it a small vessell full of Water. Now while the Snow and Salt (which have in them a great deale of aire) are melting, the aire which is pressed out every way in Wind, rakes the sides of the Vessel; and as the Wind by its motion rakes the Vessell, so the Vessell by the same motion and action congeles the Water within it.

We find by experience, that Cold is allwayes more Remisse in places where it raynes, or where the weather is cloudy (things being alike in all other respects) then where the aire is cleare. And this agreeth very well with what I have sayd before. For in cleare weather, the course of the Wind which (as I sayd even now) rakes the Superficies of the Earth, as it is free from all interruption, so also it is very strong. But when small drops of water are either rising or falling, that Wind is repelled, broken and dissipated by them; and the lesse the Wind is, the lesse is the Cold.

We find also by experience, that in deep Wells the Water freezeth not so much, as it doth upon the Superficies of the Earth. For the Wind by which Ice is made, entring into the Earth (by reason of the laxity of its parts) more or lesse, loseth some of its force,

force, though not much. So that if the Well be not deep, it will freeze; whereas if it be so deep, as that the Wind which causeth cold cannot reach it, it will not freeze.

We find moreover by experience, that Ice is lighter then Water. The cause whereof is manifest from that which I have already shewn, namely, that Aire is received in and mingled with the particles of the Water whilest it is in congeling.

8 We have seen one way of making things Hard, namely, by Congelation. Another way is thus. Having already supposed, that innumerable Atomes, some harder then others, and that have several simple motions of their own, are intermingled with the æthereal substance; it follows necessarily from hence, that by reason of the fermentation of the whole Aire (of which I have spoken in the 21 Chap.) some of those Atomes meeting with others, will cleave together, by applying themselvs to one another in such manner as is agreeable to their motions and mutual contacts; and (seeing there is no *Vacuum*) cannot be pulled asunder, but by so much force as is sufficient to overcome their Hardness.

Now there are innnmerable degrees of Hardness. As (for example) there is a degree of it in Water, as is manifest from this, that upon a plain it may be drawn any way at pleasure by ones finger. There is a greater degree of it in clammy liquors; which when they are poured out, doe in falling downwards dispose themselves into one continued thred; which thred before it be broken will by little and little diminish its thickness, till at last it be so small, as that it seems to break onely in a point; and in their separation the external parts break first from one another, and then the more internal parts successively one after another. In Wax there is yet a greater degree of Hardness. For when we would pull one part of it from another, we first make the whole mass slenderer, before we can pull it asunder. And how much the harder any thing is which we would break, so much the more force we must apply to it. Wherefore, if we go on to harder things, as Ropes, Wood, Metals, Stones, &c. reason prompteth us to believe that the same (though not alwayes sensibly) will necessarily happen; and that even the hardest things are broken asunder in the same manner, namely, by Solution of their continuity, begun in the outermost Superficies,

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and proceeding successively to the innermost parts. In like manner, when the parts of Bodies are to be separated, not by pulling them asunder, but by breaking them, the first separation will necessarily be in the convex Superficies of the bowed part of the Body, and afterwards in the concave Superficies. For in all bowing, there is in the convex Superficies an endeavour in the parts to go one from another, and in the concave Superficies to penetrate one another.

This being well understood, a reason may be given, how two Bodies which are contiguous in one common Superficies, may by force be separated without the introduction of *Vacuum*; though *Lucretius* thought otherwise, believing that such separation was a strong establishment of *Vacuum*. For a Marble Pillar being made to hang by one of its bases, if it be long enough it will by its own weight be broken asunder, and yet it will not necessarily follow that there should be *Vacuum*, seeing the solution of its continuity may begin in the Circumference, and proceed successively to the midst thereof.

Lastly, Wine is not so easily congeled as Water, because in Wine there are particles which being not fluid, are moved very swiftly, and by their motion congelation is retarded; but if the Cold prevail against this motion, then the outermost parts of the Wine will be first frozen, and afterwards the inner parts; whereof this is a signe, that the Wine which remains unfrozen in the midst will be very strong.

9 Another cause of Hardness in some things may be in this manner. If a soft Body consist of many hard particles, which by the intermixture of many other fluid particles cohære but loosely together, those fluid parts (as hath been shewn in the last Article of the 21 Chapter) will be exhaled; by which means each hard particle will apply it self to the next to it according to a greater Superficies; and consequently they will cohære more closely to one another; that is to say, the whole mass will be made Harder.

10 Again, in some things Hardness may be made to a certain degree, in this manner. When any fluid substance hath in it certain very small Bodies intermingled, which being moved with simple motion of their own, contribute like motion to the parts of the

fluid ſubſtance, and this be done in a ſmall encloſed ſpace (as in the hollow of a little Sphere, or a very ſlender Pipe) if the motion be vehement, and there be a great nnmber of theſe ſmall encloſed Bodies, two things will happen; the one, that the fluid ſubſtance will have an endeavour of dilating it ſelf at once every way; the other, that if thoſe ſmal Bodies can no where get out, then from their reflexion it will follow, that the motion of the parts of the encloſed fluid ſubſtance, which was vehement before, will now be much more vehement. Wherefore if any one particle of that fluid ſubſtance ſhould be touched & preſſed by ſome external Movent, it could not yeild but by the application of very ſenſible force. Wherefore the fluid ſubſtance which is encloſed, and ſo moved, hath ſome degree of Hardneſs. Now greater and leſs degree of Hardneſs depends upon the quantity and velocity of thoſe ſmall Bodies and upon the narrowneſs of the place both together.

11 Such things as are made Hard by ſudden heat, namely, ſuch as are hardned by fire, are commonly reduced to their former ſoft form by Maceration. For fire hardens by Evaporation, and therefore if the evaporated moiſture be reſtored again, the former nature and form is reſtored together with it. And ſuch things as are frozen with Cold, if the Wind by which they were frozen change into the oppoſite quarter, they will be unfrozen again (unleſs they have gotten a habit of new motion or endeavour by long continuance in that hardneſs). Nor is it enough to cauſe thawing, that there be a ceſſation of the Freezing Wind (for the taking away of the Cauſe doth not deſtroy a produced effect); but the thawing alſo muſt have its proper cauſe, namely, a contrary Wind, or at leaſt a Wind oppoſite in ſome degree. And this we finde to be true by experience. For if Ice be laid in a place ſo well encloſed that the motion of the Aire cannot get to it, that Ice will remain unchanged, though the place be not ſenſibly cold.

12 Of Hard Bodies, ſome may manifeſtly be bowed; others not, but are broken in the very firſt moment of their bending. And of ſuch Bodies as may manifeſtly be bended, ſome being bent, do as ſoon as ever they are ſet at liberty, Reſtore themſelves to their former poſture; others remain ſtill bent. Now if the cauſe of this Reſtitution be asked, I ſay it may be in this manner; namely, that the particles of the bended Body, whileſt it is held bent, do neverthe-

less retain their motion; and by this motion they restore it as soon as the force is removed by which it was bent. For when any thing is bent (as a plate of steel), and as soon as the force is removed restores it self again, it is evident that the cause of its restitution cannot be referred to the ambient aire; nor can it be referred to the removal of the force by which it was bent; for in things that are at rest, the taking away of impediments, is not a sufficient cause of their future Motion; there being no other cause of Motion, but Motion. The cause therefore of such Restitution is in the parts of the Steel it self. Wherefore whilest it remains bent, there is in the parts of which it consisteth, some motion, though invisible, that is to say, some endeavour at least that way by which the restitution is to be made; and therefore this endeavour of all the parts together is the first beginning of Restitution; so that the impediment being removed, that is to say, the force by which it was held bent, it will be restored again. Now the motion of the parts by which this is done, is that which I called Simple Motion, or Motion returning into it self. When therefore in the bending of a plate, the ends are drawn together, there is on one side a mutual compression of the parts; which compression is one endeavour opposite to another endeavour; and on the other side a divulsion of the parts. The endeavour therefore of the parts on one side tends to the restitution of the plate from the middle towards the ends; and on the other side, from the ends towards the middle. Wherefore the impediment being taken away, this endeavour (which is the beginning of restitution) will restore the plate to its former posture. And thus I have given a possible cause why some Bodies when they are bent Restore themselves again, which was to be done.

As for Stones, seeing they are made by the accretion of many very hard particles within the Earth; which particles have no great coherence, that is to say, touch one another in small latitude, and consequently admit many particles of aire, it must needs be that in bending of them, their internal parts will not easily be compressed by reason of their hardness. And because their coherence is not firm, as soon as the external hard particles are disjoyned, the ætherеal parts will necessarily break out, and so the Body will suddenly be broken.

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13 Those Bodies are called *Diaphanous*, upon which whilest the Beams of a lucid Body do work, the action of every one of those Beams is propagated in them in such manner as that they still retain the same order amongst themselves, or the inversion of that order; and therefore Bodies which are perfectly Diaphanous, are also perfectly homogeneous. On the contrary, an *Opacous* Body is that, which by reason of its heterogeneous nature, doth by innumerable reflexions and refractions in particles of different figures and unequal hardness, weaken the Beams that fall upon it before they reach the Eie. And of Diaphanous Bodies, some are made such by Nature from the beginning; as the substance of the Aire, and of the Water, and perhaps also some parts of Stones, unless these also be Water that has been long congeled. Others are made so by the power of Heat, which congregates homogeneons Bodies. But such as are made Diaphanous in this manner, consist of parts which were formerly Diaphanous.

14 In what manner Clouds are made by the motion of the Sunne, elevating the particles of Water from the Sea and other moist places, hath been explained in the 26th Chapter. Also how Clouds come to be frozen, hath been shewn above at the 7th Article. Now from this, that Aire may be enclosed, as it were in Caverns, and pent together more and more by the meeting of ascending and descending Clouds, may be deduced a possible Cause of *Thunder* and *Lightening*. For seeing the Aire consists of two parts, the one Æthereal, which has no proper motion of its own, as being a thing divisible into the least parts; the other Hard, namely, consisting of many hard Atomes which have every one of them a very swift simple motion of its own; whilest the Clouds by their meeting do more and more straighten such Cavities as they intercept, the Æthereal parts will penetrate and pass through their watry substance; but the hard parts will in the mean time be the more thrust together, and press one another, and consequently (by reason of their vehement motions) they will have an endeavour to rebound from each other. Whensoever therefore the compression is great enough, and the concave parts of the Clouds are (for the cause I have already given) congeled into Ice, the Cloud wil necessarily be broken; & this breaking of the Cloud produceth

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the first clap of Thunder. Afterwards, the Aire which was pent in, having now broken through, makes a concussion of the Aire without; and from hence proceeds the roaring and murmur which follows; and both the first Clap and the Murmur that follows it, make that noise which is called *Thunder*. Also from the same Aire breaking through the Clouds, and with concussion falling upon the Eie, proceeds that action upon our Eie, which causeth in us a perception of that Light which we call *Lightening*. Wherefore I have given a possible cause of *Thunder* and *Lightening*.

15 But if the Vapours which are raised into Clouds, do run together again into Water, or be congeled into Ice, from whence is it (seeing both Ice and Water are heavy) that they are sustained in the Aire? Or rather, what may the cause be, that being once elevated, they fall down again? For there is no doubt but the same force which could carry up that Water, could also sustain it there. Why therefore being once carried up, doth it fall again? I say it proceeds from the same Simple Motion of the Sunne, both that Vapours are forced to ascend, and that Water gathered into Clouds is forced to descend. For in the 21th Chapter and 11th Article I have shewn how Vapours are elevated; and in the same Chapter and 5th Article I have also shewn how by the same motion Homogeneous Bodies are congregated, & Heterogeneous dissipated; that is to say, how such things as have a like nature to that of the Earth, are driven towards the Earth; that is to say, what is the cause of the descent of Heavy Bodies. Now if the action of the Sun be hindered in the raising of vapours, and be not at all hindered in the casting of them down, the Water will descend. But a Cloud cannot hinder the action of the Sunne in making things of an earthly nature descend to the Earth, though it may hinder it in making Vapours ascend For the lower part of a thick Cloud is so covered by its upper part, as that it cannot receive that action of the Sunne by which Vapours are carried up, because Vapours are raised by the perpetual fermentation of the Aire, or by the separating of its smallest parts from one another, which is much weaker when a thick Cloud is interposed, then when the Skie is cleere. And therefore whensoever a Cloud is made thick enough, the water which

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would not descend before, will then descend, unless it be kept up by the agitation of the Winde. Wherefore I have rendred a possible cause, both why the Clouds may be sustained in the Aire, and also why they may fall down again to the Earth; which was propounded to be done.

16 Granting that the Clouds may be frozen, it is no wonder if the Moon have been seen eclipsed at such time as she hath been almost two degrees above the Horizon, the Sunne at the same time appearing in the Horizon; for such an Eclipse was observed by *Mestline* at *Tubing* in the year 1590. For it might happen that a frozen Cloud was then interposed between the Sunne and the Eie of the Observer. And if it were so, the Sunne which was then almost two Degrees below the Horizon, might appear to be in it, by reason of the passing of his Beams through the Ice. And it is to be noted, that those that attribute such refractions to the *Atmosphere*, cannot attribute to it so great a refraction as this. Wherefore not the *Atmosphere*, but either Water in a continued Body, or else Ice must be the cause of that refraction.

17 Again, granting that there may be Ice in the Clouds, it will be no longer a wonder that many Sunnes have sometimes appeared at once. For Looking-glasses may be so placed, as by reflections to shew the same object in many places. And may not so many frozen Clouds serve for so many Looking-glasses? and may they not be fitly disposed for that purpose? Besides, the number of Appearances may be encreased by refractions also; and therefore it would be a greater wonder to me, if such *Phænomena* as these should never happen.

And were it not for that one *Phænomenon* of the new Starre which was seen in *Cassiopæa*, I should think *Comets* were made in the same manner, namely, by Vapours drawn not onely from the Earth, but from the rest of the Planets also, and congeled into one continued Body. For I could very well from hence give a reason both of their Haire, and of their motions. But seeing that Starre remained sixteen whole moneths in the same place amongst the fixed Starres, I cannot believe the matter of it was Ice. Wherefore I leave to others, the disquisition of the cause of Comets; concerning which, nothing that hath hitherto been published, (besides

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the bare Histories of them) is worth considering.

18 The Heads of Rivers may be deduced from Rain-water, or from melted Snowes, very easily; but from other causes, very hardly, or not at all. For both Rain-water, and melted Snowes run down the descents of Mountains; and if they descend onely by the outward Superficies, the Showres or Snowes themselves may be accounted the Springs or Fountains; but if they enter the Earth & descend within it, then wheresoever they break out, there are their Springs. And as these Spings make small streams, so, many small streams running together make Rivers. Now there was never any Spring foũd, but where the Water wch flowed to it, was either further, or at least as farre from the center of the Earth, as the Spring it self. And whereas it has bin objected by a great Philosopher, that in the top of *Mount-Cenis* (which parts Savoy from Piemont) there Springs a River which runs down by *Susa*; it is not true. For there are above that River, for two miles length, very high hils on both sides, which are almost perpetually covered with Snow; from which, innumerable little streams running down do manifestly supply that River with water sufficient for its magnitude.

Chap. XXIX.

Of Sound, Odour, Savour, and Touch.

1 The definition of Sound, and the distinctions of Sounds. 2 The cause of the degrees of Sounds. 3 The difference between Sounds Acute and Grave. 4 The difference between Clear and Hoarse Sounds, whence. 5 The Sound of Thunder and of a Gunne, whence it proceeds. 6 Whence it is, that Pipes by blowing into them have a clear Sound. 7 Of Reflected Sound. 8 From whence it is that Sound is Uniform and Lasting. 9 How Sound may be helped aud hindered by the Wind. 10 Not onely Aire, but other Bodies how hard soever they be, conveigh Sound. 11 The causes of Grave and Acute Sounds, and of Concent. 12 Phanomena for Smelling. 13 The first Organ and the generation of Smelling. 14 How it is helped by Heat and by Wind. 15 Why such Bodies are least smelt, which have least intermixture of Aire in them. 16 Why Odorous things become more Odorous by being bruised. 17 The first Organ of Tasting; and why some Savours cause Nauseousness. 18 The first Organ of Feeling; and how we come to the knowledge of such Objects as are common to the Touch and other Senses.

SOUND *is Sense generated by the action of the Medium, when its motion reacheth the Eare and the rest of the Organs of Sense.* Now the motion of the Medium is not the Sound it self, but the cause of it. For the Phantasme which is made in us, that is to say, the Reaction of the Organ is properly that which we call *Sound.*

The principal distinctions of Sounds are these; First, that one Sound is stronger, another Weaker. Secondly, that one is more Grave, another more Acute. Thirdly, that one is Clear, another Hoarse. Fourthly, that one is Primary, another Derivative. Fifthly, that one is Uniform, another not. Sixthly, that one is more Durable, another less Durable. Of all which distinctions the members may be subdistinguished into parts distinguishable almost infinitely. For the variety of Sounds seems to be not much less then that of Colours.

As Vision, so Hearing is generated by the motion of the Medium, but not in the same manner. For Sight is from Pressure, that is, from an Endeavour; in which there is no perceptible progression of any of the parts of the Medium; but one part urging or thrusting on an other, propagateth that action successively to any distance whatsoever; whereas the motion of the Medium by which Sound is made, is a Stroke. For when we Hear, the Drumme of the Eare (which is the first Organ of Hearing) is stricken; and the Drumme being stricken, the *Pia Mater* is also shaken, and with it the Arteries which are inserted into it; by which the action being propagated to the Heart it self, by the reaction of the Heart a Phantasm is made which we call Sound; and (because the reaction tendeth outwards) we think it is without.

2 And seeing the effects produced by Motion, are greater or lesse, not onely when the Velocity is greater or less, but also when the Body hath greater or less Magnitude though the Velocity be the same; a Sound may be greater or lesse both these wayes. And because neither the greatest nor the least Magnitude or Velocity can be given, it may happen, that either the motion may be of so small velocity, or the Body it self of so small magnitude, as to produce no Sound at all; or either of them may be so great, as to take away the Faculty of Sense by hurting the Organ.

From hence may be deduced possible causes of the strength and weakness of Sounds in the following *Phænomena*.

The first whereof is this, That if a man speak through a Trunk which hath on end applyed to the mouth of the Speaker, and the other to the eare of the Hearer, the Sound will come stronger then it would do through the open Aire. And the cause (not onely the possible, but the certain and manifest cause) is this, that the Aire which is moved by the first breath, and carried forwards in the Trunk, is not diffused, as it would be in the open Aire, and is consequently brought to the eare almost with the same velocity with which it was first breathed out; Whereas in the open Aire, the first motion diffuseth it self every way into Circles, such as are made by the throwing of a Stone into a standing water, where the velocity grows less and less as the Undulation proceeds further and further from the beginning of its motion.

The second is this, That if the Trunk be short, and the

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end which is applyed to the mouth be wider then that which is applyed to the eare, thus alſo the Sound will be ſtronger then if it were made in the open aire. And the cauſe is the ſame; namely, that by how much the wider end of the Trunk is leſs diſtant from the beginning of the Sound, by ſo much the leſs is the diffuſion.

The third, That it is eaſier for one that is within a Chamber, to heare what is ſpoken without, then for him that ſtands without, to hear what is ſpoken within. For the Windows and other inlets of the moved Aire, are as the wide end of the Trunk. And for this reaſon ſome creatures ſeem to hear the better, becauſe Nature ha s beſtowed upon them wide and capacious Ears.

The fourth is this, That though he which ſtandeth upon the Sea ſhore, cannot heare the Colliſion of the two neereſt waves, yet neverthleſs he hears the roaring of the whole Sea. And the cauſe ſeems to be this, that though the ſeveral colliſions move the Organ, yet they are not ſeverally great enough to cauſe Senſe; whereas nothing hinders but that all of them together may make Sound.

3 That Bodies when they are ſtricken do yeild ſome a more Grave, others a more Acute Sound, the cauſe may conſiſt in the difference of the times in which the parts ſtricken and forced out of their places, return to the ſame places again. For in ſome Bodies, the reſtitution of the moved parts is quick, in others ſlow. And this alſo may be the cauſe why the parts of the Organ which are moved by the Medium, return to their reſt again, ſometimes ſooner, ſometimes later. Now by how much the Vibrations, or the reciprocal motions of the parts are more freqnent, by ſo much doth the whole Sound made (at the ſame time) by one ſtroke, conſiſt of more, and conſequently of ſmaller parts. For what is Acute in Sound, the ſame is Subtle in Matter; and both of them, namely, Acute Sound, and Subtle Matter conſiſt of very ſmall parts, that of Time, and this of the Matter it ſelf.

The third diſtinction of Sounds cannot be conceived clearly enough by the names I have uſed of *Clear* and *Hoarſe*, nor by any other that I know; and therefore it is needful to explain them by examples. When I ſay *Hoarſe*, I underſtand Whiſpering and Hiſſing, and whatſoever is like to theſe, by what appellation ſoever it be expreſſed. And Sounds of this kind ſeem to be made by the force of ſome ſtrong Wind, raking rather then ſtriking ſuch hard

Bodies, as it falls upon. On the contrary, when I uſe the word *Clear*, I do not underſtaud ſuch a Sound as may be eaſily and diſtinctly heard, for ſo Whiſpers would be Clear, but ſuch as is made by ſomewhat that is Broken; and ſuch as is Clamor, Tinkling, the Sound of a Trumpet, &c. and (to expreſs it ſignificantly in one word), Noiſe. And ſeeing no Sound is made but by the concourſe of two Bodies at the leaſt, by which conconrſe it is neceſſary that there be as well Reaction as Action, that is to ſay, one motion oppoſite to another; it follows, that according as the proportion between thoſe two oppoſite motions is diverſified, ſo the Sounds which are made will be different from one another. And whenſoever the proportion between them is ſo great, as that the motion of one of the Bodies be inſenſible if compared with the motion of the other, then the Sound will not be of the ſame kind; as when the Wind falls very obliquely upon a hard Body, or when a hard Body is carried ſwiftly through the Aire; for then there is made that Sound which I call a Hoarſe Sound, in Greek σύριγμος. Therefore the breath blown with violence from the mouth makes a Hiſſing, becauſe in going out it rakes the Superficies of the Lips, whoſe reaction againſt the force of the breath is not Senſible. And this is the cauſe why the Winds have that Hoarſe Sound. Alſo if two Bodies how hard ſoever, be rubbed together with no great preſſure, they make a Hoarſe Sound. And this Hoarſe Sound when it is made (as I have ſaid) by the Aire raking the Superficies of a hard Body, ſeemeth to be nothing but the dividing of the Aire into innumerable and very ſmall Files. For the aſperity of the Superficies doth by the eminencies of its innumerable parts divide or cut in pieces the Aire that ſlides upon it.

4 *Noiſe*, or that which I call Clear Sound, is made two wayes; one, by two Hoarſe Sounds, made by oppoſite motions; the other, by Colliſion, or by the ſuddain pulling aſunder of two Bodies, whereby their ſmall particles are put into commotion, or being already in commotion, ſuddenly reſtore themſelves again; which motion making impreſſion upon the Medium, is propagated to the Organ of Hearing. And ſeeing there is in this Colliſion, or divulſion, an endeavour in the particles of one Body, oppoſite to the endeavour of the particles of the other Body, there will alſo be made in the Organ of Hearing a like oppoſition of endeavours, that is to

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ſay, of motions, and conſequently the Sound ariſing from thence, will be made by two oppoſite motions, that is to ſay, by two oppoſite Hoarſe Sounds in one and the ſame part of the Organ. For (as I have already ſaid) a Hoarſe Sound ſuppoſeth the ſenſible motion of but one of the Bodies. And this oppoſition of motions in the Organ is the cauſe why two Bodies make a Noyſe, when they are either ſuddenly ſtricken againſt one another, or ſuddenly broken aſunder.

5 This being granted; and ſeeing withall, that Thunder is made by the vehement eruption of the Aire out of the cavities of congeled Clouds; the cauſe of the great Noyſe or Clap, may be the ſuddain breaking aſunder of the Ice. For in this action it is neceſſary, that there be not onely a great concuſſion of the ſmall particles of the broken parts, but alſo that this Concuſſion (by being communicated to the Aire) be carried to the Organ of Hearing, & make impreſſion upon it. And then, from the firſt reaction of the Organ proceeds that firſt and greateſt Sound, which is made by the colliſion of the parts whilſt they reſtore themſelvs. And ſeeing there is in all Concuſſion a reciprocation of Motion forwards and backwards in the parts ſtricken, (for oppoſite motions cannot extinguiſh one another in an inſtant, as I have ſhewn in the 11th Art. of the 8th Chap.) it follows neceſſarily, that the Sound will both continue, and grow weaker and weaker, till at laſt the action of the reciprocating aire grow ſo weak, as to be unperceptible. Wherefore a poſſible cauſe is given both of the firſt fierce Noyſe of the Thunder; and alſo of the Murmur that follows it.

The cauſe of the great Sound from a diſcharged piece of Ordnance, is like that of a Clap of Thunder. For the Gunpowder being fired, doth in its endeavour to go out, attempt every way the ſides of the metal in ſuch manner, as that it enlargeth the Circumference all along, and withall ſhortneth the axis; ſo that whileſt the peece of Ordnance is in diſcharging, it is made both wider and ſhorter then it was before; and therefore alſo preſently after it is diſcharged its wideneſs will be diminiſhed, and its length encreaſed again by the reſtitution of all the particles of the matter of which it conſiſteth to their former poſition. And this is done with ſuch motion of the parts, as are not onely very vehement, but alſo oppoſite to one another; which motions

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being communicated to the Aire, make impression upon the Organ, and by the reaction of the Organ create a Sound; which lasteth for some time, as I have already shewn in this Article.

I note by the way (as not belonging to this place) that the possible cause why a Gun recoyles when it is shot off, may be this; That being first swoln by the force of the fire, and afterwards restoring it self; from this restitution there proceeds an endeavour from all the sides towards the cavity; and consequently this endeavour is in those parts which are next the breech; which being not hollow, but solid, the effect of the restitution is by it hindered and diverted into the length; and by this means both the breech and the whole Gun is thrust backwards; and the more forcibly by how much the force is greater by which the part next the breech is restored to its former posture; that is to say, by how much the thiner is that part. The cause therefore why Gunnes recoyle, some more, some less, is the difference of their thickness towards the breech; & the greater that thickness is, the less they recoyl; and contrarily.

6 Also the cause why the Sound of a Pipe, which is made by blowing into it, is neverthelesse Clear, is the same with that of the Sound which is made by collision. For if the breath when it is blown into a Pipe doe onely rake its concave Superficies, or fall upon it with a very sharp angle of incidence, the Sound will not be Clear, but Hoarse. But if the angle be great enough, the percussion which is made against one of the hollow sides, will be reverberated to the opposite side; and so successive repercussions will be made from side to side, till at last the whole concave Superficies of the Pipe be put into motion; which motion will be reciprocated, as it is in Collision; and this reciprocation being propagated to the Organ, from the reaction of the Organ will arise a Cleare Sound, such as is made by Collision, or by breaking asunder of hard Bodies.

In the same manner it is with the Sound of a Mans voice. For when the breath passeth out, without interruption, and doth but lightly touch the cavities through which it is sent, the Sound it maketh is a Hoarse Sound. But if in going out it strike strongly upon the *Larinx*, then a Clear Sound is made, as in a Pipe. And the same breath, as it comes in divers manners to the Palate, the Tongue, the Lips, the Teeth, and other Organs of Speech, so the Sounds,

Sounds into which it is articulated become different from one another.

7 I call that *Primary Sound*, which is generated by motion from the ſounding Body to the Organ in a ſtraight line without reflexion; and I call that *Reflected Sound*, which is generated by one or more reflexions; being the ſame with that we call *Echo*, and is iterated as often as there are reflexions made from the Object to the Eare. And theſe reflexions are made by Hils, Wals, and other reſiſting Bodies, ſo placed, as that they make more or fewer reflexions of the motion, according as they are themſelves more or fewer in number; and they make them more or leſs frequently, according as they are more or leſs diſtant from one another. Now the cauſe of both theſe things is to be ſought for in the ſituation of the reflecting Bodies, as is uſually done in Sight. For the Lawes of Reflexion are the ſame in both, namely, that the Angles of Incidence and Reflexion be equal to one another. If therefore in a hollow Elliptique Body whoſe inſide is well poliſhed, or in two right Parabolical Solids which are joyned together by one common baſe, there be placed a Sounding Body in one of the Burning Points, & the Ear in the other, there will be heard a Sound by many degrees greater then in the open Aire; and both this, and the burning of ſuch combuſtible things, as being put in the ſame places are ſet on fire by the Sun-beams, are effects of one and the ſame cauſe. But as when the viſible Object is placed in one of the Burning Points, it is not diſtinctly ſeen in the other, becauſe every part of the Object being ſeen in every line which is reflected from the Concave Superficies to the Eie, makes a confuſion in the Sight; ſo neither is Sound heard articulately and diſtinctly when it comes to the Eare in all thoſe reflected lines. And this may be the reaſon, why in Churches which have arched rooffs, though they be neither Elliptical nor Parabolical; yet becauſe their figure is not much different from theſe, the voice from the Pulpit will not be heard ſo articulately as it would be if there were no vaulting at all.

8 Concerning the *Uniformity* and *Duration* of Sounds, both which have one common cauſe, we may obſerve, that ſuch Bodies as being ſtricken yeild an unequal or harſh Sound, are very heterogeous, that is to ſay, they conſiſt of parts which are very unlike both in figure and hardneſs, ſuch as are Wood, Stones, and others not a

few

ſew. When theſe are ſtricken, there follows a concuſſion of their internal particles, and a reſtitution of them again. But they are neither moved alike, nor have they the ſame action upon one another; ſome of them recoyling from the ſtroke whileſt others which have already finiſhed their recoylings are now returning;by which means they hinder and ſtop on another. And from hence it is that their motions are not only unequal and harſh, but alſo that their reciprocations come to be quickly extinguiſhed. Whenſoever therfore this motion is propagated to the Eare, the Sound it makes is Unequal and of ſmall Duration. On the contrary, if a Body that is ſtricken,be not onely ſufficiently hard,but have alſo the particles of which it conſiſteth like to one another both in hardneſs and figure, (ſuch as are the particles of Glaſs and Metals, which being firſt melted do afterwards ſettle and harden) the Sound it yeildeth, will (becauſe the motions of its parts and their reciprocations are like and Uniform) be Uniform and pleaſant, and be more or leſs Laſting according as the Body ſtricken hath gteater or leſs magnitude. The poſſible cauſe therefore of Sounds Uniform, and Harſh, and of their longer or ſhorter Duration, may be one and the ſame likeneſs and unlikeneſs of the internal parts of the Sounding Body, in reſpect both of their figure and hardneſs.

Beſides, if two plain Bodies of the ſame matter, and of equal thickneſs, do both yeild an Uniform Sound, the Sound of that Body which hath the greateſt extent of length will be the longeſt heard. For the motion which in both of them hath its beginning from the point of percuſſion, is to be propagated in the greater Body through a greater Space, and conſequently that propagation will require more time; and therefore alſo the parts which are moved will require more time for their return. Wherefore all the reciprocations cannot be finiſhed but in longer time; and being carried to the Eare, will make the Sound laſt the longer. And from hence it is manifeſt, that of hard Bodies which yeild an Uniform Sound, the Sound laſteth longer which comes from thoſe that are round and hollow, then from thoſe that are plain, if they be like in all other reſpects. For in circular lines the action which begins at any point, hath not frō the figure any end of its propagation, becauſe the line in which it is propagated returns again to its beginning; ſo that the figure hinders not but that the motion may

have

have infinite progreſſion; whereas in a plain, every line hath its magnitude finite, beyond which the action cannot proceed. If therefore the matter be the ſame, the motion of the parts of that Body whoſe figure is round and hollow, wil laſt longer, then of that which is plain.

Alſo, if a ſtring which is ſtretched, be faſtned at both ends to a hollow Body, and be ſtricken, the Sound will laſt longer then if it were not ſo faſtned; becauſe the trembling or reciprocation which it receives from the ſtroke, is by reaſon of the connexion communicated to the hollow Body; and this trembling, if the hollow Body be great, will laſt the longer by reaſon of that greatneſs. Wherefore alſo (for the reaſon above mentioned) the Sound will laſt the longer.

9 In Hearing, it happens (otherwiſe then in Seeing) that the action of the Medium is made ſtronger by the Wind when it blows the ſame way, and weaker when it blows the contrary way. The cauſe whereof cannot proceed from any thing but the different generation of Sound and Light. For in the generation of Light, none of the parts of the Medium between the object and the Eie are moved from their own places to other places ſenſibly diſtant; but the action is propagated in ſpaces imperceptible; ſo that no contrary Wind can diminiſh, nor favourable Winde encreaſe the Light, unleſs it be ſo ſtrong as to remove the Object further off, or bring it nearer to the Eie. For the Wind, that is to ſay, the aire moved, doth not by its interpoſition between the object and the eie, worke otherwiſe then it would doe if it were ſtil and calme. For where the preſſure is perpetuall, one part of the aire is no ſooner carried away, but another by ſucceeding it receives the ſame impreſſion which the part carried away had received before. But in the generation of Sound, the firſt colliſion or breaking aſunder, beateth away & driveth out of its place the neareſt part of the aire, and that to a conſiderable diſtance, and with conſiderable velocity; and as the circles grow (by their remoteneſſe) wider and wider, ſo the aire being more & more diſſipated, hath alſo its motion more & more weakned. Whenſoever therfore the air is ſo ſtricken as to cauſe Sound, if the Wind fall upon it, it will move it all, neerer to the Eare if it blow that way, and further from it if it blow the contrary way; ſo that

ding as it blowes from or towards the Object, ſo the Sound which is heard will ſeeme to come from a neerer or remoter place; and the action by reaſon of the unequall diſtances be ſtrengthened or debilitated.

From hence may be underſtood the reaſon, why the voice of ſuch as are ſaid to ſpeake in their bellies, though it be uttered neer hand, is neverthleſſe heard by thoſe that ſuſpect nothing, as if it were a great way off. For having no former thought of any determined place from which the voice ſhould proceed, and judging according to the greateſſe, if it be weake they thinke it a great way off, if ſtrong neer. Theſe *Ventriloqui* therefore by forming their voice, not (as others) by the emiſſion of their breath, but by drawing it inwards, doe make the ſame appear ſmall and weake; which weakneſſe of the voice deceives thoſe that neither ſuſpect the artifice, nor obſerve the endeavour which they uſe in ſpeaking; and ſo inſtead of thinking it weake they thinke it farre off.

10 As for the Medium which conveighs Sound, it is not Aire onely. For Water, or any other Body how hard ſoever may be that Medium. For the Motion may be propagated perpetually in any hard continuous Body; but by reaſon of the difficulty with which the parts of hard Bodies are moved, the motion in going out of hard matter makes but a weak impreſſion upon the Aire. Neverthelesſ if one end of a very long and hard beam be ſtricken, & the eare be applyed at the ſame time to the other end, ſo that when the action goeth out of the beam, the aire which it ſtriketh may immediately be received by the eare, and be carried to the *Tympanum*, the Sound will be conſiderably ſtrong.

In like manner, if in the night (when all other noyſe, which may hinder Sound, ceaſeth) a man lay his eare to the ground, he will hear the Sound of the ſteps of Paſſengers, though at a great diſtance; becauſe the motion which by their treading they communicate to the earth, is propagated to the eare by the uppermoſt parts of the earth which receiveth it from their feet.

11 I have ſhewn above, that the difference between Grave and Acute Sounds conſiſteth in this, that by how much the ſhorter the time is, in which the reciprocations of the parts of a Body ſtricken are made, by ſo much the more Acute will be the Sound.

Now

Now by how much a Body of the ſame bigneſs, is either more heavy, or leſs ſtretched, by ſo much the longer will the reciprocations laſt; and therefore heavier, and leſs ſtretched Bodies (if they be like in all other reſpects) will yeild a Graver Sound then ſuch as be lighter and more ſtretched.

As for the *Concent* of Sounds, it is to be conſidered, that the reciprocation or Vibration of the Aire (by which Sound is made) after it hath reached the Drumme of the eare, imprinteth a like Vibration upon the air that is encloſed within it; by which means the ſides of the Drumme within, are ſtricken alternately. Now the Concent of two Sounds conſiſts in this, that the *Tympanum* receive its ſtroke from both the Sounding Bodies in equal, & equally frequēt ſpaces of time; So that when two ſtrings make their Vibrations in the ſame times, the Concent they produce, is the moſt exquiſite of all other. For the ſides of the *Tympanum*, (that is to ſay, of the Organ of Hearing) will be ſtricken by both thoſe Vibrations together at once, on one ſide or other. For example, if the two equal ſtrings A B and C D be ſtricken together, and the latitudes of their Vibrations E F and G H be alſo equal, and the points E, G, F & H be in the concave ſuperficies of the *Tympanum*, ſo that it receive ſtrokes from both the ſtrings together in E and G, and again together in F and H, the Sound which is made by the Vibrations of each ſtring, will be ſo like, that it may be taken for the ſame Sound, and is called *Uniſon*; which is the greateſt Concord. Again, the ſtring A B retaining ſtill its former Vibration E F, let the ſtring C D be ſtretched, till its Vibration have double the ſwiftneſs it had before, and let E F be divided equally in I. In what time therefore the ſtring C D makes one part of its Vibration from G to H, in the ſame time the ſtring A B will make one part of its Vibration from E to I; and in what time the ſtring C D hath made the other part of its Vibration back from

H to G, in the ſame time another part of the Vibration of the ſtring A B will be made from I to F. But the points F and G are both in the ſides of the Organ, and therefore they will ſtrike the Organ both together, not at every ſtroke, but at every other ſtroke. And this is the neereſt Concord to Uniſon, and makes that Sound which is called an *Eighth*. Again, the Vibration of the ſtring A B remayning ſtill the ſame it was, let C D be ſtretched till its Vibration be ſwifter then that of the ſtring A B, in the proportion of 3 to 2, and let E F be divided into three equal parts in K and L. In what time therefore the ſtring C D makes one third part of its Vibration, which third part is from G to H, the ſtring A B will make one third part of its Vibration, that is to ſay, two thirds of E F, namely, E L. And in what time the ſtring C D makes another third part of its Vibration, namely H G, the ſtring A B will make another third part of its Vibration, namely, from L to F, and back again from F to L. Laſtly, whileſt the ſtring C D makes the laſt third part of its Vibration, that is, from G to H, the ſtring A B will make the laſt third part of its Vibration from L to E. But the points E and H are both in the ſides of the Organ. Wherefore at every third time the Organ will be ſtricken by the Vibration of both the ſtrings together, and make that Concord which is called a *Fifth*.

12 For the finding out of the cauſe of *Smels*, I ſhall make uſe of the evidence of theſe following *Phænomena*. Firſt, that Smelling is hindred by Cold, and helped by Heat. Secondly, that when the Wind bloweth from the Object, the Smel is the ſtronger; and contrarily when it bloweth from the Sentient towards the Object, the weaker; both which *Phænomena* are by experience manifeſtly found to be true in Doggs which follow the track of Beaſts by the Sent. Thirdly, that ſuch Bodies as are leſs pervious to the fluid Medium, yeild leſs Smell then ſuch as are more pervious; as may be ſeen in Stones and Metals, which compared with Plants and Living Creatures, and their Parts, Fruits and Excretions, have very little or no Smell at all. Fourthly, that ſuch Bodies as are of their own nature Odorous, become yet more Odorous when they are bruiſed. Fifthly, that when the breath is ſtopped (at leaſt in Men) nothing can be Smelt. Sixthly, that the ſenſe of Smelling is alſo taken

away

away by the stopping of the Nostrils, though the mouth be left open.

13 By the fourth and fifth *Phænomenon* it is manifest, that the first and immediate Organ of Smelling is the innermost cuticle of the Nostrils, and that part of it which is below the passage common to the Nostrils and the Palate. For when we draw breath by the Nostrils, we draw it into the Lungs. That breath therefore which conveighs Smels, is in the way which passeth to the Lungs, that is to say, in that part of the Nostrils which is below the passage through which the breath goeth. For nothing is Smelt, neither beyond the passage of the breath within, nor at all without the Nostrils.

And seeing that from different Smels there must necessarily proceed some mutation in the Organ, and all mutation is motion; it is therefore also necessary that in Smelling, the parts of the Organ, that is to say, of that internal cuticle, and the nerves that are inserted into it, must be diversly moved by different Smels. And seeing also that it hath been demonstrated, that nothing can be moved but by a Body that is already moved and contiguous; and that there is no other Body contiguous to the internal membrane of the nostrils, but breath, that is to say, attracted aire, and such little solid invisible Bodies (if there be any such) as are intermingled with the aire; it follows necessarily, that the cause of Smelling is either the motion of that pure aire or æthereal Substance, or the motion of those small Bodies. But this motion is an effect proceding from the Object of Smell; and therefore either the whole Object it self, or its several parts must necessarily be moved. Now we know, that Odorous Bodies make Odour though their whole bulk be not moved. Wherefore the cause of Odour is the motion of the invisible parts of the Odorous Body. And these invisible parts do either go out of the Object, or else retaining their former situation with the rest of the parts, are moved together with them, that is to say, they have simple and invisible motion. They that say there goes something out of the Odorous Body, call it an *Effluvium*; which *Effluvium* is either of the æthereal substance, or of the small Bodies, that are intermingled with it. But that all variety of Odours should proceed from the

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Effluviums of those small Bodies that are intermingled with the æthereal substance, is altogether incredible, for these considerations; First, that certain Unguents, though very little in quantity, do neverthelesse send forth very strong Odours, not onely to a great distance of place, but also to a great continuance of time, and are to be Smelt in every point both of that place and time; so that the parts issued out are sufficient to fil ten thousand times more space, then the whole Odorous Body is able to fill; which is impossible. Secondly, that whether that issuing out be with straight or with crooked motion, if the same quantity should flow from any other Odorous Body with the same motion, it would follow, that all Odorous Bodies would yeild the same Smell. Thirdly, that seeing those *Effluviums* have great Velocity of motion (as is manifest from this, that noysome Odours proceeding from caverns are presently Smelt at a great distance) it would follow, that by reason there is nothing to hinder the passage of those *Effluviums* to the Organ, such motion alone were sufficient to cause Smelling. Which is not so; for we cannot Smell at all, unless we draw in our breath through our Nostrils. Smelling therefore is not caused by the *Effluvium* of Atomes; nor, for the same reason is it caused by the *Effluvium* of æthereal substance; for so also we should Smell without the drawing in of our breath. Besides the æthereal substance being the same in all Odorous Bodies, they would always affect the Organ in the same manner, and consequently the Odours of all things would be like.

It remains therefore, that the cause of Smelling must consist in the Simple motion of the parts of Odorous Bodies, without any efflux or diminution of their whole substance. And by this motion, there is propagated to the Organ by the intermediate aire, the like motion, but not strong enough to excite Sense of it self without the attraction of aire by respiration. And this is a possible cause of Smelling.

14 The cause why Smelling is hindred by Cold, and helped by Heat, may be this, that Heat (as hath been shewn in the 21 Chapter) generateth Simple motion; and therefore also wheresoever it is already, there it will encrease it; and the cause of Smelling being encreased the Smell it self will also be encreased. As for the cause why

why the Wind blowing from the Object makes the Smell the stronger, it is all one with that for which the attraction of aire in respiration doth the same. For he that draws in the aire next to him, draws with it by succession that aire in which is the Object. Now this motion of the aire is Wind, and when another Wind bloweth from the Object, will be encreased by it.

15 That Bodies which cōtain the least quantity of air, as Stones and Metals, yeild less Smell then Plants and Living Creatures, the cause may be, that the motion which causeth Smelling, is a motion of the fluid parts onely; which parts, if they have any motion from the hard parts in which they are contained, they communicate the same to the open aire, by which it is propagated to the Organ. Where therefore there are no fluid parts, as in Metals; or where the fluid parts receive no motion from the hard parts, as in Stones, which are made hard by accretion, there can be no Smell. And therefore also the Water, whose parts have little or no motion, yeildeth no Smell. But if the same Water, by Seeds, and the heat of the Sunne, be together with particles of Earth raised into a Plant, and be afterwards pressed out again, it will be Odorous, as Wine from the Vine. And as Water passing through plants is by the motion of the parts of those plants made an Odorous liquour; so also of aire passing through the same plants whilest they are growing, are made Odorous aires. And thus also it is with the Juices and Spirits which are bred in Living Creatures.

16 That Odorous Bodies may be made more Odorous by Contrition, proceeds from this, that being broken into many parts which are all Odorous, the aire which by respiration is drawn from the Object towards the Organ, doth in its passage touch upon all those parts, and receives their motion. Now the aire toucheth the superficies onely; and a Body having less superficies whilest it is whole, then all its parts together have after it is reduced to powder, it follows that the same Odorous Body yeildeth less Smell whilest it is whole, then it will do after it is broken into smaller parts. And thus much of Smels.

17 The *Tast* follows; whose generation hath this difference from that of the Sight, Hearing and Smelling, that by these we have Sense of remote Objects; whereas we Tast nothing but what is contiguous, and doth immediately touch either the Tongue or

Palate,

Palate,or both. From whence it is evident, that the cuticles of the Tongue and Palate,and the Nerves inserted into them are the first Organ of Tast; and (because from the concussion of the parts of these, there followeth necessarily a concussion of the *Pia Mater*) that the action communicated to these,is propagated to the Brain, and from thence to the farthest Organ, namely,the Heart; in whose reaction consisteth the nature of Sense.

Now that Savours (as well as Odours) doe not onely move the Brain,but the Stomack also,as is manifest by the loathings that are caused by them both, they that consider the Organ of both these Senses will not wonder at all; seeing the Tongue, the Palate & the Nostrils have one and the same continued cuticle,derived from the *Dura Mater*.

And that *Effluviums* have nothing to doe in the Sense of Tasting, is manifest from this, that there is no Tast where the Organ and the Object are not contiguous.

By what variety of motions the different kinds of Tasts (which are innumerable) may be distinguished, I know not. I might (with others) derive them from the divers figures of those Atomes, of which whatsoever may be Tasted consisteth; or from the diverse motions which I might (by way of Supposition) attribute to those Atomes; conjecturing (not without some likelyhood of truth) that such things as tast Sweet, have their particles moved with slow circular motion, and their figures Spherical, which makes them smooth and pleasing to the Organ; that Bitter things have circular motion, but vehement, and their figures full of Angles, by which they trouble the Organ; and that Sowre things have straight and reciprocal motion, and their figures long and small, so that they cut and wound the Organ. And in like manner I might assigne for the causes of other Tasts such several motions and figures of Atomes as might in probability seem to be the true causes. But this would be to revolt from Philosophy to Divination.

18 By the *Touch*, we feel what Bodies are Cold or Hot, though they be distant from us. Others, as Hard, Soft, Rough and Smooth, we cannot feel, unless they be contiguous. The Organ of Touch, is every one of those membranes, which being continued from the *Pia Mater*, are so diffused throughout the whole Body, as that

that no part of it can be pressed, but the *Pia Mater* is pressed together with it. Whatsoever therefore presseth it, is felt as Hard or Soft, that is to say, as more or less Hard. And as for the Sense of Rough, it is nothing else but innumerable perceptions of Hard and Hard succeeding one another by short intervals both of time and place. For we take notice of Rough and Smooth, as also of Magnitude and Figure, not onely by the Touch, but also by Memory. For though some things are touched in one Point; yet Rough and Smooth, like Quantity and Figure, are not perceived but by the Flux of a Point, that is to say, we have no Sense of them without Time; and we can have no Sense of Time, without Memory.

CHAP. XXX.
Of Gravity.

1 A Thick Body doth not contain more Matter (unless also more Place) then a Thinne. 2 That the Descent of Heavy Bodies proceeds not from their own Appetite, but from some Power of the Earth. 3 The difference of Gravities proceedeth from the difference of the Impetus *with which the Elements whereof Heavy Bodies are made do fall upon the Earth. 4 The cause of the Descent of Heavy Bodies. 5 In what proportion the Descent of Heavy Bodies is accelerated. 6 Why those that Dive do not when they are under Water, feel the waight of the Water above them. 7 The Waight of a Body that floateth, is equal to the Waight of so much Water as would fill the space which the immersed part of the Body takes up within the Water. 8 If a Body be Lighter then Water, then how big soever that Body be, it will float upon any quantity of Water, how little soever. 9 How Water may be lifted up aud forced out of a Vessel by Air. 10 Why a Bladder is Heavier when blown full of aire, then when it is empty. 11 The cause of the ejection upwards of Heavy Bodies from a Wind-Gun. 12 The cause of the ascent of Water in a Weather-glass. 13 The cause of motion upwards in Living Creatures. 14 That there is in Nature a kind of Body Heavier then Aire, which neverthelesss is not by Sense distinguishable from it. 15 Of the cause of Magnetical vertue.*

1 IN the 21 Chapter I have defined Thick and Thinne (as that place required) so, as that by Thick was signified a more Resisting Body, and by Thinne a Body less Resisting; following the custome of those that have before me discoursed of Refraction. Now if we consider the true and vulgar signification of those words, we shall find them to be Names Collective, that is to say, Names of Multitude; as Thick to be that which takes up more parts of a space given, & Thinne that which contains fewer parts of the same magnitude, in the same space, or in a space equal to it. Thick therefore is the same with *Frequent*, as a Thick Troop; And Thinne the same with *Unfrequent*, as a Thinne Rank, Thinne of Houses; not that there is more matter in one place then in another equal

qual place, but a greater quantity of some named Body. For there is not less matter or Body indefinitely taken, in a Desert, then there is in a City; but fewer Houses, or fewer Men. Nor is there in a Thick Rank a greater quantity of Body, but a greater number of Souldiers, then in a Thinne. Wherefore the multitude & paucity of the parts contained within the same space, do constitute *Density* and *Rarity*, whether those parts be separated by *Vacuum*, or by Aire. But the consideration of this is not of any great moment in Philosophy; and therefore I let it alone, and pass on to the search of the causes of *Gravity*.

2 Now we call those Bodies *Heavy*, which (unless they be hindred by some force) are carried towards the center of the Earth, and that by their own accord, for ought we can by Sense perceive to the contrary. Some Philosophers therefore have been of opinion, that the Descent of Heavy Bodies proceeded from some internal Appetite, by which when they were cast upwards, they descended again, as moved by themselves, to such place as was agreeable to their nature. Others thought they were attracted by the Earth. To the former I cannot assent, because I think I have already clearly enough demonstrated, that there can be no beginning of motion, but from an external & moved Body; and consequently, that whatsoever hath motion or endeavour towards any place, will alwayes move or endeavour towards that same place, unless it be hindered by the reaction of some external Body. Heavy Bodies therefore being once cast upwards, cannot be cast down again but by external motion. Besides, seeing inanimate Bodies have no Appetite at all, it is ridiculous to think that by their own innate Appetite they should to preserve themselves (not understanding what preserves them) forsake the place they are in, and transferre themselves to another place; whereas Man (who hath both Appetite and understanding) cannot for the preservation of his own life, raise himselfe by leaping above three or four feet from the ground. Lastly, to attribute to created Bodies the power to move themselves, what is it else then to say that there be creatures which have no dependance upon the Creator? To the later, who attribute the Descent of Heavy Bodies to the attraction of the Earth; I assent. But by what motion this is done, hath not as yet been explained by any man. I shall therefore in this place say somewhat

of the manner, and of the way by which the Earth by its action attracteth Heavy Bodies.

3 That by the supposition of simple motion in the Sunne, homogeneous Bodies are congregated, and heterogeneous dissipated, has already been demonstrated in the 5th Article of the 21 Chapter. I have also supposed, that there are intermingled with the pure Air, certain little Bodies, or (as others call them) Atomes, which by reason of their extreme smalness are invisible, and differing from one another in *Consistence, Figure, Motion & Magnitude*; from whence it comes to pass, that some of them are congregated to the Earth, others to other Planets, and others are carried up and down in the spaces between. And seeing those which are carried to the Earth, differ from one another in Figure, Motion and Magnitude, they will fall upon the Earth, some with greater, others with less *Impetus*. And seeing also that we compute the several degrees of Gravity no otherwise then by this their falling upon the Earth with greater or less *Impetus*; it follows, that we conclude those to be the more Heavy that have the greater *Impetus*, and those to be less Heavy that have the less *Impetus*. Our enquiry therefore must be, by what means it may come to pass, that of Bodies which descend from above to the Earth, some are carried with greater, others with less *Impetus*; that is to say, some are more Heavy then others. We must also enquire, by what means such Bodies as settle upon the Earth, may by the Earth it self be forced to ascend.

4 Let the Circle made upon the center C (in the 2d figure) be a great Circle in the Superficies of the Earth, passing through the points A and B. Also let any Heavy Body, as the stone A D be placed any where in the plain of the Æquator; and let it be conceived to be cast up from A D perpendicularly, or to be carried in any other line to E, and supposed to rest there. Therefore how much space soever the stone took up in AD, so much space it takes up now in E. And because all place is supposed to be full, the space A D will be filled by the aire which flows into it first from the neerest places of the Earth, and afterwards successively from more remote places. Upon the center C let a Circle be understood to be drawn through E; and let the plain space which is between the Superficies of the Earth and that Circle, be divided into plain Orbs equal and concentrique; of which, let that be the first which is

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contained by the two perimeters that paſs through A & D. Whilſt therefore the aire which is in the firſt Orbe, filleth the place AD, the Orbe it ſelf is made ſo much leſs, and conſequently its latitude is leſs then the ſtraight line A D. Wherefore there will neceſſarily deſcend ſo much aire from the Orbe next above. In like manner, for the ſame cauſe, there will alſo be a deſcent of aire from the Orbe next above that; and ſo by Succeſſion from the Orbe in which the Stone is at reſt in E. Either therefore the Stone it ſelf, or ſo much aire will deſcend. And ſeeing aire is by the diurnal revolution of the Earth more eaſily thruſt away, then the Stone, the aire which is in the Orbe that contains the Stone will be forced further upwards then the Stone. But this, without the admiſſion of *Vacuum*, cannot be, unleſs ſo much aire deſcend to E from the place next above; which being done, the Stone will be thruſt downwards. By this means therefore the Stone now receives the beginning of its Deſcent, that is to ſay, of its Gravity. Furthermore, whatſoever is once moved, will be moved continually (as hath been ſhewn in the 19th Article of the 8th Chapter) in the ſame way, and with the ſame celerity, except it be retarded or accelerated by ſome external Movent. Now the aire (which is the onely Body that is interpoſed between the Earth A and the ſtone above it E) will have the ſame action in every point of the ſtraight line E A, which it hath in E. But it depreſſed the ſtone in E; and therefore alſo it will depreſs it equally in every point of the ſtraight line E A. Wherefore the ſtone will deſcend from E to A with accelerated motion. The poſſible cauſe therefore of the Deſcent of Heavy Bodies under the Æquator, is the Diurnal motion of the Earth. And the ſame demonſtration will ſerve, if the ſtone be placed in the plain of any other Circle parallel to the Æquator. But becauſe this motion hath by reaſon of its greater ſlowneſs, leſs force to thruſt off the aire in the parallel Circles then in the Æquator, and no force at all at the Poles, it may well be thought (for it is a certain conſequent) that Heavy Bodies deſcend with leſs and leſs velocity, as they are more & more remote from the Æquator; & that at the Poles themſelves they wil either not deſcend at all, or not deſcend by the Axis; which whether it be true or falſe, Experience muſt determine. But it is hard to make the experiment, both becauſe the times of their deſ-
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cents cannot be easily measured with sufficient exactness, and also because the places neer the Poles are inaccessible. Neverthelesse, this we know, that by how much the neerer we come to the Poles, by so much the greater are the Flakes of the Snow that falls; and by how much the more swiftly such Bodies descend as are fluid and dissipable, by so much the smaller are the particles into which they are dissipated.

5 Supposing therefore this to be the cause of the Descent of Heavy Bodies; it will follow, that their motion will be accelerated in such manner, as that the spaces which are transmitted by them in the several times, will have to one another the same proportion which the odd numbers have in succession from Unity. For if the straight line E A be divided into any number of equal parts, the Heavy Body descending, will (by reason of the perpetual action of the Diurnal motion) receive from the aire in every one of those times, in every several point of the streight line E A, a several new and equal impulsion; and therefore also in every one of those times it will acquire a several and equal degree of celerity. And from hence it follows, by that which *Galilæus* hath in his Dialogues of Motion demonstrated, that Heavy Bodies descend in the several times with such differences of transmitted spaces as are equal to the differences of the square numbers that succeed one another from Unity; which square numbers being 1, 4, 9, 16, &c. their differences are 3, 5, 7, &c; that is to say, the odd numbers which succeed another from Unity. Against this cause of Gravity which I have given, it will perhaps be objected, that if a Heavy Body be placed in the bottom of some hollow Cylinder of Iron or Adamant, and the bottom be turned upwards, the Body will descend, though the aire above cannot depress it, much less accelerate its motion. But it is to be considered, that there can be no Cylinder or Cavern, but such as is supported by the Earth, and being so supported, is together with the Earth carried about by its diurnal Motion. For by this means the bottom of the Cylinder will be as the Superficies of the Earth; and by thrusting off the next and lowest aire, will make the uppermost aire depress the Heavy Body which is at the top of the Cylinder, in such manner as is above explicated.

6 The Gravity of Water being so great as by experience wee find it is, the reason is demanded by many, why those that Dive,

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how deep ſoever they go under water, do not at all feel the weight of the water which lyes upon them. And the cauſe ſeems to be this, that all Bodies by how much the Heavier they are, by ſo much the greater is the endeavour by which they tend downwards. But the Body of a Man is Heavier then ſo much water as is equal to it in magnitude, and therefore the endeavour downwards of a Mans Body is greater then that of water. And ſeeing all endeavour is motion, the Body alſo of a Man will be carried towards the bottom with greater Velocity then ſo much water. Wherefore there is greater Reaction from the bottom; and the Endeavour upwards is equal to the endeavour downwards, whether the water be preſſed by water, or by another Body which is Heavier then water. And therefore by theſe two oppoſite equal endeavours, the endeavour both ways in the water is taken away; and conſequently, thoſe that Dive are not at all preſſed by it.

Coroll. From hence alſo it is manifeſt, that water in water hath no Waight at all, becauſe all the parts of water (both the parts above, and the parts that are directly under) tend towards the bottom with equal endeavour and in the ſame ſtraight lines.

7 If a Body float upon the water, the waight of that Body is equal to the waight of ſo much water as would fill the place which the immerſed part of the Body takes up within the water.

Let E F (in the 3d figure) be a Body floating in the water A B C D; and let the part E be above, and the other part F under the water. I ſay the waight of the whole Body E F is equal to the waight of ſo much water as the Space F will receive. For ſeeing the waight of the Body E F forceth the water out of the ſpace F, and placeth it upon the Superficies A B, where it preſſeth downwards; it follows, that from the reſiſtance of the bottom there will alſo be an endeavour upwards. And ſeeing again that by this endeavour of the water upwards, the Body E F is lifted up; it follows, that if the endeavour of the Body downwards be not equal to the endeavour of the water upwards, either the whole Body E F will (by reaſon of that inequality of their endeavours or moments) be raiſed out of the water, or elſe it will deſcend to the bottom. But it is ſuppoſed to ſtand ſo, as neither to aſcend nor deſcend. Wherefore there is an *Æquilibrium* between the two endeavours; that is to ſay, the waight of the Body E F is equal to the

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waight of ſo much water as the Space F will receive; Which was to be proved.

8 From hence it follows, that any Body of how great magnitude ſoever, provided it conſiſt of matter leſs Heavy then water, may neverthele ſs float upon any quantity of water how little ſoever.

Let A B C D (in the 4th figure) be a veſſel; and in it let E F G H be a Body conſiſting of matter which is leſs Heavy then water; and let the ſpace A G C F be filled with water. I ſay the Body EF G H will not ſink to the bottom D C. For ſeeing the matter of the Body E F G H is leſs Heavy then Water, if the whole ſpace without A B C D were full of Water, yet ſome part of the Body E F G H, as E F I K would be above the Water; and the waight of ſo much water as would fill the ſpace I G H K would be equal to the waight of the whole Body E F G H; and conſequently G H wonld not touch the bottom D C. As for the ſides of the veſſel, it is no matter whether they be hard, or fluid; for they ſerve onely to terminate the Water; which may be done as well by water as by any other matter how hard ſoever; and the water without the Veſſel is terminated ſomewhere, ſo as that it can ſpread no further. The part therefore E F I G will be extant above the water A G C F which is contained in the veſſel. Wherefore the Body E F G H will alſo float upon the water AG C F, how little ſoever that water be; which was to be demonſtrated.

9 In the 4th Article of the 26th Chapter, there is brought for the proving of *Vacuum*, the experiment of water encloſed in a veſſel; which water, the Orifice above being opened, is ejected upwards by the impulſion of the aire. It is therefore demanded (ſeeing water is Heavier then aire) how that can be done. Let the 2d. figure of the ſame 26th Chap. be conſidered, where the water is with great force injected by a Syringe into the ſpace F G B. In that injection, the aire (but pure aire) goeth with the ſame force out of the veſſel through the injected water. But as for thoſe ſmall Bodies which formerly I ſuppoſed to be intermingled with aire, & to be moved with ſimple motion, they can not together with the pure air penetrate the water; but remayning behind are neceſſarily thruſt together into a narrower place, namely into the ſpace which is above the water F G. The motions therefore of thoſe ſmall Bodies will be leſs and leſs free, by how much the quantity of the injected

jectedwater is greater and greater;so that by their motions falling upon one another the same small Bodies will mutually compress each other, and have a perpetual endeavour of regayning their liberty, and of depressing the water that hinders them. Wherefore, as soone as the orifice above is opened, the water which is next it will have an endeavour to ascend; and will therefore necessarily go out. But it cannot go out, unless at the same time there enter in as much aire; and therefore both the water will go out, and the aire enter in, till those small Bodies which were left within the vessel have recovered their former liberty of motion; that is to say, till the vess l be again filled with aire, and no water be left of sufficient height to stop the passage at B. Wherefore I have shewn a possible cause of this *Phænomenon*; namely, the same with that of Thunder. For as in the generation of Thunder, the small Bodies enclosed within the Clouds by being too closely pent together, do by their motion break the Clouds, and restore themselves to their natural liberty; so here also the small Bodies enclosed within the space which is above the straight line F G, do by their own motion expel the water as soon as the passage is opened above. And if the passage be kept stopped, and these small Bodies be more vehemently compressed by the perpetual forcing in of more water,they will at last break the vessel it self with great noise.

10 If Aire be blown into a hollow Cylinder, or into a Bladder, it will encrease the waight of either of them a little, as many have found by experience,who with great accurateness have tried the same. And it is no wonder, seeing(as I have supposed)there are intermingled with the common aire a great number of small hard Bodies which are Heavier then the pure aire. For the æthereal substance, being on all sides equally agitated by the motion of the Sunne, hath an equal endeavour towards all the parts of the Universe; and therefore it hath no Gravity at all.

11 We find also by experience,that by the force of air enclosed in a hollow Canon, a bullet of lead may with considerable violence be shot out of a Gunne of late invention, called the *Wind-Gun*.In the end of this Canon there are two holes with their Valves on the inside, to shut them close; one of them serving for the admission of aire, and the other for the letting of it out. Also to that

end which serves for the receiving in of aire, there is joyned another Canon of the same metal and bignesse, in which there is fitted a Rammer, which is perforated, and hath also a Valve opening towards the former Canon. By the help of this Valve the Rammer is easily drawn back, and letteth in aire from without; and being often drawn back and returned again with violent strokes, it forceth some part of that aire into the former Canon, so long, till at last the resistance of the enclosed aire is greater then the force of the stroke. And by this means men think there is now a greater quantity of aire in the Canon then there was formerly, though it were full before. Also the aire thus forced in, how much soever it be, is hindered from getting out again by the foresaid Valves, which the very endeavour of the aire to get out doth necessarily shut. Lastly, that Valve being opened which was made for the letting out of the aire, it presently breaketh out with violence, & driveth the bullet before it with great force and velocity.

As for the cause of this, I could easily attribute it (as most men do) to Condensation, and think that the aire, which had at the first but its ordinary degree of Rarity, was afterwards by the forcing in of more aire condensed, and last of all rarified again by being let out and restored to its natural liberty. But I cannot imagine how the same place can be alwayes full, and neverthelesse contain sometimes a greater, sometimes a less quantity of matter; that is to say, that it can be fuller then full. Nor can I conceive, how Fulness can of it self be an efficient cause of motion. For both these are impossible. Wherefore we must seek out some other possible cause of this *Phænomenon*. Whilst therefore the Valve wch serves for the letting in of aire is opened by the first stroke of the Rammer, the aire within doth with equal force resist the entering of the aire from without; so that the endeavours between the internal and external aire are opposite, that is, there are two opposite motions, whilest the one goeth in and the other cometh out; but no augmentation at all of aire within the Canon. For there is driven out by the stroke as much pure aire which passeth between the Rammer and the sides of the Canon, as there is forced in of aire impure by the same stroke. And thus by many forcible strokes the quantity of small hard Bodies will be encreased.

sed within the Canon, and their motions also will grow stronger and stronger as long as the matter of the Canon is able to endure their force; by which if it be not broken, it will at least be urged every way by their endeavour to free themselves; and as soon as the Valve which serves to let them out is opened, they will fly out with violent motion, and carry with them the bullet which is in their way. Wherefore I have given a possible cause of this *Phænomenon.*

12 Water, contrary to the custome of Heavy Bodies, ascendeth in the Weather-glasse; but it doth it when the aire is cold; for when it is warme it descendeth againe. And this Organ is called a *Thermometer*, or *Thermoscope*, because the degrees of Heat and Cold are measured and marked by it. It is made in this manner. Let A B C D (in the 5th figure) be a vessel full of water, and E F G a hollow Cylinder of glasse, closed at E, and open at G. Let it be heated, and set upright within the water to F; and let the open end reach to G. This being done; as the aire by little and little grows colder, the water will ascend slowly within the Cylinder from F towards E; till at last the externall and internall aire coming to be both of the same temper, it will neither ascend higher, nor descend lower, till the temper of the aire be changed. Suppose it therefore to be setled any where, as at H. If now the heat of the aire be augmented, the water will descend below H; and if the heat be diminished, it will ascend above it. Which though it be certainely known to be true by experience, the cause neverthelesse hath not as yet been discovered.

In the 6 and 7 articles of the 27th chapter (where I consider the cause of Cold) I have shewne, that fluid Bodies are made colder by the pressure of the aire, that is to say, by a constant Wind that presseth them. For the same cause it is, that the Superficies of the water is pressed at F; and having no place to which it may retire from this pressure besides the cavity of the Cylinder between H and E, it is therefore necessarily forced thither by the Cold, and consequently it ascendeth more or lesse, according as the Cold is more or lesse encreased. And againe, as the Heat is more intense, or the Cold more remisse, the same water will be depressed more or lesse by its own Gravity, that is to say, by the cause of Gravity above explicated.

13 Also Living creatures,though they be Heavy,can by Leaping, Swimming & Flying raise themselvs to a certain degree of height. But they cannot do this except they be supported by some resisting Body, as the Earth, the Water and the Aire. For these motions have their beginning from the contraction (by the helpe of the Muscles) of the Body animate. For to this contraction there succeedeth a distension of their whole Bodies; by which distension, the Earth, the Water or the Aire which supporteth them, is pressed; and from hence, by the reaction of those pressed Bodies, Living Creatures acquire an endeavour upwards, but such, as by reason of the Gravity of their Bodies is presently lost againe. By this endeavour therefore it is, that Living creatures rayse themselues up a little way by Leaping, but to no great purpose; but by Swimming & Flying they raise themselves to a greater height; because before the effect of their endeavour is quite extinguished by the Gravity of their bodies, they can renew the same endeavour againe.

That by the power of the Soule, without any antecedent contraction of the Muscles, or the helpe of something to support him, any man can be able to raise his Body upwards, is a childish conceipt. For if it were true, a man might raise himselfe to what height he pleased.

14 The diaphanous Medium which surrounds the Eie on all sides, is invisible; Nor is Aire to be seen in Aire, nor Water in Water, nor any thing but that which is more opacous. But in the confines of two diaphanous Bodies, one of them may be distinguished from the other. It is not therefore a thing so very ridiculous for ordinary people to think all that Space empty, in which we say is Aire; it being the worke of Reason to make us conceive that the Aire is any thing. For by which of our Senses is it, that we take notice of the Aire, seeing we neither See, nor Hear, nor Tast, nor Smell, nor Feel it to be any thing? When we feel Heat,we do not impute it to the Air,but to the Fire;nor do we say the aire is Cold,but we our selves are Cold; and when we feel the Wind, we rather think something is comming, then that any thing is already come. Also we do not at al feel the waight of water in water, much less of air in air. That we come to know that to be a Body which we call Aire, it is by Reasoning; but it is from one

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Reaſon onely, namely,becauſe it is impoſſible for remote Bodies to work upon our Organs of Senſe but by the help of Bodies intermediate,without which we could have no ſenſe of them, till they came to be contiguous. Wherefore,from the Senſes alone,without reaſoning from effects, we cannot have ſufficient evidence of the nature of Bodies.

For there is under-ground in ſome Mines of Coles, a certain matter of a middle nature between Water and Aire, which neverthleſs cannot by Senſe be diſtinguiſhed from aire; for it is as Diaphanous as the pureſt aire; and as farre as Senſe can judge, equally penetrable. But if we look upon the effect, it is like that of water. For when that matter breaks out of the Earth into one of thoſe Pits, it fils the ſame either totally,or to ſome degree; and if a Man, or Fire be then let down into it, it extinguiſhes them in almoſt as little time as water would do. But for the better underſtanding of this *Phænomenon*, I ſhall deſcribe the 6th figure. In which, let A B repreſent the pit of the Mine; and let part thereof, namely, C B. be ſuppoſed to be filled with that matter. If now a lighted Cădle be let down into it below C,it wil as ſuddenly be extinguiſhed,as if it were thruſt into water. Alſo if a grate filled with coles throughly kindled and burning never ſo brightly,be let down; as ſoon as ever it is below C, the fire will begin to grow pale, and ſhortly after (loſing its light) be extinguiſhed, no otherwiſe then if it were quenched in water. But if the grate be drawn up again preſently, whileſt the coles are ſtill very hot, the fire will by little and little be kindled again,and ſhine as before.There is indeed between this matter & water this conſiderable difference,that it neither wetteth, nor ſticketh to ſuch things as are put down into it, as water doth; which by the moiſture it leaveth, hindereth the kindling again of the matter once extinguiſhed. In like manner, if a Man be let down below C, he will preſently fall into a great difficulty of breathing, and immediatly after into a ſwoun, and die, unleſs he be ſuddenly drawn up again. They therefore that go down into theſe pits, have this cuſtome, that as ſoon as ever they feel themſelves ſick, they ſhake the rope,by which they were let down,to ſignifie they are not well, and to the end that they may ſpeedily be pulled up again. For if a man be drawn out too late,

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void of ſenſe and motion, they digg up a Turff, and put his face and mouth into the freſh earth; by which means (unleſs he be quite dead) he comes to himſelf again by little and little, and recovers life by the breathing out (as it were) of that ſuffocating matter which he had ſucked in whileſt he was in the pit; almoſt in the ſame manner as they that are drowned come to themſelves again by vomiting up the water. But this doth not happen in all Mines, but in ſome onely; and in thoſe not alwayes, but often. In ſuch Pits as are ſubject to it, they uſe this remedy. They dig another pit, as D E, cloſe by it, of equal depth; and joyning them both together with one common channel E F, they make a Fire in the bottom E, which carries out at D the aire contained in the pit D E; and this draws with it the aire contained in the channel E F; which in like manner is followed by the noxious matter contained in C B; & by this means the pit is for that time made healthful. Out of this Hiſtory (which I write onely to ſuch as have had experience of the truth of it, without any deſigne to ſupport my Philoſophy with Stories of doubtful credit) may be collected the following poſſible cauſe of this *Phænomenon*; namely, that there is a certain matter, fluid, & moſt tranſparent, and not much lighter then water; which breaking out of the Earth fills the Pit to C; and that in this matter, as in water, both Fire and Living creatures are extinguiſhed.

15 About the nature of Heavy Bodies, the greateſt difficulty ariſeth from the contemplation of thoſe things which make other Heavy Bodies aſcend to them; ſuch are *Jet*, *Amber*, and the *Loadſtone*. But that which troubles men moſt is the Loadſtone, which is alſo called *Lapis Herculeus*; a ſtone, though otherwiſe deſpicable, yet of ſo great power, that it taketh up Iron from the Earth, and holds it ſuſpended in the aire, as *Hercules* did *Antæus*. Neverthéleſs, we wonder at it ſomewhat the leſs, becauſe we ſee Jet draw up Straws, which are Heavy Bodies, though not ſo Heavy as Iron. But as for Jet, it muſt firſt be excited by rubbing, that is to ſay, by motion to and fro; whereas the Loadſtone hath ſufficient excitation from its own nature, that is to ſay, from ſome internal principle of motion peculiar to it ſelf. Now whatſoever is moved, is moved by ſome contiguous and moved Body, as hath been formerly demonſtrated. And from hence it follows evidently, that the firſt endeavour

which

which Iron hath towards the Loadſtone, is cauſed by the motion of that aire which is contiguous to the Iron. Alſo that this motion is generated by the motion of the next aire, and ſo on ſucceſſively, till by this ſucceſſion we find that the motion of all the intermediate air taketh its beginning from ſome motion which is in the Loadſtone it ſelf; which motion (becauſe the Loadſtone ſeems to be at reſt) is inviſible. It is therefore certain, that the attractive power of the Loadſtone is nothing elſe but ſome motiō of the ſmalleſt particles thereof. Suppoſing therefore that thoſe ſmall Bodies of which the Loadſtone is (in the bowels of the Earth) compoſed, have by nature ſuch motion or endeavour as was above attributed to Jet, namely a reciprocal motiō in a line too ſhort to be ſeen, both thoſe ſtones wil have one & the ſame cauſe of attraction. Now in what manner, and in what order of working this cauſe produceth the effect of attraction, is the thing to be enquired. And firſt we know, that when the ſtring of a Lute or Viol is ſtricken, the Vibration, that is, the reciprocal motion of that ſtring in the ſame ſtraight Line, cauſeth like Vibration in another ſtring which has like tenſion. We know alſo, that the dregs or ſmall ſands which ſink to the bottom of a Veſſel, will be raiſed up from the bottom by any ſtrong and reciprocal agitation of the water ſtirred with the hand or with a ſtaff. Why therefore ſhould not reciprocal motion of the parts of the Loadſtone contribute as much towards the moving of Iron? For if in the Loadſtone there be ſuppoſed ſuch reciprocal motion, or motion of the parts forwards and backwards, it will follow, that the like motion will be propagated by the aire to the Iron, and conſequently that there will be in all the parts of the Iron the ſame reciprocations or motions forwards and backwards. And from hence alſo it will follow, that the intermediate aire between the Stone and the Iron, will by little and little be thruſt away; and the aire being thruſt away, the Bodies of the Loadſtone and the Iron will neceſſarily come together. The poſſible cauſe therefore why the Loadſtone and Jet draw to them, the one Iron, the other Strawes, may be this, that thoſe attracting Bodies have reciprocal motion either in a ſtraight line, or in an Elliptical line, when there is nothing in the nature of the attracted Bodies which is repugnant to ſuch a motion.

But:

But why the Loadſtone (if with the help of Cork it float at liberty upon the top of the water) ſhould from any poſition whatſoever ſo place it ſelf in the plain of the Meridian, as that the ſame points which at one time of its being at reſt reſpect the Poles of the Earth, ſhould at all other times reſpect the ſame Poles, the cauſe may be this, That the reciprocal motion which I ſuppoſed to be in the parts of the Stone, is made in a line parallel to the Axis of the Earth, and has been in thoſe parts ever ſince the Stone was generated. Seeing therefore the Stone whileſt it remains in the Mine, and is carried about together with the Earth by its diurnal motion, doth by length of time get a habit of being moved in a line which is perpendicular to the line of its reciprocal motion, it will afterwards, though its axis be removed from the parallel ſituation it had with the axis of the Earth, retain its endeavour of returning to that ſituation again; and all endeavour being the beginning of motion, and nothing intervening that may hinder the ſame, the Loadſtone will therefore return to its former ſituation. For any piece of Iron that has for a long time reſted in the plain of the Meridian, whenſoever it is forced from that ſituation, and afterwards left to its own liberty again, will of it ſelf return to lie in the Meridian again; which return is cauſed by the endeavour it acquired from the diurnal motion of the Earth in the parallel circles which are perpendicular to the Meridians.

If Iron be rubbed by the Loadſtone drawn from one Pole to the other, two things will happen; one, that the Iron will acquire the ſame direction with the Loadſtone, that is to ſay, that it will lie in the Meridian, and have its Axis and Poles in the ſame poſition with thoſe of the Stone; the other, that the like Poles of the Stone and of the Iron will avoid one another, and the unlike Poles approach one another. And the cauſe of the former may be this, that Iron being touched by motion which is not reciprocal, but drawn the ſame way from Pole to Pole, there will be imprinted in the Iron alſo an endeavour from the ſame Pole to the ſame Pole. For ſeeing the Loadſtone differs from Iron no otherwiſe then as Ore from Metal, there will be no repugnance at all in the Iron to receive the ſame motion which is in the Stone. From whence it follows, that ſeeing they are both affected alike by the diurnal motion

on of the Earth, they will both equally return to their situation in the Meridian whensoever they are put frõ the same. Also of the later this may be the cause, that as the Loadstone in touching the Irõ doth by its action imprint in the Iron an endeavour towards one of the Poles, suppose towards the North Pole; so reciprocally, the Iron by its action upon the Loadstone doth imprint in it an endeavour towards the other Pole, namely towards the South Pole. It happens therefore in these reciprocations or motions forwards and backwards of the particles of the Stone and of the Iron betwixt the North & the South, that whilest in one of them the motion is from North to South, and the return from South to North, in the other the motion wil be from South to North, & the return frõ North to South; which motions being opposite to one another, and communicated to the Air, the North Pole of the Iron (whilest the attraction is working) will be depressed towards the South Pole of the Loadstone; or contrarily the North Pole of the Loadstone will be depressed towards the South Pole of the Iron, and the Axes both of the Loadstone and of the Iron will be situate in the same straight line. The truth whereof is taught us by experience.

As for the propagation of this Magnetical vertue, not onely through the Aire, but through any other Bodies how hard so ever, it is not to be wondred at, seeing no motion can be so weak, but that it may be propagated infinitely through a space filled with Body of any hardness whatsoever. For in a full Medium, there can be no motion which doth not make the next part yeild, and that the next, and so successively without end; so that there is no effect whatsoever but to the production thereof something is necessarily contributed by the several motions of all the several things that are in the World.

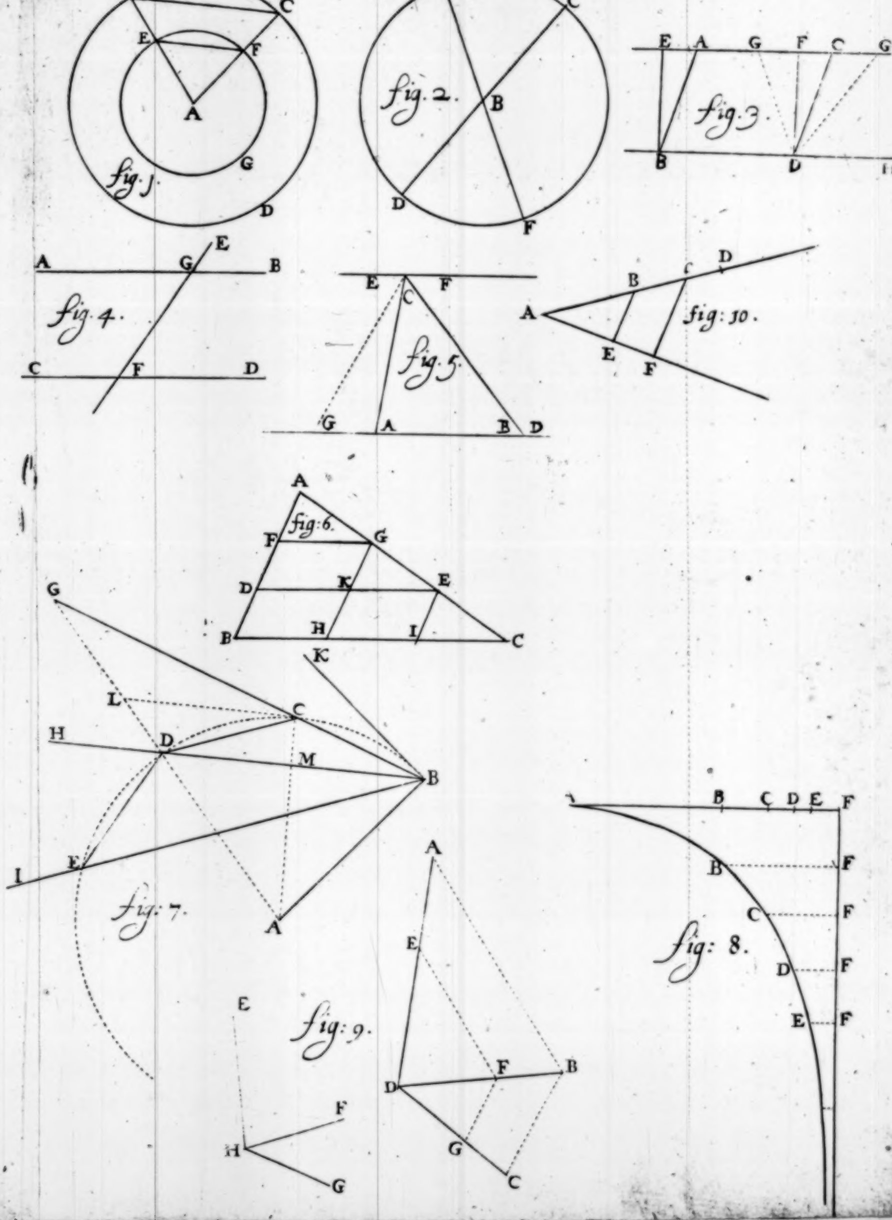
And thus much concerning the nature of Body in general; with which I conclude this my first Section of the Elements of Philosophy. In the first, second and third Parts, where the Principles of Ratiocination consist in our own Understanding, that is to say, in the legitimate use of such Words as we our selves constitute, all the Theoremes (if I be not deceived) are rightly demonstrated. The fourth Part depends upon *Hypotheses*; which unless we know them to be true, it is impossible for us to demonstrate that those

Causes which I have there explicated, are the true Causes of the things whose productions I have derived from them.

Neverthelesse, seeing I have assumed no *Hypothesis*, which is not both possible and easie to be comprehended; and seeing also that I have reasoned aright from those Assumptions, I have withall sufficiently demonstrated that they may be the true Causes; wch is the end of Physical Contemplation. If any other man from other *Hypotheses* shall demonstrate the same, or greater things, there wil be greater praise and thanks due to him then I demand for my self, provided his *Hypotheses* be such as are conceivable. For as for those that say any thing may be moved or produced by *it Self*, by *Species*, by *its own Power*, by *Substantial Forms*, by *Incorporeal Substances*, by *Instinct*, by *Anteperistasis*, by *Antipathy*, *Sympathy*, *Occult Quality*, and other empty words of Schoolmen, their saying so is to no purpose.

And now I proceed to the *Phænomena* of Mans Body; Where I shall speak of the *Opticks*, and of the *Dispositions*, *Affections*, *and Manners* of Men (if it shall please God to give me life), and shew their Causes.

AD CAP. XIV



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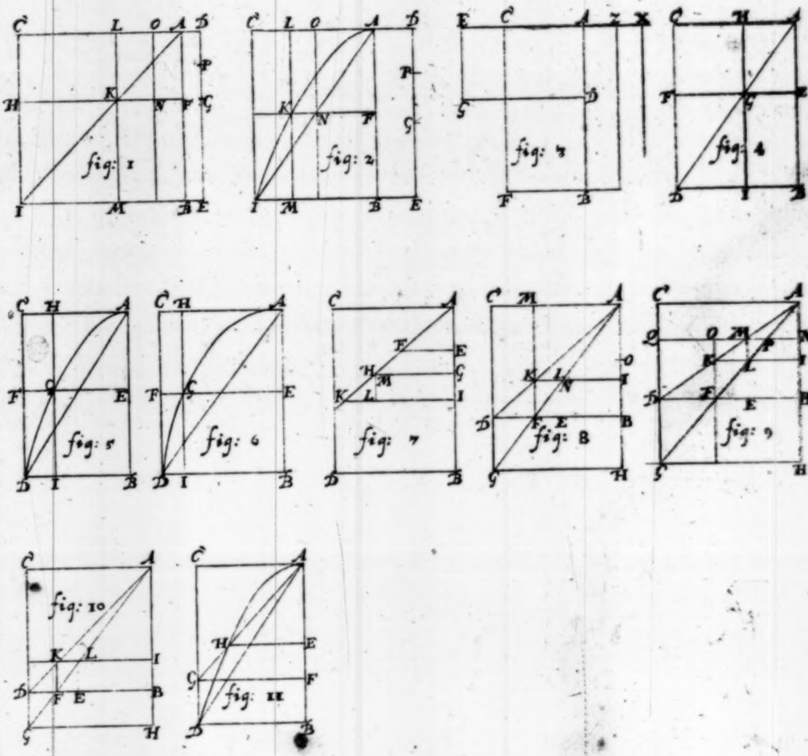

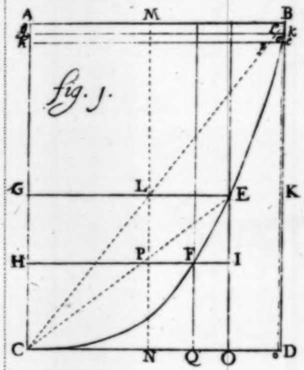

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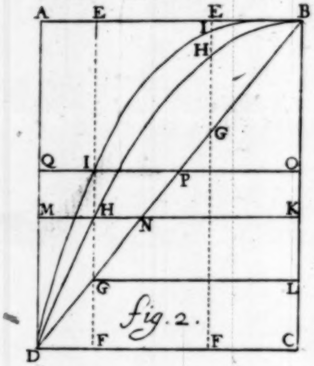

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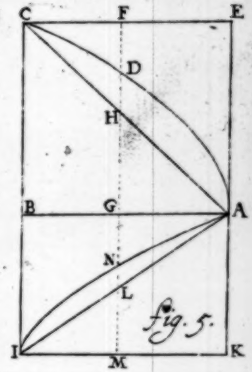

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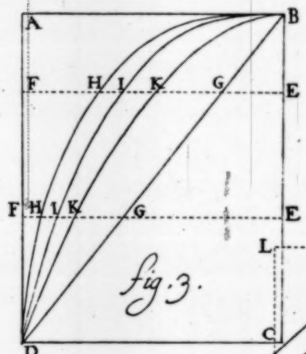

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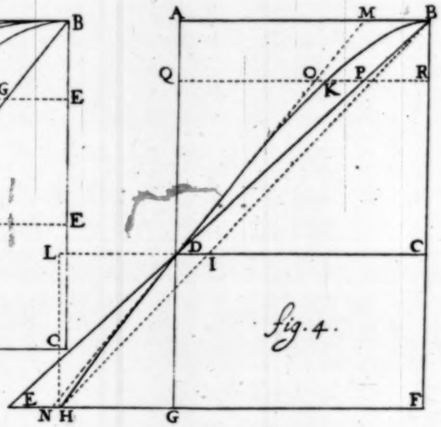

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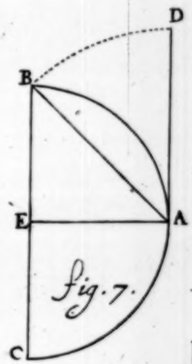

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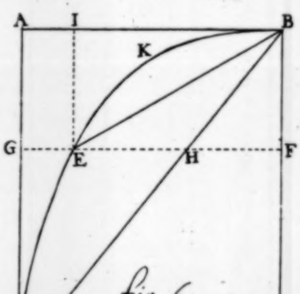

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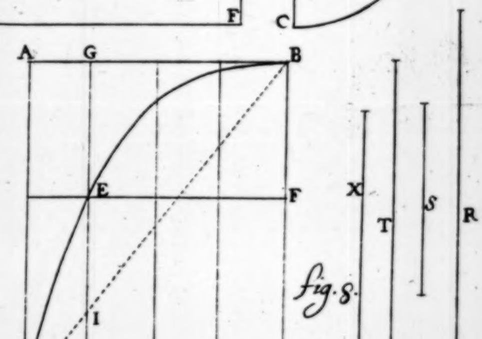

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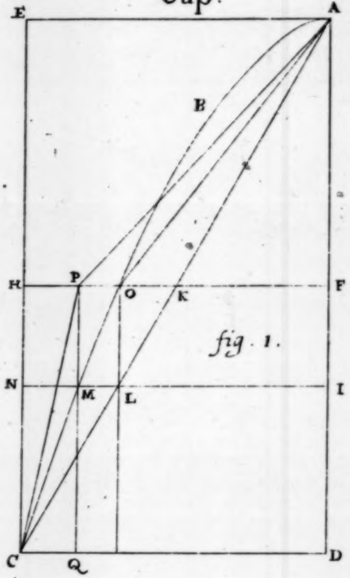

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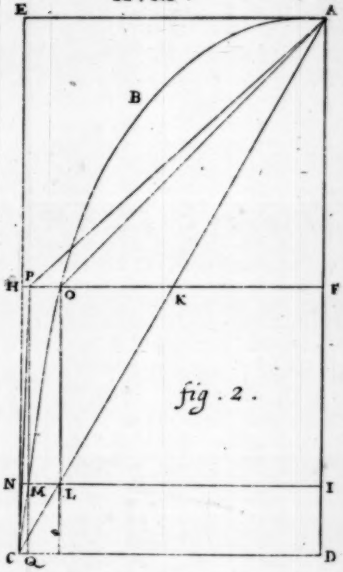

fig. 2.

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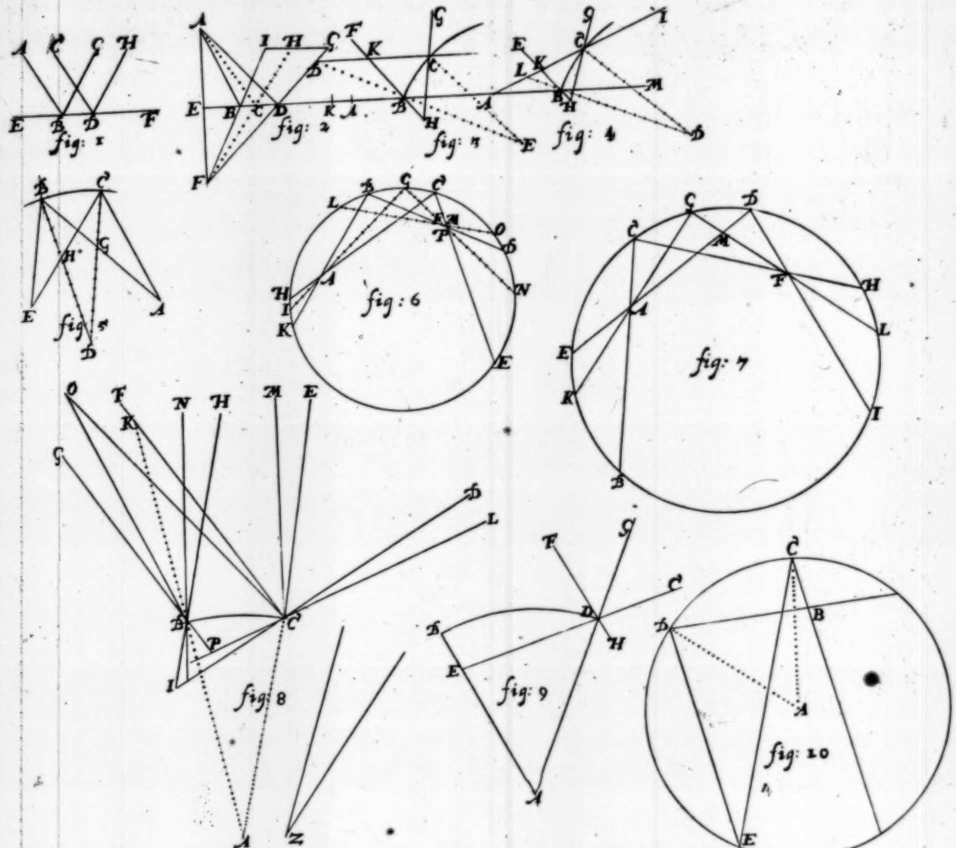

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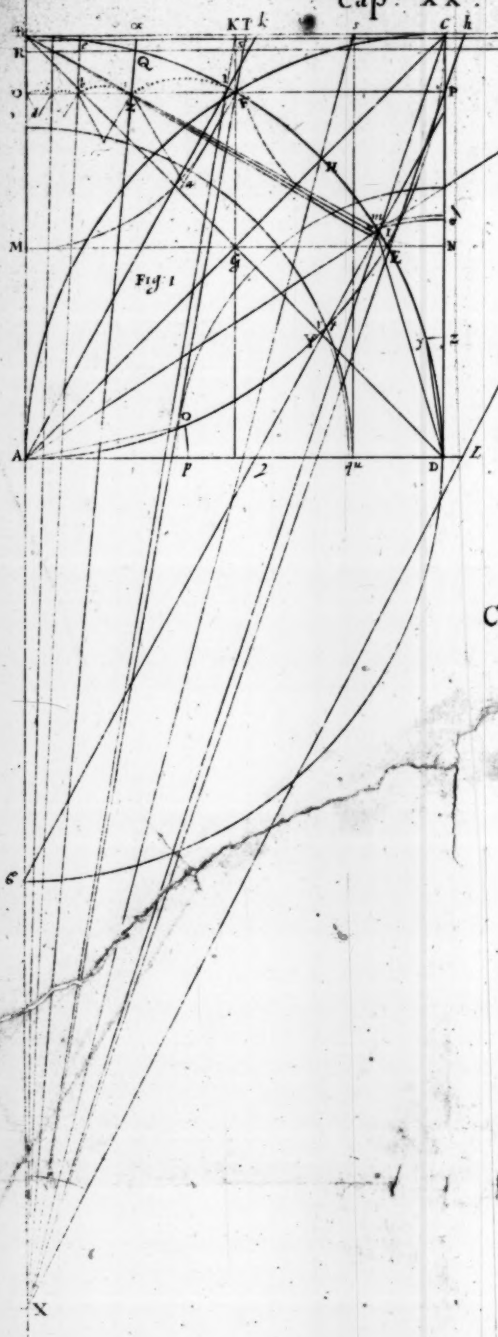
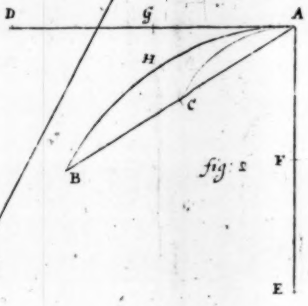
Cap: XX





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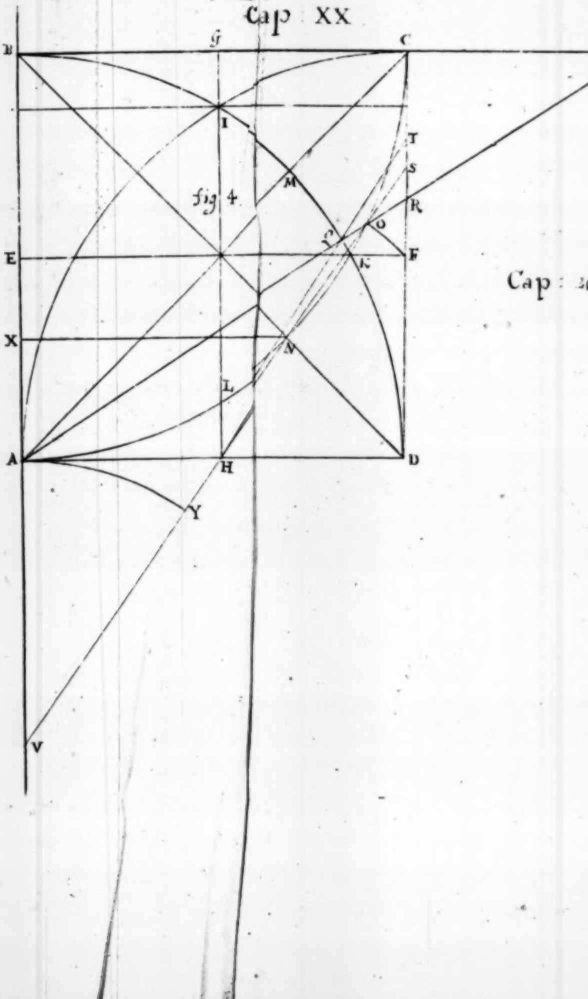

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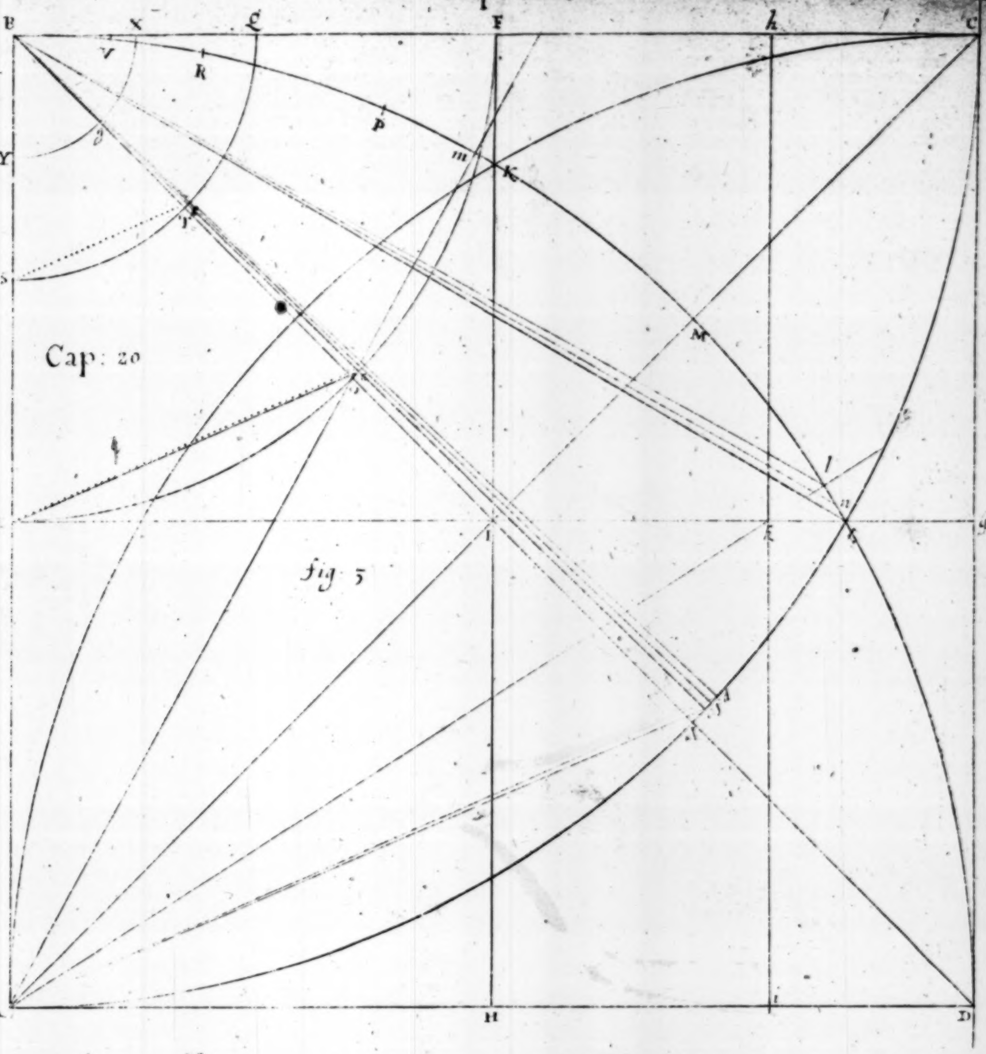

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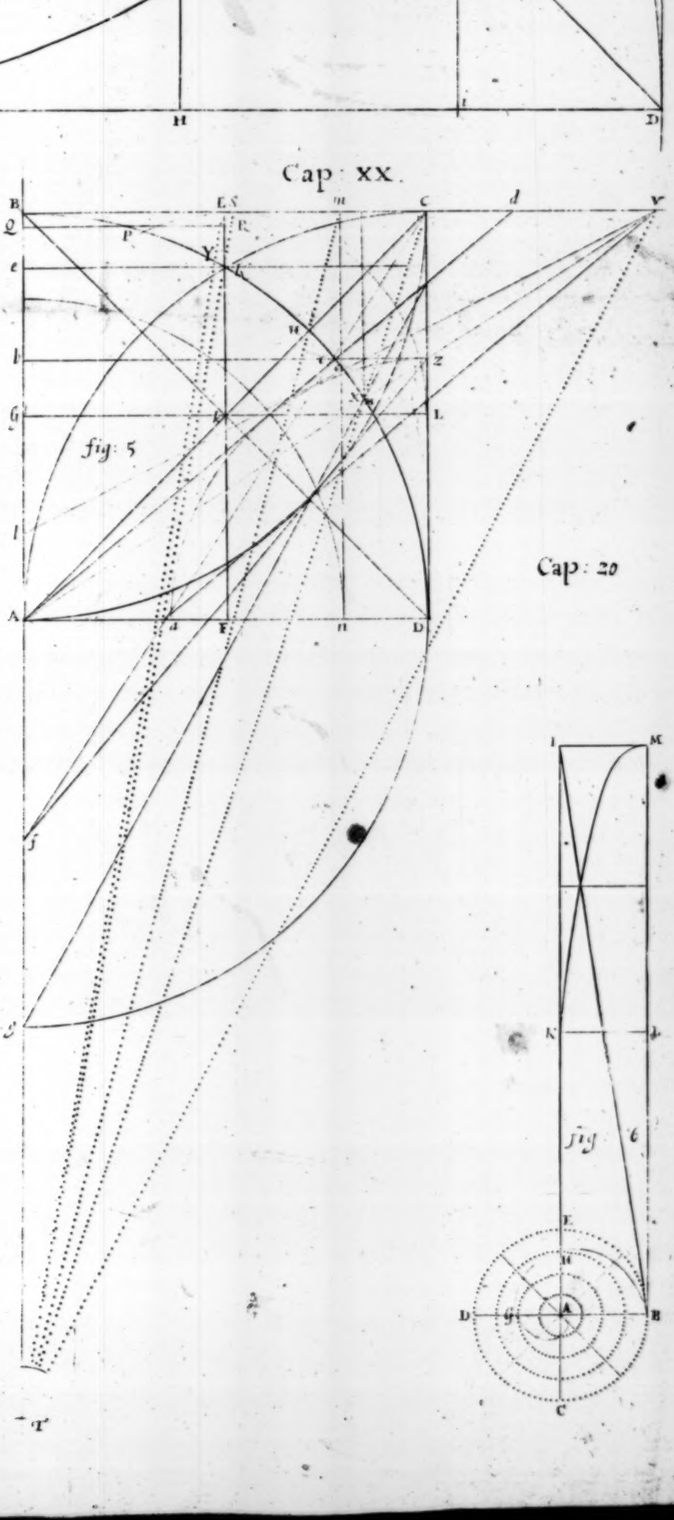

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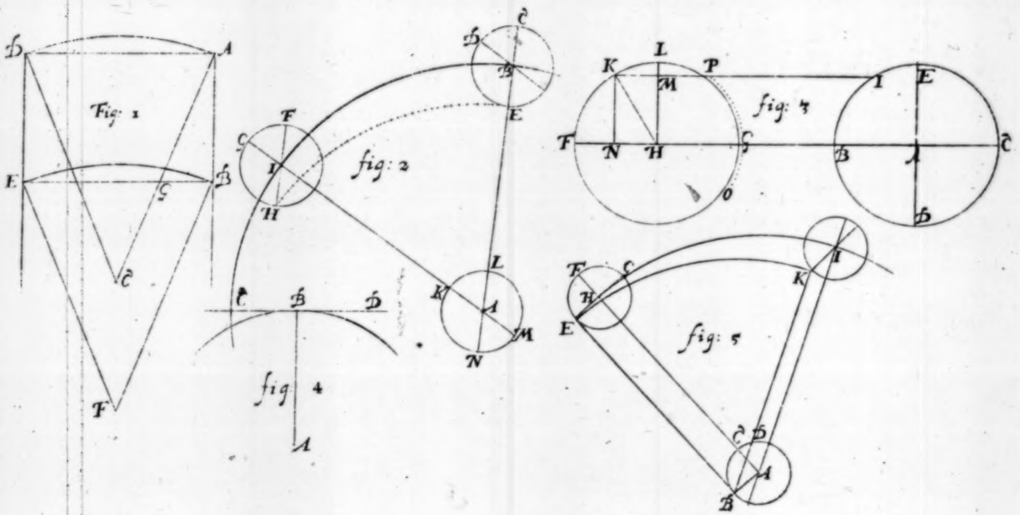

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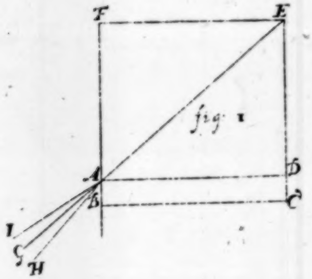

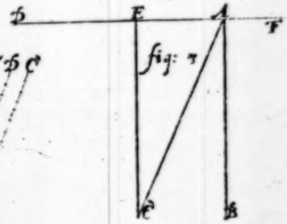

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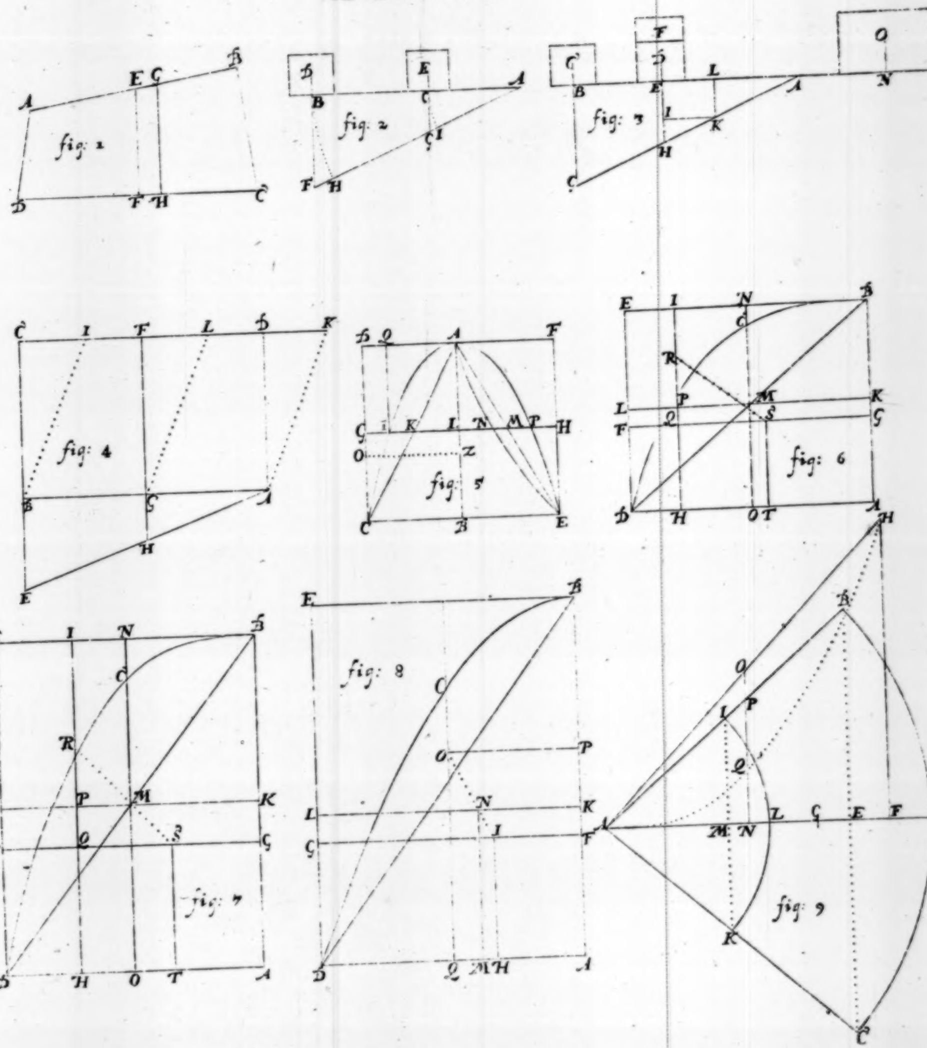

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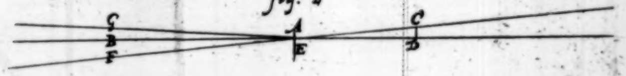

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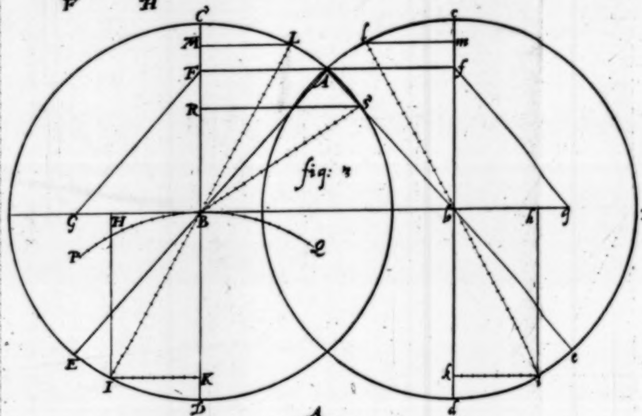

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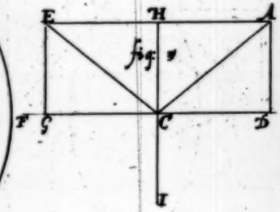

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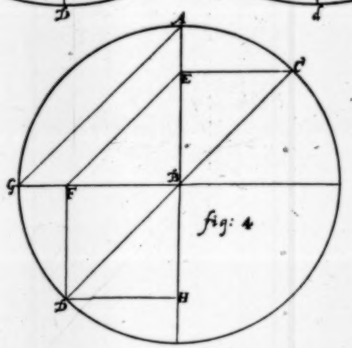

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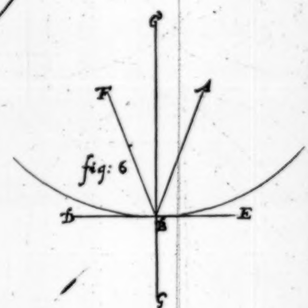

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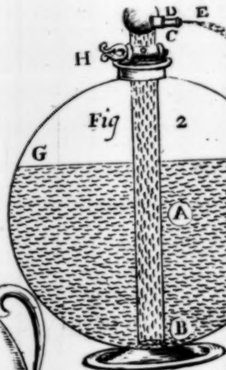

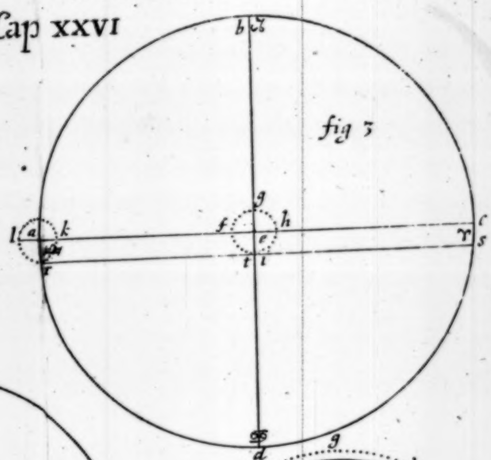

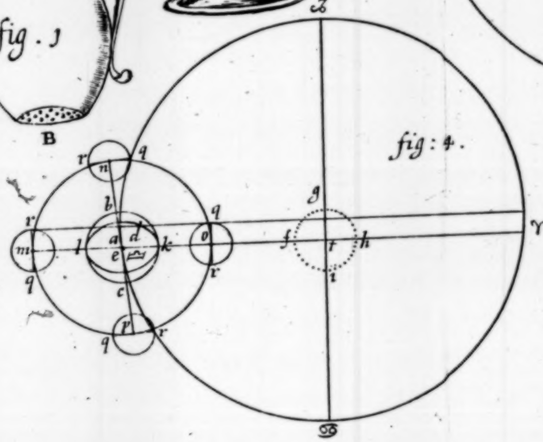

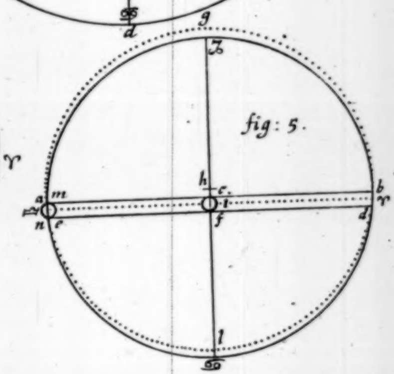

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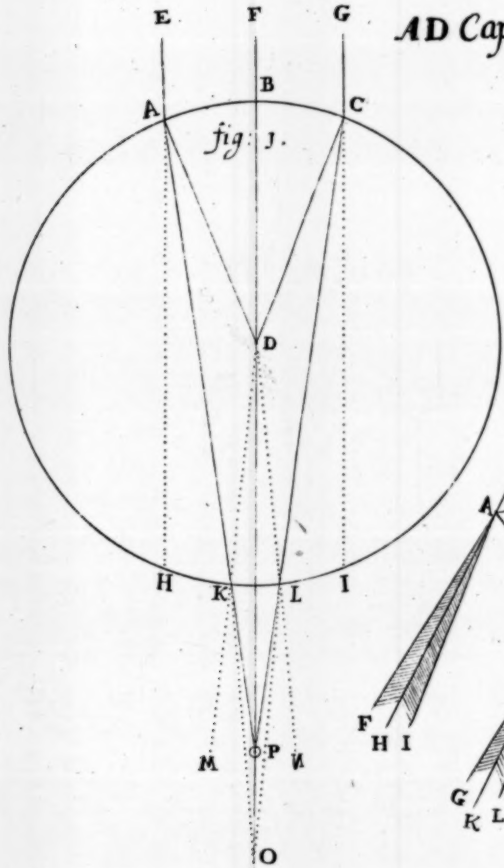

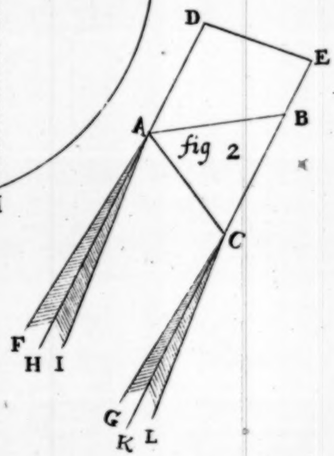

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